Softly broken lepton numbers:
an approach to maximal neutrino mixing

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ABSTRACT: In the extension of the standard model with three right-handed neutrinos and the seesaw mechanism, we assume that the generation lepton numbers are separately conserved in the Yukawa couplings, and that they are softly broken by the Majorana mass matrix of the neutrino singlets. Deviations from family-lepton-number conservation are then calculable. In this framework, it is possible to introduce symmetries such that either maximal atmospheric-neutrino mixing, or maximal solar-neutrino mixing, or both simultaneously, are achieved naturally.

I am going to report on work which has been done in collaboration with Walter Grimus, of the University of Vienna. Our paper [1] contains three ideas. The first one is that the three individual family lepton numbers L_e, L_μ, and L_τ may constitute symmetries of the lagrangian which are broken only *softly* by the Majorana masses of right-handed neutrinos. From this idea it follows that the soft breaking of the lepton numbers occurs through *super-high* (of order 10^{11} GeV or so) mass terms, instead of proceeding through small masses, as is usually argued in connection with the soft breaking of symmetries. We claim that this idea makes sense because, due to the soft breaking of the family lepton numbers, the renormalization-group equations do not induce family-lepton-number-violating Yukawa couplings; all lepton-number-breaking effects turn out to be calculable (finite) and, indeed, suppressed. The second idea consists of a model which, within the framework described above, leads to maximal atmospheric-neutrino mixing. The third idea (which is separate and distinct from the second one) is a model for maximal solar-neutrino mixing. At this point, I want to stress that when we are talking of models we thereby understand theories with well-defined particle contents, symmetries, and mechanisms of symmetry breaking. Our models are not *Ansätze*, "textures", or "patterns", rather they are self-contained theories, consistent within a quantum-field-theoretical framework.

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In order to set the stage, I start with the weak leptonic current:

$$j^{\alpha} = \left(\bar{e}, \ \bar{\mu}, \ \bar{\tau} \right) \gamma^{\alpha} \gamma_L \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{1}$$

where the matrix U is the lepton mixing matrix. We know that the mass-squared differences among the three (light) neutrinos have distinct orders of magnitude:

$$\Delta m_{\text{atm}}^2 = \left| m_3^2 - m_2^2 \right| \gg \left| m_2^2 - m_1^2 \right| = \Delta m_{\odot}^2 \,. \tag{2}$$

We also know, from the CHOOZ and Super-Kamiokande experiments, that $|U_{e3}| \ll 1$. Finally, as we have already learned in this parallel session, experiment indicates that atmospheric-neutrino mixing is maximal, $|U_{\mu3}| \approx |U_{\tau3}| \approx 1/\sqrt{2}$, and that solar-neutrino mixing should also be (close to) maximal, $|U_{e1}| \approx |U_{e2}| \approx 1/\sqrt{2}$.

Now, $|U_{e3}| \ll 1$ and maximal *solar* mixing are easy to explain or predict through a model. Indeed, it suffices, for instance, to assume an approximate $L_e - L_\mu - L_\tau$ symmetry; this has been noticed before by several authors [2, 3]. On the other hand, maximal *atmospheric* mixing is *not* easy to explain or predict by a model (as distinct from a "texture" or an *Ansatz*). This is because the mass matrix of the charged leptons is in general non-diagonal, and its diagonalization destroys the maximal mixing that may follow from the mass matrix of the neutrinos.

The proposal of Ref. [1], then, is to assume that L_e, L_μ, and L_τ are separately conserved and that they are broken only softly. It follows from this assumption that the mass matrix of the charged leptons e, μ, and τ is automatically diagonal, and this opens the way for explaining maximal atmospheric mixing, like I suggested above.

The idea of soft breaking of the individual lepton numbers is implemented within the context of the seesaw mechanism [4]. Thus, we assume that there are three right-handed neutrinos, each of them carrying one unit of either L_e, L_μ, or L_τ. We moreover assume—and this is crucial—that the scalar sector of our theory only contains doublets of the weak isospin, i.e. we allow for neither triplets nor singlets. The mass terms for the six neutrinos are then of the form

$$\frac{1}{2} \left(\nu_L^T C^{-1}, \ -\bar{\nu}_R \right) \begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ C\bar{\nu}_R^T \end{pmatrix} + \text{h.c.} \tag{3}$$

The zero upper-left submatrix follows from the absence of scalar triplets. The Dirac mass matrix M_D is *diagonal*, while the right-handed-neutrinos Majorana mass matrix M_R is *non-diagonal*. The seesaw mechanism assumes that the mass terms in M_R are super-large, and we therefore have *soft breaking of the lepton numbers through super-high mass terms*! (Notice that the mass terms contained in M_R are soft because there are no scalar singlets in our theory and M_R therefore does not arise out of any vacuum expectation value.) Now we claim that, in this framework, all lepton-flavour-changing processes are suppressed by inverse powers of the large mass scale and/or by small Yukawa couplings. We are now performing detailed calculations in order to elucidate these suppression mechanisms [5].

In the context of this framework we [1] propose our "model one", which predicts maximal *atmospheric* mixing and $U_{e3} = 0$. The model contains three scalar doublets—each of them having *lepton-flavour-diagonal* Yukawa couplings—and two Z_2 symmetries:

$$Z_2^{(1)}: \ \nu_{\mu R} \leftrightarrow \nu_{\tau R}, \ D_\mu \leftrightarrow D_\tau, \ \mu_R \leftrightarrow \tau_R, \ \phi_3 \rightarrow -\phi_3; \tag{4}$$

$$Z_2^{(2)}: \ \mu_R \rightarrow -\mu_R, \ \tau_R \rightarrow -\tau_R, \ \phi_2 \rightarrow -\phi_2, \ \phi_3 \rightarrow -\phi_3, \tag{5}$$

where D_μ and D_τ are lepton doublets, while μ_R and τ_R are charged-lepton singlets. From these two symmetries—note in particular the interchange of μ and τ indices through $Z_2^{(1)}$—follow neutrino mass matrices of the form

$$M_D = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{pmatrix}, \quad M_R = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}. \tag{6}$$

It is easy to see that, then, the effective mass matrix for the light neutrinos, $-M_D M_R^{-1} M_D$, is of exactly the same form as M_R. That form of the effective mass matrix leads to $U_{e3} = 0$ and to maximal atmospheric mixing—to $|U_{\mu 3}| = |U_{\tau 3}| = 1/\sqrt{2}$. (This fact was recognized by a number of people before us [6]; however, the main point here is to present a consistent model which justifies that form of the neutrino mass matrix.) Most important, the charged-lepton mass matrix remains diagonal, and we are able to break the $\mu \leftrightarrow \tau$ symmetry and obtain $m_\mu \neq m_\tau$. The relevant Yukawa couplings are

$$\left(\bar{D}_\mu \mu_R + \bar{D}_\tau \tau_R \right) \phi_2 \quad \text{and} \quad \left(\bar{D}_\mu \mu_R - \bar{D}_\tau \tau_R \right) \phi_3. \tag{7}$$

The simultaneous vacuum expectation values of ϕ_2 and ϕ_3 allow us to obtain $m_\mu \neq m_\tau$.

In the context of the same framework, we [1] may construct "model two", *independent* of "model one", which leads to (almost-)maximal *solar* mixing. For this purpose it is enough to assume that there is an approximate $L_e - L_\mu - L_\tau$ symmetry in M_R. That symmetry implies

$$(M_R)_{e\mu}, (M_R)_{e\tau} \gg (M_R)_{ee}, (M_R)_{\mu\mu}, (M_R)_{\tau\tau}, (M_R)_{\mu\tau}, \tag{8}$$

because $(M_R)_{e\mu}$ and $(M_R)_{e\tau}$ do not break $L_e - L_\mu - L_\tau$. If we denote by ϵ the order of magnitude of the ratio of the masses on the right-hand-side of eq. (8) over the ones on the left-hand-side, then ϵ is small and we easily obtain

$$|U_{e3}| \sim \epsilon, \tag{9}$$

$$1 - \sin^2(2\theta_\odot) \sim \epsilon^2, \tag{10}$$

$$\Delta m_\odot^2 / \Delta m_{\text{atm}}^2 \sim \epsilon^3. \tag{11}$$

With a suitable ϵ one may then, for instance, fit for the large-mixing-angle MSW solution of the solar-neutrino puzzle, which seems at present to be the experimentally preferred one.

I want to stress that *model two is an alternative to model one*: the two models may—or may not—be used together. If we use them together—by assuming model one with its three

scalar doublets and the two symmetries in eqs. (4) and (5), and simultaneously assuming that there is an approximate $L_e - L_\mu - L_\tau$ symmetry in M_R—then we shall naturally obtain bimaximal mixing. However, this is not a necessary option.

In conclusion, I have suggested that, in the context of the seesaw mechanism for suppression of the neutrino masses, the three lepton numbers L_e, L_μ, and L_τ may be separately conserved in the Yukawa couplings while they are softly broken, at a high scale, in the Majorana mass matrix of the singlet neutrinos. I have shown that this yields a framework within which self-contained, consistent theories for maximal atmospheric-neutrino mixing and/or for maximal solar-neutrino mixing may be constructed.

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