Damping of inhomogeneities in neutralino dark matter

Dominik J. Schwarz[*]

Institut für Theoretische Physik, Technische Universität Wien,
Wiedner Hauptstraße 8–10, A-1040 Wien, Austria
E-mail: `dschwarz@hep.itp.tuwien.ac.at`

Stefan Hofmann and Horst Stöcker

Institut für Theoretische Physik, Universität Frankfurt,
Postfach 11 19 32, D-60054 Frankfurt am Main, Germany
E-mail: `stehof@th.physik.uni-frankfurt.de`

ABSTRACT: The lightest supersymmetric particle, most likely the neutralino, might account for a large fraction of dark matter in the Universe. We show that the primordial spectrum of density fluctuations in neutralino cold dark matter (CDM) has a sharp cut-off due to two damping mechanisms: collisional damping during the kinetic decoupling of the neutralinos at $\mathcal{O}(10\ \mathrm{MeV})$ and free streaming after last scattering of neutralinos. The cut-off in the primordial spectrum defines a minimal mass for CDM objects in hierarchical structure formation. For typical neutralino and sfermion masses the first gravitationally bound neutralino clouds have masses above $10^{-6} M_\odot$.

1. Introduction

The evidence that there is a significant amount of cold dark matter (CDM) in the Universe has been confirmed by several measurements recently. An analysis of the temperature anisotropies of the cosmic microwave background (CMB) gives for the mass density of CDM $\Omega_{\mathrm{cdm}} h^2 = 0.12 \pm 0.05$ (weak priors) [1], while from the same analysis the mass density of baryons is much smaller, $\Omega_{\mathrm{b}} h^2 = 0.021^{+0.004}_{-0.003}$. The latter is consistent with the prediction of primordial nucleosynthesis [2]. Large galaxy redshift surveys agree with the CMB measurements. From the analysis of over 160,000 galaxies the authors of Ref. [3] find $(\Omega_{\mathrm{cdm}} + \Omega_{\mathrm{b}})h = 0.20 \pm 0.03$ and $\Omega_{\mathrm{b}}/(\Omega_{\mathrm{cdm}} + \Omega_{\mathrm{b}}) = 0.15 \pm 0.07$. These recent findings agree well with the picture of hierarchical structure formation that emerged over the years from the study of much smaller structures as galaxies, groups of galaxies and clusters. An up to date summary of the evidence for CDM is presented in [4].

---

[*]Speaker.

The characteristic feature of CDM is its non-relativistic equation of state, i.e. $P \ll \rho$, in order to ensure that large scale structure has enough time to form. The behaviour of all CDM candidates is the same at large scales (> 1 Mpc), whereas at the galactic scale and below various CDM candidates might be distinguishable. To learn more about the nature of CDM we study the small-scale structure of CDM (see also [5]).

One of the most popular CDM candidates is the neutralino $\tilde{\chi}_1^0$, which probably is the lightest supersymmetric particle. In the context of the constrained minimal supersymmetric standard model it is almost a pure bino with mass $M_{\tilde{\chi}} = \mathcal{O}(100 \text{ GeV})$ [6].

In this contribution we discuss the two damping mechanisms, collisional damping during the kinetic decoupling of neutralinos from the radiation fluid and free streaming thereafter, that are relevant for the formation of the very first structures of neutralino CDM. For more details see [7].
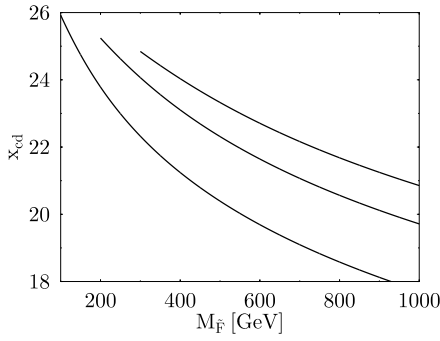
2. Chemical and kinetic decoupling

Dark matter particles that are massive and subject to weak interactions decouple chemically long before they decouple kinetically from the radiation fluid (photons and leptons) [8, 7]. This can be understood by comparing the neutralino annihilation rate $\Gamma_{\text{ann}}(\tilde{\chi} + \tilde{\chi} \to l + \bar{l}) \equiv \langle v\sigma_{\text{ann}} \rangle n_{\tilde{\chi}}$ to the rate of elastic scatterings $\Gamma_{\text{el}}(\tilde{\chi} + l \to \tilde{\chi} + l) \equiv \langle v\sigma_{\text{el}} \rangle n_l$. Since $M_{\tilde{\chi}} \gg T$ at chemical decoupling (freeze out), the number density of neutralinos $n_{\tilde{\chi}}$ is suppressed with respect to the number density of the relativistic leptons n_l. Therefore the chemical decoupling temperature T_{cd} is much larger than the temperature of kinetic decoupling T_{kd}. The dominant contribution to the elastic scattering amplitudes for bino-like neutralinos comes from slepton exchange. It was shown in [9] that the contribution of neutralino-photon scattering is negligible. To estimate the temperature of kinetic decoupling we have to calculate the relaxation time τ. The momentum transfer per elastic scattering is $\Delta p_{\tilde{\chi}}/p_{\tilde{\chi}} \sim T/M_{\tilde{\chi}} \ll 1$, thus we need many scatterings to change the momentum of an individual CDM particle by a significant amount. Kinetic decoupling occurs when the relaxation time starts to exceed the Hubble time,

$$\tau = \frac{p_{\tilde{\chi}}}{\Delta p_{\tilde{\chi}}} \frac{1}{\Gamma_{\text{el}}} \sim \frac{1}{H}, \tag{2.1}$$

from which we can determine T_{kd}. Figures 1 and 2 show the dependence of the decoupling temperatures on the sfermion mass. For reasonable masses $M_{\tilde{\chi}} = 150$ GeV and $M_{\tilde{F}} = M_{\tilde{L}} = 250$ GeV we find $T_{\text{cd}} \approx 6$ GeV and $T_{\text{kd}} \approx 40$ MeV.

3. Collisional damping

During the process of kinetic decoupling the neutralinos acquire a finite mean free path. Density inhomogeneities on scales of the diffusion length are damped by the mechanism of collisional damping. It is convenient to describe the CDM as an imperfect fluid. We have shown in [7] that the coefficient of heat conduction vanishes at the leading order in $T/M_{\tilde{\chi}}$. Thus the dominant contribution to collisional damping comes from bulk and shear

Figure 1: Chemical decoupling. $x_{\rm cd} \equiv M_{\tilde\chi}/T_{\rm cd}$ as a function of the sfermion mass for $M_{\tilde\chi} = 50, 100, 200$ GeV (bottom to top).
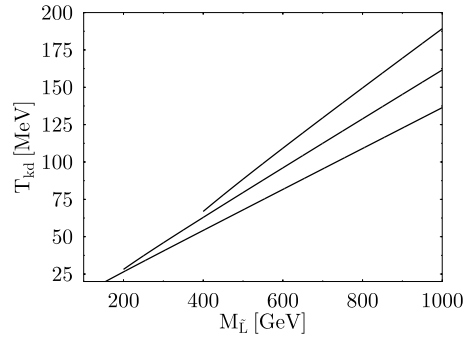
Figure 2: Kinetic decoupling. $T_{\rm kd}$ as a function of the slepton mass for $M_{\tilde\chi} = 50, 100, 200$ GeV (bottom to top).

viscosity. Since energy of the CDM fluid can be transferred to the radiation fluid, which acts here like an inner degree of freedom for the CDM particles, the bulk viscosity does not vanish. Nevertheless, the radiation fluid can be treated as a perfect fluid since $\rho_{\rm rad} \gg \rho_{\rm cdm}$ at kinetic decoupling of the neutralinos. We calculated the relevant coefficients of transport from kinetic theory in [7]. At linear order in the relaxation time the coefficients of shear and bulk viscosity become $\eta \approx n_{\tilde\chi} T \tau$ and $\zeta \approx 5\eta/3$, respectively.

The density inhomogeneities in CDM are damped exponentially below the scale $M_{\rm d}$ due to viscosity [10, 7]

$$\left(\frac{\delta\rho_{\tilde\chi}}{\rho_{\tilde\chi}}\right)_k \propto \exp\left[-\frac{3}{2}\int_0^{t_{\rm kd}} \frac{T\tau}{M_{\tilde\chi}} k_{\rm ph}^2 {\rm d}t\right] = \exp\left[-\left(\frac{M_{\rm d}}{M}\right)^{2/3}\right], \qquad (3.1)$$

where it is useful to work with the CDM mass enclosed in a sphere of radius $2\pi/k_{\rm ph}$, because the mass in CDM is time independent. In figure 3 we plot the damping mass $M_{\rm d}$ as a function of the neutralino mass for various values of the slepton mass. The damping (3.1) provides a small-scale cut-off in the primordial spectrum of density perturbations in neutralino CDM.

4. Free streaming

Once the temperature in the Universe drops below $T_{\rm kd}$ the rate of elastic scatterings is not high enough to keep the neutralinos in thermal equilibrium with the radiation fluid. The neutralinos enter the regime of free streaming. This process continues to smear out inhomogeneities, since the individual neutralinos do not move coherently. In a forthcoming paper we will show by means of kinetic theory that the damping due to free streaming goes as

$$\left(\frac{\delta\rho_{\tilde\chi}}{\rho_{\tilde\chi}}\right)_k \propto \exp\left[-\frac{T_{\rm kd}}{2m_{\tilde\chi}}\left(\frac{k_{\rm ph}}{H}\right)^2_{T=T_{\rm kd}} \ln^2\left(\frac{a}{a_{\rm kd}}\right)\right] = \exp\left[-\left(\frac{M_{\rm fs}(a)}{M}\right)^{2/3}\right]. \qquad (4.1)$$

The mass scale of damping from free streaming $M_{\rm fs}$ is written as a function of the cosmic scale factor a. The damping scale grows logarithmically with the scale factor. This calculation agrees with the estimate of the free streaming scale from the free streaming length as
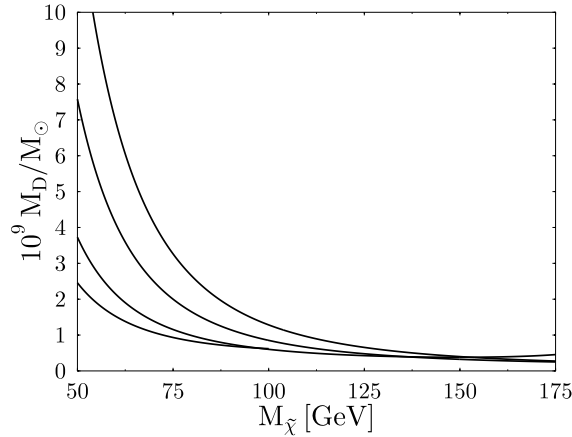
Figure 3: Mass scale of collisional damping of CDM inhomogeneities against the neutralino mass for $M_{\tilde{l}} = 150, 200, 300, 400$ GeV (bottom to top).

presented in [7] up to a numerical factor $(2\pi/\sqrt{6})^3 \approx 17$. (We previously underestimated the free streaming mass by that factor.) The ratio

$$\frac{M_{\rm fs}}{M_{\rm d}} = \left[\sqrt{\frac{5}{3}} \ln \frac{a}{a_{\rm kd}} \right]^3 \tag{4.2}$$

exceeds unity for $a > 2.2 a_{\rm kd}$, thus free streaming starts to dominate the damping from collisional damping once the Universe has doubled its size after kinetic decoupling. It is interesting to evaluate (4.2) at the time of matter-radiation equality, since this is the moment when CDM density perturbations start to grow linearly with the expansion. For a kinetic decoupling temperature $T_{\rm kd} = 40$ MeV and for $(\Omega_{\rm cdm} + \Omega_{\rm b})h^2 = 0.15$ we find $M_{\rm fs}(a_{\rm eq})/M_{\rm d} \approx 1.3 \times 10^4$, thus $M_{\rm fs}(a_{\rm eq}) \approx 5 \times 10^{-6} M_{\odot}$ for $M_{\tilde{\chi}} = 150$ GeV.

Typically the free streaming mass at the time of equality is of the order of $10^{-6} - 10^{-5} M_{\odot}$, which is in striking contrast to claims in the literature (see e.g. [5]) that the minimal mass for the first objects would be $\sim 10^{-13}(150 \text{ GeV}/M_{\tilde{\chi}})^3 M_{\odot}$. The huge difference to our result comes mainly from the false assumption that kinetic decoupling happens simultaneously with chemical decoupling.

5. Conclusions

We have shown that collisional damping and free streaming smear out all power of primordial density inhomogeneities in neutralino CDM below $\sim 10^{-6} M_{\odot}$. This implies that there is a peak (subhorizon CDM density perturbations grow logarithmically during the radiation epoch) in the power spectrum close to the cut-off and therefore we have found the minimal mass for the very first objects, if CDM is made of neutralinos. Our result does not depend in a strong way on the parameters of the supersymmetric standard model.

According to the picture of the hierarchical formation of structure these very small first objects are supposed to merge and to form larger objects, eventually galaxies and larger

structures. It is unclear whether some of the very first objects have a chance to survive. CDM simulations show structure on all scales, down to the resolution of the simulation [11]. However, the dynamic range is not sufficient to deal with the first CDM objects, so the fate of the first CDM clouds is an open issue. A cloudy distribution of CDM in the galaxy would have important implications for direct and indirect searches for dark matter.

Acknowledgments

References

[1] C.B. Netterfield, et al., *A measurement by BOOMERANG of multiple peaks in the angular power spectrum of the cosmic microwave background*, [astro-ph/0104460].

[2] D. Tytler, et al., *Review of Big Bang Nucleosynthesis and Primordial Abundances*, *Phys. Scr.* **T85** (2000) 12, [astro-ph/0001318].

[3] W.J. Percival, et al., *The 2dF Galaxy Redshift Survey: The power spectrum and the matter content of the universe*, [astro-ph/0105252].

[4] M.S. Turner, *A New Era in Determining the Matter Density*, [astro-ph/0106035].

[5] A.V. Gurevich, K.P. Zybin and V.A. Sirota, *Small-scale structure of dark matter and microlensing*, *Physics-Uspekhi* **40** (1997) 869.

[6] J. Ellis, et al., *Charginos and Neutralinos in the Light of Radiative Corrections: Sealing the Fate of Higgsino Dark Matter*, *Phys. Rev.* **D 58** (1998) 095002, [hep-ph/9801445]; *The CMSSM Parameter Space at Large tan beta*, *Phys. Lett.* **B 510** (2001) 236, [hep-ph/0102098].

[7] S. Hofmann, D.J. Schwarz and H. Stöcker, *Damping scales of neutralino cold dark matter*, *Phys. Rev.* **D 64** (2001) 083507, [astro-ph/0104173].

[8] C. Schmid, D.J. Schwarz, P. Widerin, *Amplification of Cosmological Inhomogeneities by the QCD Transition*, *Phys. Rev.* **D 59** (1999) 043517, [astro-ph/9807257].

[9] X. Chen, M. Kamionkowski, X. Zhang, *Kinetic decoupling of neutralino dark matter*, *Phys. Rev.* **D 64** (2001) 021302, [astro-ph/0103452].

[10] S. Weinberg, *Entropy Generation and the Survival of Protogalaxies in an Expanding Universe*, *Astrophys. J.* **168** (1971) 175.

[11] E. van Kampen, *The survival of subhaloes in galaxies and galaxy clusters*, [astro-ph/0008453].