PROCEEDINGS

International Europhysics Conference on HEP

Improved Determination of $\overline{B}_7$ and $\overline{B}_8$

Vincenzo Cirigliano[*]

Inst. für Theoretische Physik, Univ. of Vienna, Boltzmanngasse 5, Vienna A-1090 Austria
vincenzo@thp.univie.ac.at

John F. Donoghue and Eugene Golowich[†] [‡]

Physics Department, Univ. of Massachusetts, Amherst, MA 01003 USA
golowich@physics.umass.edu, donoghue@physics.umass.edu

Kim Maltman[§]

Department of Mathematics and Statistics, York University, 4700 Keele St., Toronto ON M3J 1P3 Canada
kmaltman@physics.adelaide.edu.au

Abstract: I report on a dispersive matrix element calculation of $\overline{B}_{7,8}$ performed within a fully rigorous theoretical framework which incorporates experimental data as a central ingredient. This approach is the first to correctly implement two-loop matching of the effective theory to the dispersive description while using the same scheme dependence as the calculation of the OPE coefficients. The numerical treatment is also completely new and provides a determination of not only central values for $\overline{B}_{7,8}$ but also legitimate estimates of their uncertainties.

1. Introduction

The Standard Model formula for ϵ'/ϵ involves in part the matrix element $\langle(\pi\pi)_{I=2}|\mathcal{Q}_8|K^0\rangle$. The ordinary definition of the associated B-parameter $B_8^{(3/2)}$ involves dividing this matrix element by its vacuum insertion value, whose determination unfortunately depends on imprecisely known light-quark masses. Instead, I will work with the modified quantities $\overline{B}_{7,8}$ in which the matrix elements are divided by 1 GeV^3. That is, if the matrix element

[*]Research supported by TMR, EC-Contract No. ERBFMRX-CT980169 (EURODAΦNE)

[†]Speaker.

[‡]Research supported by the National Science Foundation (Grant PHY-9801875)

[§]Research supported by Natural Sciences and Engineering Research Council of Canada, CSSM at the Univ. of Adelaide and Theory Group at TRIUMF.

of operator $\mathcal{Q}_k$ is expressed in units of GeV^3, then it is numercially equal to $\overline{B}_k$. Also, in this talk I work with the operator

$$\mathcal{Q}_8 \equiv \bar{s}_a \Gamma^\mu_{\rm L} d_b \left(\bar{u}_b \Gamma^{\rm R}_\mu u_a - \frac{1}{2} \bar{d}_b \Gamma^{\rm R}_\mu d_a \right) + \frac{1}{2} \bar{s}_a \Gamma^\mu_{\rm L} u_b \bar{u}_b \Gamma^{\rm R}_\mu d_a$$

to be contrasted with

$$\mathcal{Q}_8^{(3/2)} \equiv \bar{s}_a \Gamma^\mu_{\rm L} d_b \left(\bar{u}_b \Gamma^{\rm R}_\mu u_a - \bar{d}_b \Gamma^{\rm R}_\mu d_a \right) + \bar{s}_a \Gamma^\mu_{\rm L} u_b \bar{u}_b \Gamma^{\rm R}_\mu d_a$$

as used previously in Ref. [2]. In particular, one has

$$\langle (\pi\pi)_{I=2} | \mathcal{Q}_8^{(3/2)} | K^0 \rangle = 2 \langle (\pi\pi)_{I=2} | \mathcal{Q}_8 | K^0 \rangle \quad .$$

In the chiral limit, $\langle (\pi\pi)_{I=2} | \mathcal{Q}_8 | K^0 \rangle$ is proportional to vacuum matrix elements $\langle \mathcal{O}_{1,8} \rangle$, [2]

$$\begin{aligned} \lim_{p=0} \ \langle (\pi\pi)_{I=2} | \mathcal{Q}_7 | K^0 \rangle_\mu &= -\frac{2}{F_\pi^{(0)3}} \ \langle \mathcal{O}_1 \rangle_\mu \quad , \\ \lim_{p=0} \ \langle (\pi\pi)_{I=2} | \mathcal{Q}_8 | K^0 \rangle_\mu &= -\frac{2}{F_\pi^{(0)3}} \ \left[\frac{1}{3} \langle \mathcal{O}_1 \rangle_\mu + \frac{1}{2} \langle \mathcal{O}_8 \rangle_\mu \right] \quad , \end{aligned}$$

where

$$\begin{aligned} \mathcal{O}_1 &\equiv \bar{q} \gamma_\mu \frac{\tau_3}{2} q \ \bar{q} \gamma^\mu \frac{\tau_3}{2} q - \bar{q} \gamma_\mu \gamma_5 \frac{\tau_3}{2} q \ \bar{q} \gamma^\mu \gamma_5 \frac{\tau_3}{2} q \quad , \\ \mathcal{O}_8 &\equiv \bar{q} \gamma_\mu \lambda^a \frac{\tau_3}{2} q \ \bar{q} \gamma^\mu \lambda^a \frac{\tau_3}{2} q - \bar{q} \gamma_\mu \gamma_5 \lambda^a \frac{\tau_3}{2} q \ \bar{q} \gamma^\mu \gamma_5 \lambda^a \frac{\tau_3}{2} q \quad . \end{aligned}$$

In the above, the superscript '(0)' denotes evaluation in the chiral limit, $q = u, d, s$, τ_3 is a Pauli (flavor) matrix, $\{\lambda^a\}$ are the Gell Mann color matrices and the subscripts on $\mathcal{O}_1$, $\mathcal{O}_8$ refer to the color carried by their currents.

The purpose of this talk is to derive dispersive expressions for $\langle \mathcal{O}_{1,8} \rangle$, to describe their evaluation and to present *preliminary* numerical results.

2. Analytical Developments

The key ingredient is study of the amplitude

$$\mathcal{M} \equiv \frac{g_2^2}{16 F_\pi^2} \int d^4x \ \mathcal{D}(x, M_W^2) \langle 0 | T \left(V_3^\mu(x) V_{\mu,3}(0) - A_3^\mu(x) A_{\mu,3}(0) \right) | 0 \rangle \ .$$

This quantity has a familiar form - the exhange of a W-boson between two currents. However, the chiral structure of these currents differs from the LH weak currents which appear in the Standard Model. The amplitude $\mathcal{M}$ is of interest for two reasons. On the one hand, it can be cast in terms of an effective theory which involves $\langle \mathcal{O}_{1,8} \rangle_\mu$,

$$\mathcal{M} \simeq \frac{G_F}{2\sqrt{2} F_\pi^2} \left[c_1(\mu) \langle \mathcal{O}_1 \rangle_\mu + c_8(\mu) \langle \mathcal{O}_8 \rangle_\mu \right] \quad .$$

If we can determine the Wilson coefficients $c_{1,8}(\mu)$ then we will have direct access to $\langle \mathcal{O}_{1,8} \rangle_\mu$.

On the other hand, one can relate $\mathcal{M}$ to a correlator $\Delta\Pi(Q^2)$,

$$\mathcal{M} = \frac{3G_F M_W^2}{32\sqrt{2}\pi^2 F_\pi^2} \int_0^\infty dQ^2 \ \frac{Q^4}{Q^2 + M_W^2} \Delta\Pi(Q^2) \ ,$$

where $\Delta\Pi \equiv \Pi_{\rm v} - \Pi_{\rm a}$ is the difference of isospin vector and axialvector correlators. The imaginary part of $\Delta\Pi$ is proportional to a measurable spectral function $\Delta\rho$,

$$\Delta\Pi(Q^2) \equiv (\Pi_{V,3} - \Pi_{A,3})(Q^2) = \frac{1}{Q^4} \int_0^\infty ds \ \frac{s^2}{s + Q^2} \ \Delta\rho(s) \quad ,$$

where $Q^2 \equiv -q^2$ is the variable for spacelike momenta. The dependence upon $\Delta\rho$ will allow us to use ALEPH data from τ decays [3] as an aid in determining $\mathcal{M}$ and thus $\langle \mathcal{O}_{1,8} \rangle_\mu$.

2.1 Wilson Coefficients

A two-loop analysis of $c_{1,8}(\mu)$ requires matching between the effective and full theories at the scale $\mu = M_W$ using one-loop amplitudes. [2] This is followed by renormalization group (RG) evolution to smaller μ using a two-loop anomalous dimension matrix (*e.g.* see Ref. [4]). The matching and RG evolution are performed in $\overline{\rm MS}$ renormalization and contains explicit scheme dependence (NDR, HV). Special care must be taken for HV RG-evolution to allow for a weak current anomalous dimension. In order that QCD perturbation theory continue to make sense, the renormalization scale μ should be kept sufficiently high ($\mu \geq 2$ GeV). We obtain for the case of $N_c = 3$ and $n_f = 4$ the results

$$c_1^{\overline{\rm MS}}(\mu) = 1 + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \left[\frac{3A_1}{16} \ln \frac{M_W^2}{\mu^2} + \frac{1}{4} \ln^2 \frac{M_W^2}{\mu^2} \right] \quad ,$$

$$c_8^{\overline{\rm MS}}(\mu) = \frac{\alpha_s(\mu)}{\pi} \left[\frac{3}{8} \ln \frac{M_W^2}{\mu^2} - \frac{3}{8} \left(\frac{3}{2} + 2d_s \right) \right] + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \left[\frac{3A_8}{16} \ln \frac{M_W^2}{\mu^2} - \frac{1}{16} \ln^2 \frac{M_W^2}{\mu^2} \right] \quad .$$

In the above the scheme dependence appears in the constants d_s, A_1 and A_8,

	d_s	A_1	A_8	B
NDR	$-5/6$	2	205/36	$-1/6$
HV	1/6	$-10/3$	169/36	11/6

We have also listed another scheme dependent quantity B which will appear below.

2.2 Correlator $\Delta\Pi(Q^2)$

Let us partition the amplitude $\mathcal{M}$ respectively into low momentum ($\mathcal{M}_<(\mu)$) and high momentum ($\mathcal{M}_>(\mu)$) components $\mathcal{M} = \mathcal{M}_<(\mu) + \mathcal{M}_>(\mu)$, where

$$\mathcal{M}_<(\mu) = \frac{3G_F}{32\sqrt{2}\pi^2 F_\pi^2} \int_0^{\mu^2} dQ^2 \ Q^4 \ \Delta\Pi(Q^2) + \ldots$$

$$\mathcal{M}_>(\mu) = \frac{3G_F M_W^2}{32\sqrt{2}\pi^2 F_\pi^2} \int_{\mu^2}^\infty dQ^2 \ \frac{Q^4}{Q^2 + M_W^2} \Delta\Pi(Q^2)$$

We can substitute the operator product expansion (OPE) for $\Delta\Pi(Q^2)$ in the high momentum component $\mathcal{M}_>(\mu)$. Recall [5] that in the chiral limit, the leading term in the OPE for $\Delta\Pi$ carries dimension-six ($d = 6$) while the so-called higher orders comprise $d = 8, 10, \ldots$,

$$\Delta\Pi(Q^2) \sim \frac{1}{Q^6}\left[a_6(\mu) + b_6(\mu)\ln\frac{Q^2}{\mu^2}\right] + \Delta\overline{\Pi}(Q^2) \quad .$$

The quantity $\Delta\overline{\Pi}(Q^2)$ consists of all OPE contributions with $d > 6$.

2.3 Analytical Results

Upon equating the effective theory form of $\mathcal{M}$ with the correlator form, we obtain expressions in $\overline{\text{MS}}$ renormalization for the $d = 6$ part of the OPE,

$$\begin{aligned} a_6^{\overline{\text{MS}}}(\mu) &= 2\pi\langle\alpha_s\mathcal{O}_8\rangle_\mu^{\overline{\text{MS}}} + A_8\langle\alpha_s^2\mathcal{O}_8\rangle_\mu^{\overline{\text{MS}}} + A_1\langle\alpha_s^2\mathcal{O}_1\rangle_\mu^{\overline{\text{MS}}} \ , \\ b_6^{\overline{\text{MS}}}(\mu) &= -\frac{2}{3}\langle\alpha_s^2\mathcal{O}_8\rangle_\mu^{\overline{\text{MS}}} + \frac{8}{3}\langle\alpha_s^2\mathcal{O}_1\rangle_\mu^{\overline{\text{MS}}} \ . \end{aligned}$$

We also obtain two sum rules DG1 and DG2 which extend earlier versions found by Donoghue and Golowich [2]. The sum rule DG1 is given by

$$\langle\mathcal{O}_1\rangle_\mu^{\overline{\text{MS}}} - \frac{3B}{8\pi}\langle\alpha_s\mathcal{O}_8\rangle_\mu^{\overline{\text{MS}}} = \frac{3}{(4\pi)^2}\left[I_1(\mu) + H_1(\mu)\right]$$

whereas DG2 has the form

$$2\pi\langle\alpha_s\mathcal{O}_8\rangle_\mu^{\overline{\text{MS}}} + A_1\langle\alpha_s^2\mathcal{O}_1\rangle_\mu^{\overline{\text{MS}}} + A_8\langle\alpha_s^2\mathcal{O}_8\rangle_\mu^{\overline{\text{MS}}} \equiv a_6(\mu) = I_8(\mu) - H_8(\mu) \quad .$$

The quantities $I_{1,8}$ are spectral integrals,

$$I_{1,8}(\mu) \equiv \int_0^\infty ds\ K_{1,8}(s,\mu^2)\ \Delta\rho(s) \ ,$$

in which the spectral function $\Delta\rho(s)$ is multiplied by the weights $K_{1,8}(s,\mu)$,

$$K_1 \equiv s^2\ln\frac{s+\mu^2}{s} \qquad\qquad K_8 \equiv s^2\frac{\mu^2}{s+\mu^2} \quad .$$

There appear also the quantities $H_1(\mu)$ and $H_8(\mu)$,

$$H_1(\mu) \equiv -\int_{\mu^2}^\infty dQ^2\ \Delta\overline{\Pi}(Q^2) \ , \qquad H_8(\mu) \equiv \mu^6\ \Delta\overline{\Pi}(\mu^2) \quad ,$$

which each contain the OPE component $\Delta\overline{\Pi}$ associated with higher dimensions $d > 6$.

3. Numerical Analysis

Our discussion thus far has dealt with a dispersive analysis of $\langle\mathcal{O}_{1,8}\rangle$ which leads rather naturally to expressions for $\langle\mathcal{O}_{1,8}\rangle$ which involve integrals of spectral functions times certain weights. It would seem that there is nothing left to do at this point aside from the easy task of merely evaluating these integrals. However, we have found such evaluations to be highly nontrivial. This is especially the case if weights are large in regions where the spectral function is unknown. Given the difficulty of the problem, realistic error bars should be provided along with central values in such evaluations.

3.1 The Numerical Problem

The numerical problem defined by our dispersive analysis amounts to the linear system

$$\mathcal{T}ransfer \cdot \mathcal{O}utput = \mathcal{I}nput \ ,$$

where the transfer matrix is given in terms of the known (scheme dependent) constants $A_{1,8}, B$ and the strong fine structure constant $\alpha_s(\mu)$,

$$\begin{pmatrix} 16\pi^2/3 & -2\pi\alpha_s(\mu)B \\ \alpha_s^2(\mu)A_1 & \alpha_s(\mu)\left(2\pi + 2A_8\alpha_s(\mu)\right) \end{pmatrix} \ .$$

There is an output vector of the vacuum matrix elements we wish to evaluate,

$$\begin{pmatrix} \langle \mathcal{O}_1 \rangle_\mu^{\overline{\rm MS}} \\ \langle \mathcal{O}_8 \rangle_\mu^{\overline{\rm MS}} \end{pmatrix}$$

and an input vector of the spectral integrals $I_{1,8}$ and higher dimension contributions $H_{1,8}$,

$$\begin{pmatrix} I_1(\mu) + H_1(\mu) \\ I_8(\mu) - H_8(\mu) \end{pmatrix} \ .$$

3.2 Calculational Procedures

We have considered two methods for obtaining numerical solutions to our equations:

1. Direct Evaluation of the Dispersive Integrals: Although it is possible to formulate sum rules for the higher dimension quantities $H_{1,8}$, an accurate determination of such sum rules is questionable. As such, it is best to somehow avoid $H_{1,8}$ altogether. We therefore evaluate the input vector at a sufficiently high renormalization scale (say $\mu = 4$ GeV) to suppress the contributions of $H_{1,8}$ relative to $I_{1,8}$. The evaluation of $I_{1,8}$ at $\mu = 4$ GeV is performed with a procedure we call the *Residual Weight Method* (RWM), to be described in the following subsection. Finally we employ the two-loop RG anomalous dimension matrix to evolve from $\mu = 4$ GeV down to $\mu = 2$ GeV.

2. Finite Energy Sum Rules (FESR): This approach is performed at low scales, in the vicinity of $\mu = 2$ GeV. It yields values for the $d = 6$ coefficient $a_6 = I_8 - H_8$ as well as the $d > 6$ contributions which embody the higher dimension quantities $H_{1,8}$. Since the FESR method relies heavily on the OPE, we *assume* its validity here. Work is underway to test this assumption.

3.2.1 Residual Weight Method (RWM)

There are powerful constraints on $\Delta\rho(s)$ which assist in the evaluation of the spectral integrals. In the low energy region ($s \le m_\tau^2$) there is data from tau decay. In the high energy region ($s \ge s_{\rm asy}$) one has the two-loop OPE, which it suffices to write here schematically as

$$\Delta\rho(s) \sim \frac{tiny}{s^3} \qquad (s \ge s_{\rm asy})$$

Last but not least are the classical chiral sum rules (W1, W2, W3),

$$\begin{aligned} W1 &\equiv \int_0^\infty ds\ \Delta\rho(s) = F_\pi^{(0)2} \\ W2 &\equiv \int_0^\infty ds\ s\ \Delta\rho(s) = 0 \\ W3 &\equiv \int_0^\infty ds\ s \ln \frac{s}{1\ \mathrm{GeV}^2}\ \Delta\rho(s) \\ &= -\frac{16\pi^2 F_\pi^{(0)2}}{3e^2}\Delta m_\pi^{(0)2} \end{aligned}$$

which constrain $\Delta\rho(s)$ over all energy scales. Recall that superscript '(0)' indicates evaluation in the chiral limit. We employ numerical values for $F_\pi^{(0)2}$ and $\Delta m_\pi^{(0)2}$ from Ref. [6].

Consider evaluation of a spectral integral which contains weight $K(s)$,

$$I \equiv \int_0^\infty ds\ \ K(s)\ \Delta\rho(s)\ ,$$

by utilizing an auxiliary fitting function,

$$C(s) \equiv x + y\ s + z\ s \ln s$$

where x, y, z are arbitrary coefficients. The function $C(s)$ is constructed with the dual purpose of having a close relation to the chiral sum rules and also providing a good fit to the weight $K(s)$ over some specified fitting window, say $2.5 \leq s(\mathrm{GeV}^2) \leq 5.0$. We fix the coefficient y to minimize the norm $||C(s) - K(s)||$ over the fitting window.

In the RWM, we add and subtract the fitting function $C(s)$ to express the original weight $K(s)$ in the form

$$K(s) = C(s) + [K(s) - C(s)]\ \equiv\ C(s) +\ \ \Delta K(s)\ \ ,$$

where $\Delta K(s)$ is the *residual weight.* Substitution of the above expression into the spectral integral I results in a two-component form ('chiral' and 'residual') $I = I_{\mathrm{chiral}} + \Delta I$, where the chiral part is simply

$$I_{\mathrm{chiral}} = x \cdot W1 + z \cdot W3\ \ .$$

The residual component partitions naturally into a low energy part where data is accessible,

$$\Delta I_{\mathrm{data}} = \int_0^{m_\tau^2} \Delta K(s,\mu^2)\ \Delta\rho(s)\ \ ,$$

a negligible intermediate part ($\Delta\rho$ is small and ΔK is highly suppressed)

$$\Delta I_{\mathrm{int}} = \int_{m_\tau^2}^{s_{\mathrm{asy}}} \Delta K(s,\mu^2)\ \Delta\rho(s)\ \ ,$$

and a negligible asymptotic part ($\Delta\rho$ is tiny)

$$\Delta I_{\mathrm{asy}} = \int_{s_{\mathrm{asy}}}^\infty \Delta K(s,\mu^2)\ \Delta\rho(s)\ \ .$$

The final step in the RWM is to add in quadrature errors from the various components,

$$E(x,z) = [E^2_{\rm chiral} + E^2_{\rm data} + E^2_{\rm int} + E^2_{\rm asy}]^{1/2} \quad ,$$

and then to minimize the overall error function $E(x,z)$. This fixes the remaining coefficients x, z.

4. Results and Conclusions

I will cite results based on three distinct approaches:

1. RWM: Choose $\mu = 4$ GeV; neglect $H_{1,8}(4\text{ GeV})$. Use RWM to find $I_{1,8}$ at $\mu = 4$ GeV. RG-evolve $\langle \mathcal{O}_{1,8} \rangle$ down to $\mu = 2$ GeV.

2. Mainly FESR: FESR for a_6 $(= I_8 - H_8)$, H_1 at $\mu = 2$ GeV. Obtain $I_1(2\text{ GeV})$ as in Strategy I.

3. Hybrid: Use FESR for H_1, H_8 at $\mu = 2$ GeV. Obtain $I_{1,8}(2\text{ GeV})$ via Strategy I.

For the NDR and HV renormalization prescriptions, we obtain respectively

Strategy	$\overline{B}_7^{\rm NDR}$	$\overline{B}_8^{\rm NDR}$	$\overline{B}_7^{\rm HV}$	$\overline{B}_8^{\rm HV}$
RWM/RG	0.16 ± 0.10	2.22 ± 0.67	0.49 ± 0.07	2.46 ± 0.70
FESR	0.22 ± 0.034	1.51 ± 0.20	0.47 ± 0.04	1.84 ± 0.20
HYBRID	0.22 ± 0.034	1.72 ± 0.51	0.50 ± 0.12	2.07 ± 0.85

These three solution sets are seen to be mutually consistent.

4.1 Concluding Remarks

Our work [1] is seen to divide into two parts, analytical and numerical:

1. We analytically implement two-loop matching of an effective theory to a dispersive framework used in our earlier work. Our approach guarantees the same renormalization scheme dependence as used in the calculation of OPE coefficients.

2. A numerical procedure (RWM method) for evaluating the spectral integrals $H_{1,8}$ was constructed to make maximal use of ALEPH data and chiral sum rules. Assuming validity of the FESR approach led to additional numerical insights.

Finally, we compare our results given above for matrix elements of the operators $\mathcal{Q}_{7,8}$ with those from a lattice simulation, a $1/N_c$ analysis and a so-called X-boson procedure:

	$\langle(\pi\pi)_{I=2}\vert\mathcal{Q}_7\vert K^0\rangle$		$\langle(\pi\pi)_{I=2}\vert\mathcal{Q}_8\vert K^0\rangle$	
Method	NDR	HV	NDR	HV
This work	0.16 ± 0.10	0.49 ± 0.07	2.22 ± 0.67	2.46 ± 0.70
Lattice [7]	0.11 ± 0.04	0.18 ± 0.06	0.51 ± 0.10	0.57 ± 0.12
Large N_c [8]	0.11 ± 0.03	0.67 ± 0.20	3.5 ± 1.1	3.5 ± 1.1
X − boson [9]	0.26 ± 0.03	0.39 ± 0.06	1.2 ± 0.5	1.3 ± 0.6

References

[1] V. Cirigliano, J. F. Donoghue, E. Golowich and K. Maltman, "Determination of $\langle(\pi\pi)_{I=2}|\mathcal{Q}_{7,8}|K^0\rangle$ in the chiral limit," [hep-ph/0109113].

[2] J. F. Donoghue and E. Golowich, J. F. Donoghue and E. Golowich, "Dispersive calculation of $B_7^{(3/2)}$ and $B_8^{(3/2)}$ in the chiral limit," Phys. Lett. B **478**, 172 (2000) [hep-ph/9911309].

[3] R. Barate *et al.* [ALEPH Collaboration], "Measurement of the spectral functions of axial-vector hadronic tau decays and determination of $\alpha_s(m_\tau^2)$," Eur. Phys. J. C **4**, 409 (1998).

[4] A. J. Buras, M. Jamin, M. E. Lautenbacher and P. H. Weisz, "Two loop anomalous dimension matrix for $\Delta S = 1$ weak nonleptonic decays. 1. O(α_s^2)," Nucl. Phys. B **400**, 37 (1993) [hep-ph/9211304].

[5] V. Cirigliano, J. F. Donoghue and E. Golowich, "Dimension-eight operators in the weak OPE," JHEP**0010**, 048 (2000) [hep-ph/0007196].

[6] G. Amoros, J. Bijnens and P. Talavera, "QCD isospin breaking in meson masses, decay constants and quark mass ratios," Nucl. Phys. B **602**, 87 (2001) [hep-ph/0101127].

[7] A. Donini, V. Gimenez, L. Giusti and G. Martinelli, "Renormalization group invariant matrix elements of $\Delta S = 2$ and $\Delta I = 3/2$ four-fermion operators without quark masses," Phys. Lett. B **470**, 233 (1999) [hep-lat/9910017].

[8] M. Knecht, S. Peris and E. de Rafael, "A critical reassessment of Q_7 and Q_8 matrix elements," hep-ph/0102017.

[9] J. Bijnens, E. Gamiz and J. Prades, "Matching the electroweak penguins Q_7, Q_8 and spectral correlators," hep-ph/0108240.