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Quantum Decoherence and Neutrino Oscillations

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ABSTRACT: We discuss the effects of quantum decoherence for neutrino oscillations in the weak coupling limit and review some experimental bounds.

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1. Motivations

We all know that within the framework of ordinary quantum mechanics, isolated pure states can never evolve to mixed ones. However, as argued in [1], an analysis about the final fate of a black hole can lead us to introduce the possibility that such evolution takes place. In this picture, quantum fluctuations of the gravitational field, which in turn can be viewed as microblack holes, may lead to the loss of the quantum coherence also in a microscopic level. If pure states can really evolve to incoherent mixtures, quantum mechanics should be modified in some way. Introducing a phenomenological modification in the Liouville equation for the density operator of a quantum system, Ellis *et al.* [2] showed that the decoherence effects can be effectively implemented.

On the other hand, the subdynamics of any open system interacting with a "reservoir" can develop dissipation, and consequently, show an irreversible dynamics. However, this point of view presents no violation of the quantum theory at all [3]. The time evolution of an open system can be described by the so called quantum dynamics semigroups [4] as well as by the master equation formalism [5]. The former is a very general treatment to

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systems whose time evolution is not reversible, based on very few assumptions: probability conservation, entropy increasing with time and complete positivity (it is worth to say that no assumption is made about the interaction between system and reservoir). The master equation formalism is based on a procedure of elimination of degrees of freedom of the reservoir from the equations of motion. Therefore, it demands some knowledge about the interaction.

Independent of the fact that decoherence effects are assumed to be a fundamental property of a new quantum theory or simply produced by an effective description of an open system, we can ask to what extent these effects are important and can affect the dynamics of elementary particles, such as neutrinos.

2. Open systems formalism

Let H_ν be the free neutrino hamiltonian, H_E be the hamiltonian of a given reservoir, and $H_{\rm int}$ the neutrino-reservoir interaction hamiltonian. We can write down the total hamiltonian as

$$H = H_\nu \otimes 1 + 1 \otimes H_E + \epsilon H_{\rm int}, \tag{2.1}$$

where ϵ is a suitable coupling constant. There is a standard way of eliminating the reservoir degrees of freedom, so that we get at the end a reduced equation of motion describing the subdynamics of the subsystem. This is the so called master equation formalism. If there is no initial correlation between ν and E, that is, if at $t = 0$ we can write the total density operator as

$$\rho_{tot} = \rho_\nu \otimes \rho_E, \tag{2.2}$$

the effective equation of motion to the dynamics of the subsystem at the time t is given by

$$\rho_\nu(t) = Tr_E\left[e^{-iHt}(\rho_\nu \otimes \rho_E)e^{-iHt}\right], \tag{2.3}$$

where Tr_E is the operation of taking the trace over reservoir variables. Its formal definition can be found in [6].

The procedure of tracing over reservoir variables turns out to be very difficult, since we have to deal with the generalized master equations [6]. These integral equations incorporate memory effects. The situations in which memory effects can be eliminated give rise to the so called Markovian master equations [6]. There are two well known situations where Markovian master equations can be obtained: weak coupling limit and singular coupling limit. Both of these limits are related to the relative time scales of the subsystem and the reservoir. The weak coupling limit is explicitly implemented taking the limit $\epsilon \to 0$ (feasible interaction between subsystem and reservoir) and rescaling the time variable $t \to \tau = \lambda^2 t$. This is the limit of interest here.

In the weak coupling limit, the equation of motion for $\rho_\nu(t)$ is given by

$$\frac{\partial \rho_\nu(t)}{\partial t} = -i[H_\nu, \rho_\nu(t)] + W\rho_\nu(t), \tag{2.4}$$

where

$$W\rho_\nu(t) = \lim_{a\to\infty} \frac{-1}{2a}\int_{-a}^{a} ds \int_0^\infty dt' e^{iH_\nu s}\left(\mathrm{Tr}_R\left([e^{iH_0t'}H_{\rm int}e^{-iH_0t'},[H_{\rm int},\rho_{-s}(t)\otimes\rho_R]]\right)\right)e^{-iH_\nu s}, \tag{2.5}$$

with

$$\rho_{-s}(t) = e^{-iH_\nu s}\rho(t)e^{iH_\nu s} = e^{i[H_\nu,\rho(t)]s}, \quad H_0 = \lim_{\epsilon\to 0} H. \tag{2.6}$$

Since the interaction is weak, we can admit a linear interaction hamiltonian

$$H_{\rm int} = F_\mu \otimes B_\mu, \tag{2.7}$$

so that starting from (2.5) we get [4]

$$\frac{\partial\rho_\nu(t)}{\partial t} = -i[H_\nu^{eff},\rho_\nu(t)] + \frac{1}{2}\sum_j\left([A_j,\rho_\nu A_j^\dagger] + [A_j\rho_\nu, A_j^\dagger]\right), \tag{2.8}$$

where H_ν^{eff} is an effective hamiltonian for the neutrinos which incorporate dissipative contributions, and A_j are limited operators depending on F_μ and B_μ. However, (2.8) is essentially the infinitesimal generator of a quantum dynamical semigroup as derived by Lindblad [7] based on the few assumptions already mentioned previously. Therefore, the master equation formalism and quantum dynamical semigroups lead essentially to the same equations of motion to the density operator, provided that we apply the weak coupling limit in the master equation formalism.

3. Current constraints in two generations

Let us give an explicit parameterization of (2.8) when $\rho_\nu(t)$ describes neutrinos in the two generation framework. Expanding the operators H_ν^{eff}, $\rho_\nu(t)$ and A_j of (2.8) in an orthonormal basis of matrices $\{F_\mu\}_\mu$ of $M_2(\mathbf{C})$ (the space of 2×2complex matrices), equation (2.8) is transformed into a system of coupled first order differential equations for the components of $\rho_\nu(t)$

$$\dot\rho_\mu = -2\sum_{\nu\delta}\epsilon_{\mu\nu\delta}h_\mu\rho_\nu + \sum_\nu L_{\mu\nu}\rho_\nu\sigma_\mu, \tag{3.1}$$

where

$$[L_{\mu\nu}] = -2\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & b & \gamma & \beta \\ 0 & c & \beta & \alpha \end{bmatrix}, \tag{3.2}$$

that is, the quantum decoherence effects in a two generation framework can be parameterized by six phenomenological parameters. However these parameters are no all independent if we adopt the condition of complete positivity [3]. In the simplest case, we can work with a single parameter γ, so that the flavour conversion probability can be written as

$$P(\nu_\alpha\to\nu_\beta)(t) = Tr[\rho_\alpha(t)\rho_\beta] = \frac{1}{2}\sin^2 2\theta[1-e^{-2\gamma L}\cos(2\Delta L)], \tag{3.3}$$

where the approximation $t \approx L$ was done and $\Delta = (m_2^2 - m_1^2)/4E$. In [8] we made some *Ansätze* for the possible energy dependence of the parameter γ

$$\gamma = \gamma_0 E^n, \qquad n = -1, 0, 1, 2, \tag{3.4}$$

and we have analyzed experimental data from terrestrial neutrino oscillation experiments CHOOZ, CHORUS, E776 and CCFR in order to extract constraints on the decoherence parameter γ. Three channels were studied: $\nu_\mu \to \nu_\tau$, $\nu_\mu \to \nu_e$ and $\nu_e \to \nu_\tau$. In figures 1,2 and 3, we can see some results of the statistical data analysis. For further details see [8].

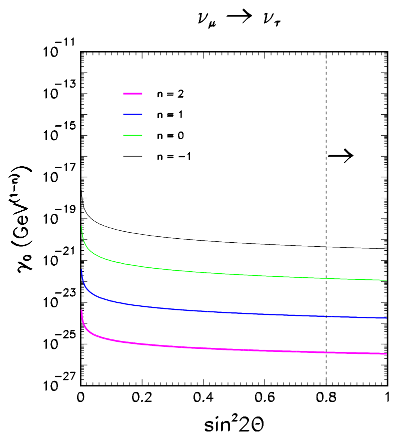

Figure 1: Limits on γ_0 as a function of $\sin^2 2\theta$ in the $\nu_\mu \to \nu_\tau$ mode. The excluded region at 99% C.L. is the one to the right of each curve. All the limits were obtained with CHORUS data.

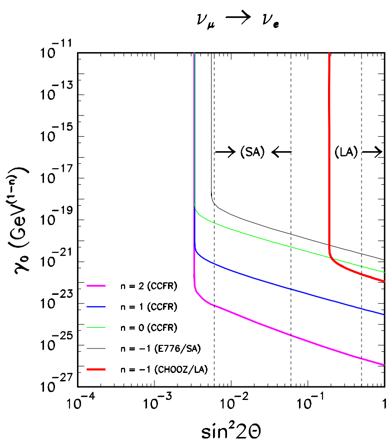

Figure 2: Limits on γ_0 as a function of $\sin^2 2\theta$ in the $\nu_\mu \to \nu_e$ mode at 99% C.L..

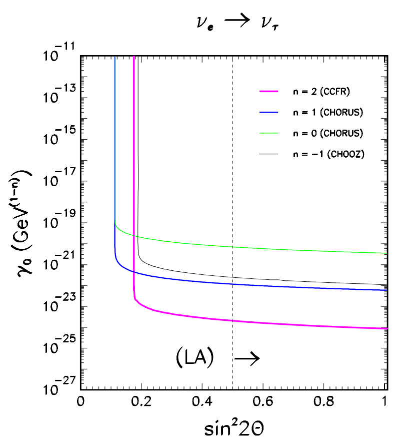

Figure 3: Limits on γ_0 as a function of $\sin^2 2\theta$ in the $\nu_e \to \nu_\tau$ mode at 99% C.L..

Considering all the three families of neutrinos, the map $L_{\mu\nu}$ can be parameterized by 36 parameters (quite complicated!). However, inspired by the two generation case, we can adopt $L_{\mu\nu}$ diagonal. In this case we are left with 8 parameters satisfying, in addition, the inequalities imposed by complete positivity. However, further considerations (as in the two generation case [3]) can reduce, in the weak coupling limit, this number to two.

4. Conclusions

We discuss stringent experimental constraints for recent terrestrial neutrino experiments on quantum decoherence effects in neutrino system. Our bounds are valid for neutrino mass square differences compatible with the atmospheric (ANP), the solar (LA=Large Amplitude solution, SA=Small Amplitude solution) and, in many cases, the LSND scale.

In the $\nu_\mu \to \nu_\tau$ mode (ANP solution range), we have established the following bounds: $\gamma_0 < (5.6 - 4.3) \times 10^{-21}$ GeV^2 for $n = -1$, $\gamma_0 < (1.6 - 1.2) \times 10^{-22}$ GeV for $n = 0$, $\gamma_0 < (3.2 - 2.4) \times 10^{-24}$ for $n = 1$ and $\gamma_0 < (4.0 - 3.1) \times 10^{-26}$ GeV^{-1} for $n = 2$, at 99% C.L. In spite of the fact that these limits are much less restrictive than the ones given in Ref. [9] from analyzing atmospheric neutrinos, they are valuable to be known. In the $\nu_\mu \to \nu_e$ mode (LA), we have established the following bounds: $\gamma_0 < (2.5 - 1.2) \times 10^{-22}$ GeV^2 for $n = -1$, $\gamma_0 < (6.0 - 3.1) \times 10^{-22}$ GeV for $n = 0$, $\gamma_0 < (5.5 - 3.0) \times 10^{-24}$ for $n = 1$ and $\gamma_0 < (2.2 - 1.2) \times 10^{-26}$ GeV^{-1} for $n = 2$, at 99% C.L. From these constraints, we concluded that for $n \gtrsim 1$ one is discouraged to try to extract better limits from the solar neutrino data itself. In the $\nu_\mu \to \nu_e$ mode (SA), we have established the following limits: $\gamma_0 < (6.0 - 0.27) \times 10^{-19}$ GeV^2 for $n = -1$, $\gamma_0 < (7.0 - 0.6) \times 10^{-20}$ GeV for $n = 0$, $\gamma_0 < (7.0 - 0.5) \times 10^{-22}$ for $n = 1$ and $\gamma_0 < (8.0 - 0.3) \times 10^{-24}$ GeV^{-1} for $n = 2$, at 99% C.L. In the case $n \gtrsim 2$, the solar neutrino data will give weaker bounds than ours. Besides, for $n \gtrsim 0$, our constraints are stronger than what we could obtain with LSND data. In the $\nu_e \to \nu_\tau$ mode (LA), we have established the following limits: $\gamma_0 < (2.5 - 1.1) \times 10^{-22}$ GeV^2 for $n = -1$, $\gamma_0 < (1.0 - 0.5) \times 10^{-20}$ GeV for $n = 0$, $\gamma_0 < (1.3 - 0.7) \times 10^{-22}$ for $n = 1$ and $\gamma_0 < (2.0 - 1.0) \times 10^{-24}$ GeV^{-1} for $n = 2$ at 99% C.L.

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