

A Non-SUSY Model for Neutrino Oscillation, Baryogenesis and Neutrinoless Double Beta Decay

Holger Bech Nielsen*, Yasutaka Takanishi

*The Niels Bohr Institute, Blegdamsvej 17,
DK-2100 Copenhagen Ø, Denmark
E-mails: hbech@nbi.dk, yasutaka@nbi.dk*

ABSTRACT: We fitted the neutrino oscillations, baryogenesis and neutrinoless double beta decay using Anti-GUT model.

KEYWORDS: Fermion Masses, Neutrino Oscillations, Baryogenesis, Neutrinoless beta decay.

1. Introduction

The Anti-GUT model [1] is based on a large gauge group, $\prod_{i=1,2,3} (SMG_i \times U(1)_{B-L,i})$, where i denoted as the generation numbers, *i.e.*, each family has its own Standard Model (SM) gauge group with additional gauged Baryon number minus Lepton number, $B - L$. The fermions are the 45 SM Weyl ones plus three right-handed neutrinos.

This gauge group characterised as the largest subgroup of the $U(48)$ transforming the Weyl fermions without anomalies, neither mixed nor gauge ones, and not unifying irreducible representation in SM. It is broken down by the six Higgs fields of Table 1 to the SM at the weak scale. The Weinberg-Salam Higgs field denoted as ϕ_{WS} is also in the table.

The right-handed neutrinos are mass-protected by the total gauge group $B - L$ (diagonal subgroup of the three family $B - L$'s) and obtain the see-saw scale mass given by the vacuum expectation value (VEV) of the Higgs field, ϕ_{B-L} , which breaks $B - L$ quantum charge by two units.

The remaining Higgs fields have VEV one or two order magnitude under the Planck scale, except the Higgs field, S , the VEV of which is almost of the order of the Planck scale.

*Speaker.

	SMG_1	SMG_2	SMG_3	$U_{B-L,1}$	$U_{B-L,2}$	$U_{B-L,3}$
ϕ_{WS}	$\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{2}{3}$	1	$-\frac{1}{3}$
S	$\frac{1}{6}$	$-\frac{1}{6}$	0	$-\frac{2}{3}$	$\frac{2}{3}$	0
W	$-\frac{1}{6}$	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	-1	$\frac{1}{3}$
ξ	$\frac{1}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$	0
T	0	$-\frac{1}{6}$	$\frac{1}{6}$	0	0	0
χ	0	0	0	0	-1	1
ϕ_{B-L}	$-\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{2}{3}$	$-\frac{2}{3}$	2

Table 1: All $U(1)$ quantum charges in Anti-GUT model. SMG_i denote the SM gauge groups.

In the table we have only Abelian quantum numbers of the Higgs fields; it is understood that non-Abelian representations are taken to be smallest ones obeying the following rules:

$$d_i/2 + t_i/3 + y_i/2 = 0 \pmod{1} . \quad (1.1)$$

Here t_i is the triality being 1 for $\underline{3}$, -1 for $\overline{3}$ and 0 for $\underline{1}$ or $\underline{8}$, respectively. The duality d_i is 0 for integer spin $SU(2)_i$ representations while 1 for half-integer $SU(2)_i$ spin.

Each generation gauge group is subgroup of $SO(10)$, *i.e.*, $\times_{i=1,2,3} (SMG_i \times U(1)_{B-L,i}) \subset SO(10)^3$, and the fermion representations could be extended to $SO(10)$ spinor representation but our Higgs field representations could not be enrolled as $SO(10)$ representations.

The philosophy of “dull” Model¹ on which the present model a long way is based means that we seek a model beyond SM which as similar to SM as possible - only deviating from the latter when phenomenologically required. We could for example hold that having several repetitions of SM gauge field systems, one for each generation, is rather little new compared to the SM. It might be phenomenologically required to have more gauge fields to make families have different masses order of magnitude-wise.

To make as weak assumptions as possible we allow everything to go on at Planck scale, actually we assume that all Yukawa coupling constants are order one complex numbers, so that we can treat them as random numbers. Therefore, we can predict from this model everything *only* with order of magnitude accuracy.

2. Mass matrices and results for masses and mixing angles

Effective Yukawa couplings appear due to the transition of using our several Higgs fields [2]; we have investigated numerical corrections [3] due to the different orders of Higgs fields VEVs being attached to the exchange chain of propagators – left-right transition for the Weyl particles. In the neutrino sector according to the see-saw mechanism we have to calculate Dirac- and Majorana-mass matrices:

$$M_{\text{eff}} \approx M_\nu^D M_R^{-1} (M_\nu^D)^T . \quad (2.1)$$

¹We thank to C. Jarlskog for question concerning this philosophy.

Here we present all mass matrices:

$$\begin{aligned}
M_U &\simeq \frac{\langle\phi_{ws}\rangle}{\sqrt{2}} \begin{pmatrix} 6\sqrt{35}SWT^2\xi^2 & 6\sqrt{10}SWT^2\xi & 6\sqrt{35}S^2W^2T\xi \\ 12\sqrt{105}S^2WT^2\xi^3 & 2\sqrt{3}WT^2 & 2\sqrt{15}SW^2T \\ 2\sqrt{35}S^3\xi^3 & \sqrt{2}S & \sqrt{6}WT \end{pmatrix}, \\
M_D &\simeq \frac{\langle\phi_{ws}\rangle}{\sqrt{2}} \begin{pmatrix} 6\sqrt{35}SWT^2\xi^2 & 6\sqrt{10}SWT^2\xi & 2\sqrt{105}S^2T^3\xi \\ 2\sqrt{15}WT^2\xi & 2\sqrt{3}WT^2 & 2\sqrt{5}ST^3 \\ 6\sqrt{210}SW^2T^4\xi & 2\sqrt{210}SW^2T^4 & \sqrt{6}WT \end{pmatrix}, \\
M_E &\simeq \frac{\langle\phi_{ws}\rangle}{\sqrt{2}} \begin{pmatrix} 6\sqrt{35}SWT^2\xi^2 & 60\sqrt{14}S^3WT^2\xi^3 & 60\sqrt{2002}S^3WT^4\xi^3\chi \\ 6\sqrt{30030}S^4WT^2\xi^5 & 2\sqrt{3}WT^2 & \sqrt{210}WT^4\chi \\ 36\sqrt{3233230}S^7W^2T^4\xi^5 & 30\sqrt{14}S^3W^2T^4 & \sqrt{6}WT \end{pmatrix}, \\
M_\nu^D &\simeq \frac{\langle\phi_{ws}\rangle}{\sqrt{2}} \begin{pmatrix} 6\sqrt{35}SWT^2\xi^2 & 60\sqrt{14}S^3WT^2\xi^3 & 60\sqrt{154}S^3WT^2\xi^3\chi \\ 6\sqrt{35}S^2WT^2\xi & 2\sqrt{3}WT^2 & 2\sqrt{15}WT^2\chi \\ 6\sqrt{70}S^2WT\xi\chi & 2\sqrt{6}WT\chi & \sqrt{6}WT \end{pmatrix}, \\
M_R &\simeq \langle\phi_{B-L}\rangle \begin{pmatrix} 2\sqrt{210}S^3\chi^2\xi^2 & \sqrt{15}S\chi^2\xi & \sqrt{6}S\chi\xi \\ \sqrt{15}S\chi^2\xi & \sqrt{6}S\chi^2 & \sqrt{\frac{3}{2}}S\chi \\ \sqrt{6}S\chi\xi & \sqrt{\frac{3}{2}}S\chi & S \end{pmatrix},
\end{aligned}$$

	Fitted	Experimental
m_u	3.1 MeV	4 MeV
m_d	6.6 MeV	9 MeV
m_e	0.76 MeV	0.5 MeV
m_c	1.29 GeV	1.4 GeV
m_s	390 MeV	200 MeV
m_μ	85 MeV	105 MeV
M_t	179 GeV	180 GeV
m_b	7.8 GeV	6.3 GeV
m_τ	1.29 GeV	1.78 GeV
V_{us}	0.21	0.22
V_{cb}	0.023	0.041
V_{ub}	0.0050	0.0035
J_{CP}	1.04×10^{-5}	$2-3.5 \times 10^{-5}$

Table 2: A fit including averaging over $\mathcal{O}(1)$ factors. All quark masses are running masses at 1 GeV except the top quark mass which is the pole mass.

new Higgs fields, ω and ρ , which have quantum numbers

$$\omega = \left(\frac{1}{6}, -\frac{1}{6}, 0, 0, 0, 0\right) \text{ and } \rho = \left(0, 0, 0, -\frac{1}{3}, \frac{1}{3}, 0\right). \quad (2.3)$$

where M_U up-type, M_D down-type, M_E charged lepton, M_ν^D Dirac-neutrino M_R Majorana-neutrino mass matrix, respectively. Each matrix elements are understood to be further multiplied by order one random numbers.

We present the best fit using following VEVs for charged fermion quantities in Table 2 and neutrino quantities in Table 3:

$$\begin{aligned}
\langle\phi_{ws}\rangle &= \frac{246}{\sqrt{2}} \text{ GeV}, \quad \langle\phi_{B-L}\rangle = 2.74 \times 10^{12} \text{ GeV}, \\
\langle S \rangle &= 0.721, \quad \langle W \rangle = 0.0945, \quad \langle T \rangle = 0.0522, \\
\langle \xi \rangle &= 0.0331, \quad \langle \chi \rangle = 0.0345, \quad (2.2)
\end{aligned}$$

where the VEVs, except the Weinberg-Salam Higgs and $\langle\phi_{B-L}\rangle$, are presented in the Planck unit.

Under the impression of the combination of Day Night effect at Super-Kamiokande and the first results from SNO we are able to get new version of this type of model concerning the first to second transition - replacing the Higgs fields, ξ and S , by

They fit large mixing angle MSW solution [4].

	Fitted	Experimental
$\frac{\Delta m_{\odot}^2}{\Delta m_{\text{atm}}^2}$	$5.8_{-5}^{+30} \times 10^{-3}$	$1.5_{-0.7}^{+1.5} \times 10^{-3}$
$\tan^2 \theta_{\odot}$	$8.3_{-6}^{+21} \times 10^{-4}$	$(0.33 - 2.5) \times 10^{-3}$
$\tan^2 \theta_{e3}$	$4.3_{-3}^{+11} \times 10^{-4}$	2.6×10^{-2}
$\tan^2 \theta_{\text{atm}}$	$0.97_{-0.7}^{+2.5}$	$0.43 - 1.0$

Table 3: The numerical results of the ratio of mass squared differences and solar, atmospheric and CHOOZ mixing angles.

SM, even at temperatures so high that sphalerons allows violation of B and L separately.

The CP violation is parameterised by ϵ_i in the decay of the i th flavour right-handed neutrino [6]:

$$\epsilon_i \equiv \frac{\Gamma_{N_{R_i}\ell} - \Gamma_{N_{R_i}\bar{\ell}}}{\Gamma_{N_{R_i}\ell} + \Gamma_{N_{R_i}\bar{\ell}}} = \frac{\sum_{j \neq i} \text{Im}[\langle (M_{\nu}^D)^\dagger M_{\nu}^D \rangle_{ji}^2] [f(\frac{M_j^2}{M_i^2}) + g(\frac{M_j^2}{M_i^2})]}{4\pi \langle \phi_{WS} \rangle^2 \langle (M_{\nu}^D)^\dagger M_{\nu}^D \rangle_{ii}} , \quad (3.1)$$

where f comes from the one-loop vertex contribution and g comes from the self-energy contribution, which can be calculated in perturbation theory if Majorana masses satisfy the condition, $|M_i - M_j| \gg |\Gamma_i - \Gamma_j|$:

$$f(x) = \sqrt{x} \left[1 - (1+x) \ln \frac{1+x}{x} \right] , \quad g(x) = \frac{\sqrt{x}}{1-x} . \quad (3.2)$$

After creation of $B - L$ asymmetry there can be significant wash-out; for it we should use the quantities:

$$K_i \equiv \frac{\Gamma_i}{2H} \Big|_{T=M_i} = \frac{M_{\text{Planck}}}{1.66 \langle \phi_{WS} \rangle^2 8\pi g_{*i}^{1/2}} \frac{\langle (M_{\nu}^D)^\dagger M_{\nu}^D \rangle_{ii}}{M_i} \quad (i = 1, 2, 3) , \quad (3.3)$$

where Γ_i is the width of the flavour i Majorana neutrino, M_i is its mass and g_{*i} is the number of the degree of freedom at temperature M_i and in non-SUSY case approximately 100. The numerical results of our best fitting case (Table 2 and 3) gives

$$|\epsilon_3| = 6.8 \times 10^{-9} , \quad K_3 = 1.06 \quad (3.4)$$

$$|\epsilon_2| = 6.0 \times 10^{-9} , \quad K_2 = 4.29 \quad (3.5)$$

$$|\epsilon_1| = 4.8 \times 10^{-10} , \quad K_1 = 19.8 . \quad (3.6)$$

Furthermore, we need the correction from the obtained K_i – dilution factors – containing the effect of the sphaleron processes being given in various ranges of K_i as:

$$10 \lesssim K_i \lesssim 10^6 : \quad \kappa_i = -\frac{0.3}{K_i (\ln K_i)^{\frac{3}{5}}} \quad (3.7)$$

$$0 \lesssim K_i \lesssim 10 : \quad \kappa_i = -\frac{1}{2\sqrt{K_i^2 + 9}} \quad (3.8)$$

Using the approximation that there is no exchange between the different “flavour $B-L$ ” quantum numbers produced by the three different ν_{R_i} ’s so that we use only dilution with K_i for the ν_{R_i} produced $B-L$ we got from the abovementioned quantities the Baryogenesis via lepton number [7]:

$$Y_B = \sum_{i=1}^3 \kappa_i \frac{\epsilon_i}{g_{*i}} = 1.5_{-1.2}^{+5.8} \times 10^{-11} . \quad (3.9)$$

That meant that we ignored *e.g.* that resonance scattering via the lightest see-saw neutrino as the resonance could contribute to the $B-L$ wash-out in the flavour combination produced by the decay of ν_{R_3} . Really since the couplings of the three see-saw neutrinos are not orthogonal in flavour-space this hypothesis is not valid² and we might rather crudely estimate an effective K_3 to be used for the ν_{R_3} decay products as an average of the three K_i ’s. This would lead to a decrease of our prediction of eq. (3.9) by a factor of the order 6:

$$Y_B \approx 2_{-1.7}^{+10} \times 10^{-12} . \quad (3.10)$$

Note that the version [4] which predicts large mixing angle MSW gives *not unexpectedly* bigger Baryon number production thus improving agreement with experimental data.

4. Neutrinoless double beta decay

From the fitted quantities, namely, neutrino mass and their mixing angles we can calculate so-called “effective Majorana mass parameter” being defined by

$$|\langle m \rangle| \equiv \sum_{i=1}^3 U_{ei}^2 m_i = 5.9_{-2.8}^{+5.3} \times 10^{-5} \text{ eV} \quad (4.1)$$

where m_i is the mass of the Majorana neutrino ν_i and U_{ei} are the elements of the MNS neutrino mixing matrix. The result satisfies the recent experimental limits. Really, it is clear that a model, which predicts small mixing angle MSW for solar neutrino puzzle and neutrino mass spectra being hierarchical pattern, obeys the limit of experiments of neutrinoless double beta decay.

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