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The excitation spectrum of the Wilson surface and QCD string theory

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ABSTRACT: The low energy excitation spectrum of the critical Wilson surface is discussed between the roughening transition and the continuum limit of lattice QCD. The fine structure of the spectrum is interpreted within the framework of two-dimensional conformal field theory.

1. INTRODUCTION

There exists great interest and considerable effort to explain quark confinement in Quantum Chromodynamics (QCD) from the string theory viewpoint. The ideas of 't Hooft, Polyakov, Witten, and others, and recent glueball spectrum or QCD string tension calculations in AdS theories are some illustrative examples of these activities. In a somewhat complementary approach, the search for a microscopic mechanism to explain quark confinement in the QCD vacuum continues with vigorous effort.

We believe that a deeper understanding of the string theory connection with large Wilson surfaces will require a precise knowledge of the surface excitation spectrum and the determination of the universality class of Wilson surface criticality in the continuum limit of lattice QCD. This approach will also require a consistent description of the conformal properties of the gapless Wilson surface excitation spectrum. In this short progress report we summarize our ab initio on-lattice calculations (a more extended status report

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was published recently[1]). In collaboration with Mike Peardon, we have also studied the spectrum of a "closed" flux loop across periodic slab geometry (Polyakov line) by choosing appropriate boundary conditions and operators for selected excitations *without* static sources[2].

2. QCD STRING FORMATION

The first attempt at a comprehensive determination of the rich energy spectrum of the gluon excitations between static sources in the fundamental representation of $SU(3)_c$ in D=4 dimensions was reported earlier[3, 4] for quark-antiquark separations r ranging from 0.1 fm to 4 fm. The extrapolation of the full spectrum to the continuum limit is summarized in Fig. 1 with very different characteristic behavior on three separate physical scales.

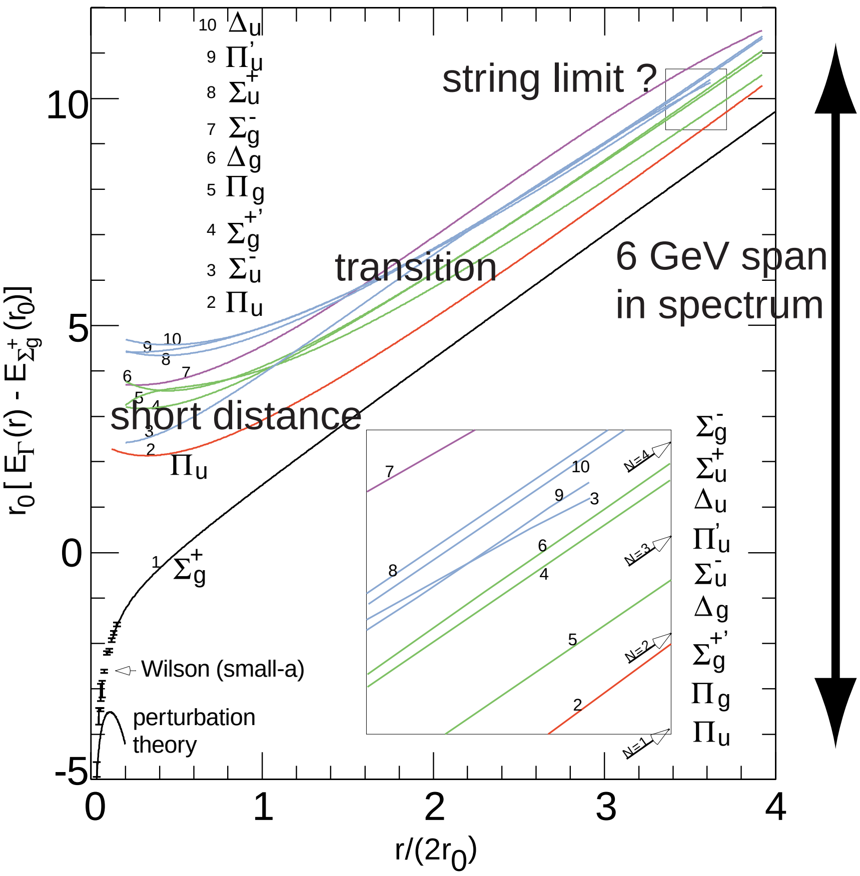

Figure 1: Continuum limit extrapolations are shown for the excitation energies where an arbitrary constant is removed by subtraction. Color coding in postscript is added to the numerical labelling of the excitations (N=0,black:1), (N=1,red:2), (N=2,green:4,5,6), (N=3,blue:3,8,9,10), and (N=4,cyan:7). The five groups represent the expected quantum numbers of a string in its ground state (N=0) and the first four excited states (N=1,2,3,4). The arrows in the inset represent the expected locations of the four lowest massless string excitations (N=1,2,3,4) which have to be compared with the energy levels of our computer simulations.

Nontrivial short distance physics dominates for $r \leq 0.3$ fermi. The transition region towards string formation is identified on the scale 0.5 fm $\leq$ 2.0 fm. String formation and the onset of string-like ordering of the excitation energies occurs in the range between 2 fm and 4 fm where we reach the current limit of our technology.

To display the fine structure with some clarity, error bars are not shown in Fig. 1. Our earlier results are compatible with extended new runs on our dedicated UP2000 Alpha cluster which was built to increase the statistics more than an order of magnitude. The notation and the origin of the quantum numbers used in the classification of the energy levels are explained in earlier publications[3, 4]. Following Sommer[5], the physical scale is set by r_0 which, to a good approximation, is $r_0 = 0.5$ fm.

We also established that the main features of string formation with three separate scales is remarkably universal, independent of the gauge groups SU(2) and SU(3), and space-time dimensions D=3 and D=4. *Although the level ordering is approximately string-like in all cases at large separation, there is a surprising and rather universal fine structure in the spectrum with large displacements from the expected massless Goldstone levels.*

We believe that the fine structure of the spectrum can be explained within the framework of two-dimensional conformal field theory.

3. CONFORMAL FIELD THEORY

One of our extensive tests included a detailed study of the Wilson surface excitation spectrum of the D=3 SU(2) gauge model of QCD_3. The Abelian subgroup Z(2) of SU(2) is expected to play an important role in the microscopic mechanisms of quark confinement suggesting that Wilson surface physics of the D=3 Z(2) gauge spin model should have qualitative and quantitative similarities with the theoretically more difficult QCD_3 case. In the critical region of the Z(2) model we have a rather reasonable description of continuum string formation based on the excitation spectrum of a semiclassical defect line (soliton) of the equivalent Φ^4 field theory. The surface physics of the Z(2) gauge model is closely related to the BCSOS model by universality argument and a duality transformation: *the two surface spectra should agree asymptotically.*

3.1 BCSOS Surface Spectrum

The body-centered solid-on-solid (BCSOS) model is obtained by the SOS condition (accurate to a few percent around the roughening transition) on the fluctuating interface in the body-centered cubic Ising model[6]. This model can be mapped into the six-vertex formulation for which the Bethe Ansatz equations are known[7]. It follows from the Bethe Ansatz solution that the surface has a roughening phase transition at $T_R = J/(k_B \cdot 2\ln 2)$ which is of the Kosterlitz-Thouless type. For $T < T_R$ the interface is smooth with a finite mass gap in its excitation spectrum. For $T \geq T_R$ the mass gap vanishes and the surface exhibits a massless excitation spectrum.

We determined the low energy part of the full surface spectrum from direct diagonalization of the transfer matrix of the BCSOS model and from the numerical solution of the Bethe Ansatz equations. A periodic boundary condition was used, which corresponds to the spectrum of a periodic Polyakov line in the Z(2) gauge model. With a flux of period L we used exact diagonalization for $L \leq 18$, and the the Bethe Ansatz equations up to L=1024. The following picture emerges from the calculation for large L values in the massless Kosterlitz-Thouless (KT) phase. The ground state energy of the flux is given by

$$\mathrm{E_0(L)} = \sigma_\infty \cdot \mathrm{L} - \frac{\pi}{6\mathrm{L}}\mathrm{c} + \mathrm{o(1/L)} \ , \tag{3.1}$$

where σ_∞ is the string tension, c designates the conformal charge, which is found to be c=1 to very high accuracy, consistent with the fact that we are in the KT phase. The o(1/L) term designates the corrections to the leading 1/L behavior; they decay faster than 1/L. At the critical point of the roughening transition, the corrections can decay very slowly, like $1/(\mathrm{lnL}^3 \cdot \mathrm{L})$. Away from the critical point, the corrections decay faster than 1/L in power-like fashion. *These finite size (or equivalent cut-off effects) in the fine structure of the spectrum are dominated by the Sine-Gordon operator in conformal perturbation theory*[8].

For each operator O_α which creates states from the vacuum with quantum numbers α, there is a tower excitation spectrum above the ground state,

$$\mathrm{E}^\alpha_{\mathrm{j,j'}}(\mathrm{L}) = \mathrm{E_0(L)} + \frac{2\pi}{\mathrm{L}}(\mathrm{x}_\alpha + \mathrm{j} + \mathrm{j'}) + \mathrm{o(1/L)} \ , \tag{3.2}$$

where the nonnegative integers j,j' label the conformal tower and x_α is the anomalous dimension of the operator O_α. The momentum of each excitation is given by

$$\mathrm{P}^\alpha_{\mathrm{j,j'}}(\mathrm{L}) = \frac{2\pi}{\mathrm{L}}(\mathrm{s}_\alpha + \mathrm{j} - \mathrm{j'}) \ , \tag{3.3}$$

where s_α is the spin of the operator O_α.

The surface excitation spectrum described by Eqs. (3.1, 3.2, 3.3) is not a simple massless string spectrum with obvious geometric interpretation. There are excitations with noninteger values of the anomalous dimensions x_α which continuously vary with the Ising coupling J. In fact, we found an infinite sequence of operators which excite surface states with fractional multiples of $2\pi/\mathrm{L}$, instead of integer multiples of $2\pi/\mathrm{L}$, as expected in a naive string picture. This sequence can be labelled by anomalous dimensions

$$\mathrm{x}^\mathrm{G}_{\mathrm{n,m}} = \frac{\mathrm{n}^2}{4\pi\mathrm{K}} + \pi\mathrm{K}\mathrm{m}^2 \ , \tag{3.4}$$

where n,m are nonnegative integers and the constant K depends in a known way on the BCSOS coupling constant J. The physical interpretation of the rather peculiar excitations of the rough gapless surface will be discussed elsewhere. Here it is sufficient to note that the spectrum is related to a free compactified Gaussian field, but the field configuration allow for line defects, presumably related to dislocations of the fluctuating rough surface.

4. D=3 QCD STRING THEORY

If the $Q\overline{Q}$ pair is located along one of the principal axes on the lattice in some spatial direction, the Wilson surface at strong coupling is *smooth* in technical terms. This implies the existence of a mass gap in its excitation spectrum, as seen for example in the strong coupling tests of our simulation technology. As the coupling weakens, a roughening transition is expected in the surface at some finite gauge coupling $\mathrm{g} = \mathrm{g_R}$ where the gap in the excitation spectrum vanishes with the characteristics of the Kosterlitz-Thouless phase transition. The correlation length in the surface diverges at the critical point $\mathrm{g_R}$ of the roughening transition and it is expected to remain infinite for any value of the gauge coupling when $\mathrm{g} \leq \mathrm{g_R}$. At the roughening transition, the bulk behavior differs from that of the continuum theory which is located in the vicinity of $\mathrm{g} = 0$. The low energy excitation spectrum of the Wilson surface for $\mathrm{g} \leq \mathrm{g_R}$ and not far from $\mathrm{g_R}$, in the domain of the critical KT phase, should be essentially identical to Eqs. (3.1-3.3) of our BCSOS spectrum.

A change in the structure of the low energy spectrum of the Wilson surface should occur from the KT universality class into a new universality class of continuum $\mathrm{QCD_3}$ string theory as we take the $\mathrm{g} \to 0$ continuum limit in the bulk. We expect that the Wilson surface remains gapless from the roughening transition point g_R to the continuum limit $\mathrm{g} \to 0$. In this scenario the critical behavior of the Wilson surface will exhibit *crossover* from the Kosterlitz-Thouless class into the new universality class of continuum QCD string theory. The other scenario where a physical mass gap develops in the surface is not excluded, although it would imply new phase transitions in the Wilson surface which is unlikely. The precise determination of the expected *crossover* behavior remains the subject of our future investigations.

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