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Gravity and gauge theory: a new connection

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Abstract: The duality between theories of gravity on anti-de Sitter space and conformal field theories in one less dimension (AdS/CFT) has provided a new geometrical tool for understanding the strong coupling dynamics of gauge theories—including confinement. It has also led to new insights into string theory and the dynamics of D-branes. I present a brief overview of these developments.

1. Introduction

An old question of quantum gravity is, "What are the quantum microstates of black holes?" This question seems a reasonable point of attack in an otherwise incomprehensible subject, since black holes certainly have entropy (due, presumably, to these microstates), and at least for large black holes there is no region of large curvatures outside the horizon (so semi-classical methods might get us somewhere). But it defeated generations of theorists up until a few years ago. Even today we have partial answers from string theory only for a rather restricted type of black hole. To appreciate the difficulties, note that Schwarzschild black holes have negative specific heat: if you add more energy (or mass), they get cooler. It is hard to construct natural-looking quantum systems that lead to this property (although of course it is possible in principle: "all" that is needed is a super-exponential growth of the density of states with energy). We might therefore be inclined to focus our attention at first on the easier case of near-extremal black holes, like the Reissner-Nordstrom solution with an electric charge nearly as great (in Planck units) as its mass. Such solutions have positive specific heat, so at least our first naive obstacle is removed.

It turns out that most examples of black holes in string theory where we do have an understanding of the quantum microstates share this property of positive specific heat. In fact, the focus in string theory is on "black branes," where the horizon has infinite extent

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in one or more directions. There is a considerable literature, starting with [1], where detailed properties of black branes are matched up with a manifestly unitary description of microstates based on D-branes constructions in string theory. In some ways I regard the correspondence between anti-de Sitter space and conformal field theory (AdS/CFT) as an outgrowth of this literature; but it is more as well, because the unitary description of black brane microstates turns out to be in terms of non-abelian gauge theories at strong coupling. Although the original D-brane constructions led to *conformally invariant* gauge theories, it has proven possible to tweak these constructions in such a way that the gauge theory confines in the infrared. Thus from the exploration of black hole microstates we are led to a startlingly different question, but almost as old and at least as important: "In what ways can string theory elucidate the phenomena of strong interactions?"

AdS/CFT is a field that many individuals have contributed to significantly, and it is impossible to give an adequate reference list in the space I have. I will therefore settle for listing here the papers that have the most to do with the actual content of my talk: [2, 3, 4, 5, 6, 7, 8, 9, 10]. A more extensive bibliography (though slightly out of date by now) can be found in [11].

2. D3-branes

A startlingly large percentage of recent developments in string theory can trace their roots back, at least partially, to the discovery of the role of D-branes in string theory [12]. D-branes are places in spacetime where strings can end, but they can be shown to have a "life of their own" in the sense that they have tension and dynamical fluctuations. They also can be described purely in geometrical terms as solutions to supergravity theories in ten-dimensions. These supergravity theories are the low-energy descriptions of the simplest string vacua. One of them, type IIB supergravity, admits the solution

$$\begin{aligned} ds_{10}^2 &= H^{-1/2}(-h dt^2 + d\vec{x}^2) + H^{1/2}\left(\frac{dr^2}{h} + r^2 d\Omega_5^2\right) \\ H &= 1 + \frac{L^4}{r^4} \qquad h = 1 - \frac{r_0^4}{r^4} \qquad \frac{L^8}{G_{10}} \sim N^2 \,. \end{aligned} \tag{2.1}$$

This is called the near-extremal black D3-brane solution. It carries N Dirac units of charge under a five-form generalization of the electromagnetic field, F_5. It is translationally invariant in the three Euclidean directions labelled $\vec{x}$, and if $r_0 = 0$ it enjoys also a Lorentz symmetry $SO(3,1)$ along the $(t, \vec{x})$ directions. These are the directions in which the non-abelian gauge theory will "live." All the other dimensions arise from string theory, but they play a role in describing the gauge theory dynamics—particularly the radial direction, as we shall see.

The geometry (2.1) has a black hole horizon at $r = r_0$ that's extended in the $\vec{x}$ directions. You might satisfy yourself that this is true by pretending $H = 1$, dropping the $\vec{x}$ coordinates, and demonstrating that the result is just the Schwarzschild solution in the remaining seven dimensions. Black holes have temperature and entropy: for example, $S = A_{\rm horizon}/4G_{10}$, where $A_{\rm horizon}$ is the area of the horizon. Assembling the relevant formulas of semi-classical general relativity, we obtain a free energy

$$F_{\text{SUGRA}} = -\frac{\pi^2}{8} N^2 V_3 T^4 \,, \tag{2.2}$$

where V_3 is the volume in the $\vec{x}$ directions (infinite actually, but obviously we want to factor it out), and the subscript "SUGRA" means that we computed using semi-classical type IIB supergravity. Let us observe that (2.2) is the form that one would get for a CFT with roughly N^2 degrees of freedom: for example, N^2 massless scalar bosons have a free energy that is not too different from (2.2). This first hint of a "dual CFT description" was reported in [2], and was noted independently by A. Strominger.

To go further, we need to know more about D-branes. One big contribution of [12] was the observation that places where strings can end (the formal definition of a D-brane) automatically carry charge. In the case of D3-branes (where the 3 just tells you that it's an object extended in three spatial dimensions as well as in time), the charge pertains to the five-form gauge field strength F_5, and one D-brane carries one Dirac unit of charge. So to get something like (2.1), one should put N D3-branes together and heat them up to the Hawking temperature of the horizon. When D3-branes come together, strings with one end on one brane and the other end on another can be very short, so very light. These strings interact with one another just like gluons in a non-abelian gauge theory. But there are also fermions and scalars on the branes, also coming from short strings. All these particles are colored (in the sense of QCD colors) just like gluons because we can imagine labelling the D3-branes arbitrarily (say as red, green, blue, and so on) and then calling a particle red-anti-green if it stretched from the green brane to the red one. They interact according to what is called $\mathcal{N} = 4$ super-Yang-Mills theory with gauge group $U(N)$. This is a well-studied quantum field theory that has the special property of being exactly conformally invariant: its beta functions vanish identically.

There are $O(N^2)$ degrees of freedom on the brane, because this is how many different types of gluons there are. Free field counting leads to the free energy

$$F^{\text{free}}_{U(N)\ SYM} = -\frac{\pi^2}{6} N^2 V_3 T^4 = \frac{4}{3} F_{SUGRA} \,. \tag{2.3}$$

To understand why the 4/3 is not a worry, we should note that there are loop corrections to the field theory result. These have been computed at low order and shown to increase the magnitude of the free energy. Actually, the supergravity computation corresponds to *strong coupling* in the field theory, a fact that we can appreciate from the following identifications (accurate up to 2's and π's):

$$\frac{L^2}{\alpha'} \sim \sqrt{g_{YM}^2 N} \qquad \frac{L^8}{G_{10}} \sim N^2 \,. \tag{2.4}$$

Here L is the characteristic length scale of the solution (2.1), α' is the square of the typical length of a string, and g_{YM} is the Yang-Mills coupling constant. The 't Hooft parameter $g_{YM}^2 N$ controls the perturbative expansion on the gauge theory side, while taking N large corresponds to suppressing non-planar diagrams. Small $g_{YM}^2 N$ is where free field counting should be nearly right, but the semi-classical supergravity approximation used in (2.2) is supposed to work when $L^2 \gg \alpha'$. We can't yet prove in field theory that the 4/3 appears at large $g_{YM}^2 N$; but this is at least a reasonable outcome. The N^2 dependence in both (2.3) and (2.2) is already considerable encouragement to the view that the classical supergravity solution captures a lot of the physics of N coincident D3-branes.

3. AdS/CFT

Clearly we would like to do better by computing something via supergravity and via gauge theory that can be compared directly. Felicitously, there are such quantities: absorption cross-sections of low-energy particles impinging on the D3-branes. For instance, we might send in a graviton perpendicular to the D3-brane worldvolume, and with a polarization such that it warps the world-volume as it hits. This deformation is carried out in the field theory through the stress-energy tensor, $T_{\mu\nu}$. An inclusive amplitude for absorption of the graviton can be calculated from a cut in the two point function of $T_{\mu\nu}$. The cross-section is

$$\sigma = G_{10} \frac{N^2}{4} \omega^3 \,, \tag{3.1}$$

where ω is the energy of the graviton. Through a chain of Ward identities and an application of the Adler-Bardeen theorem, it's possible to show that the two-point function of the stress-energy is *exact at one loop*, and hence independent of $g_{YM}^2 N$. So it's something that we can compare directly to a computation in supergravity, where a low-energy graviton impinges on the geometry (2.1) (strictly, (2.1) with $h = 1$, since we want zero temperature for this calculation) and has some amplitude of falling through the horizon. This calculation leads to *exactly* the same result (3.1), so we can claim that there's been a meaningful test of the equivalence of gravitational and gauge theory descriptions.

A crucial point—one of several realized by Maldacena in a paper [5] which marks the inception of the AdS/CFT correspondence in its "modern" form—is that the ambient flat space that surrounds the D3-branes can be dispensed with. The geometry close to the D3-branes (at zero temperature) is $AdS_5 \times S^5$, where S^5 is a five-sphere and AdS_5 is anti-de Sitter space: a solution of the five-dimensional Einstein equations with a negative cosmological constant. So the basic claim of AdS/CFT, in its simplest form, is that string theory on $AdS_5 \times S^5$ is equivalent to the non-abelian gauge theory that describes low-energy fluctuations on N coincident D3-branes.

In the graviton absorption thought-experiment, as long as the energy of the graviton is small compared to $1/L$, the nearly flat region, where $r \gg L$, participates in a purely kinematic fashion. We could as well slice off the geometry at $r = L$ and imagine controlling the boundary conditions in an arbitrary way. That's practically what the graviton absorption setup does: the graviton coming in from infinity results in some perturbation in the boundary conditions for a short while, and the absorption cross-section is a measure of how the system reacts. In gauge theory terms, these boundary conditions amount to perturbing the lagrangian by $h_{\mu\nu} T^{\mu\nu}$, where $h_{\mu\nu}$ is the graviton wave-function on the artificial boundary at $r = L$.

At this point we'd like to make a quantitative statement of AdS/CFT. We've worked toward it from two points of attack: the entropy calculation and the absorption cross-section. A way to unify these results, as well as greatly extend them, is to conjecture that the partition functions of string theory on AdS_5 and gauge theory on $\mathbf{R}^{3,1}$ coincide, provided we set the right boundary conditions on AdS_5 and add the corresponding source terms to the gauge theory lagrangian. In symbols,

$$Z^{AdS_5}_{\substack{\text{string}\\ \text{theory}}}[\phi_0] = Z^{\mathbf{R^{3,1}}}_{\substack{\text{gauge}\\ \text{theory}}}[\phi_0] \,, \tag{3.2}$$

where

$$Z^{\mathbf{R^{3,1}}}_{\substack{\text{gauge}\\ \text{theory}}}[\phi_0] = \left\langle \exp \int d^4x \, \phi_0(x) \mathcal{O}(x) \right\rangle \qquad Z^{AdS_5}_{\substack{\text{string}\\ \text{theory}}}[\phi_0] \approx \exp \left(- \underset{\phi \to \phi_0}{\text{extremum}} \, I_{\text{SUGRA}}[\phi] \right) . \tag{3.3}$$

Now, (3.2) and (3.3) should be regarded very much as schematic descriptions of more precise formulas. As soon as I explain the various constituent pieces, it should become clear how at least some things can be written more explicitly. The gauge theory partition function is a completely well-defined object: it's the vacuum-to-vacuum amplitude of some exponential of source terms, just like one uses to develop Feynman diagrams. Only here, the sources $\phi_0(x)$ are all for color singlet operators $\mathcal{O}(x)$: things like the stress tensor, or $\operatorname{tr} F^2$, which have no charge under the gluon fields. The string theory partition function takes as its arguments the same data $\phi_0(x)$, only now these functions are regarded as specifying the boundary data on the boundary of AdS_5 (or at $r = L$, in our previous and somewhat imprecise picture). It is understood in (3.2) and (3.3) that there are fields in string theory, denoted ϕ, which are defined on the whole of AdS_5 and are asymptotic to $\phi_0(x)$ as one approaches the boundary. There are lots of field ϕ, and most of them aren't scalars: for instance there's the graviton field. We would have to tweak our formulas a little for each field, so that for instance the graviton couples as $h^{\mu\nu} T_{\mu\nu}$, at least in a gauge where the other components of h_{MN} in the bulk of AdS_5 vanish. And there could be some couplings of bilinears in the boundary values ϕ_0 to color singlet operators, which are rather poorly understood. Further difficulties arise from the fact that the string theory partition function is difficult to define in the best of circumstances, and in AdS_5 it is particularly difficult because of the presence of the field F_5 (a so-called Ramond-Ramond background). In (3.3), I have short-circuited all such difficulties by simply making a saddle-point supergravity approximation to the string theory partition function as the exponential of minus the supergravity action. This is correct for Euclidean signature, but many interesting physical quantities can be obtained most easily through Wick-rotation. Making the saddle-point supergravity approximation is a controlled approximation when L is much larger than the string length ($\sqrt{\alpha'}$) and the Planck length ($G_{10}^{1/8}$).

Despite the ambiguities and caveats noted in the previous paragraph, (3.2) and (3.3) serve as the standard working definition of AdS/CFT [6, 7]. They give a precise relation between a theory of gravity in the bulk of anti-de Sitter space, and a conformal field theory on its boundary, which is Minkowski space. These equations include the statement that the free energy of the near-extremal D3-brane should coincide with the free energy of the gauge theory: one needs only recall $Z = e^{-F/T}$. And they imply the cross-section (3.1): a second variation in $h_{\mu\nu}$ allows one to extract the formula we gave earlier from either side of the duality. More generally, if the supergravity action includes a piece

$$I_{\text{scalar}} = \int_{AdS_5} d^5x \, \sqrt{g} \left[\frac{1}{2}(\partial\phi)^2 + \frac{1}{2} m^2 L^2 \phi^2 \right] , \tag{3.4}$$

describing a massive scalar, then one may easily extract from the gravity side of the duality a two-point function

$$\langle \mathcal{O}(x)\mathcal{O}(0) \rangle \propto \frac{1}{x^{2\Delta}} \qquad \text{where} \quad m^2 L^2 = \Delta(\Delta - 4) \tag{3.5}$$

for the corresponding color singlet scalar operator $\mathcal{O}$, of conformal dimension Δ, on the boundary. (Similar formulas obtain for fields with spin in the bulk and operators with Lorentz indices on the boundary).[1] There is a well-developed dictionary between fields of supergravity and operators in the gauge theory, where the correspondence between the graviton and the stress tensor is just the most obvious entry.

This is an opportune moment to explain why I suddenly stopped mentioning the S^5 part of the string theory geometry. One reason is that the dictionary between fields and operators is simpler when we perform a Kaluza-Klein reduction on the S^5: then each field in the five-dimensional theory corresponds to an operator of definite dimension. A better reason is that the correspondence as phrased in (3.2) and (3.3) is believed to be far more general than $AdS_5 \times S^5$: in fact, *any* stable string theory vacuum with an AdS_d factor is believed to be dual to *some* conformal field theory in dimension $d-1$. Even better: the string theory vacuum can be merely asymptotically AdS_d, and the correspondence should still hold, only now the field theory is a conformal field theory perturbed by relevant operators. Perhaps the most general statement of the duality that can be made is that any background of a consistent, quantizable theory of gravity, which has a timelike boundary at infinity, should be physically equivalent, essentially through the formulas (3.2) and (3.3) or close variants thereof, to a unitary quantum theory without gravity that lives on the boundary. The correspondence is best understood in cases which are close relatives of the original anti-de Sitter space setup; however it has been inventively applied to the study of other things, like the linear dilaton background, little string theories, duality cascades, and singularity resolution. The formulas (3.2) and (3.3) seem to be remarkably reliable guides: they render precise the general intuition that geometry embodies dynamics. The correspondence between bulk and boundary theories also embodies a theoretical notion called holography that was proposed some years ago by 't Hooft and Susskind.

4. RG flows and a c-theorem

Because of lack of space, I cannot do full justice to the topic of RG flows in AdS/CFT. Let us focus on an example. The lagrangian of $\mathcal{N} = 4$ super-Yang-Mills theory, in $\mathcal{N} = 1$

[1] The astute reader may observe that for some values of Δ, for instance $\Delta = 2$, we have $m^2 L^2 < 0$. This is OK, because tachyons can be consistently quantized in anti-de Sitter space provided they satisfy a certain bound [13].

superfield notation, is

$$\begin{gathered}\mathcal{L} = \int d^4\theta \, \mathrm{tr}\, \Phi_i^\dagger e^V \Phi_i + \int d^2\theta \left[\mathrm{tr}\, W_\alpha^2 + W(\Phi)\right] + h.c. \\ W(\Phi) = \mathrm{tr}\, \Phi_3[\Phi_1, \Phi_2]\,,\end{gathered} \tag{4.1}$$

where the three chiral superfields Φ_i as well as the vector superfield V are in the adjoint representation of $SU(N)$.[2] The Φ_i are colored boundary superfields, not to be confused with the bulk scalars ϕ. Many renormalization group flows may be studied by perturbing the lagrangian (4.1) by a relevant operator. One that is particularly simple to study is an addition of $\mathrm{tr}\, \frac{1}{2}m\Phi_3^2$ to the superpotential $W(\Phi)$. This mass term for Φ_3 preserves $\mathcal{N} = 1$ supersymmetry, and it turns out that the RG flow terminates at an infrared fixed point (still with $\mathcal{N} = 1$ supersymmetry) where Φ_1 and Φ_2 have dimensions $\Delta = 3/4$ rather than 1, so that $\Delta = 3/2$ for color singlet operators like $\mathrm{tr}\, \Phi_1\Phi_2$. It may further be shown, via supersymmetry and 't Hooft anomaly matching, that if we define a "central charge" in a fixed point theory (i.e. a conformal theory) via

$$\langle T_{\mu\nu}(x) T_{\alpha\beta}(0)\rangle = c(\partial_\mu\partial_\nu\partial_\alpha\partial_\beta - \text{contractions}) \left(\frac{1}{x^4}\right)\,, \tag{4.2}$$

where the contractions are designed to make the right hand side reflect the known properties of $T_{\mu\nu}$ (namely conservation and tracelessness), then in the renormalization group flow from $\mathcal{N} = 4$ super-Yang-Mills to the $\mathcal{N} = 1$ infrared fixed point, c decreases by a factor of 27/32. All this is standard in the literature on supersymmetric field theories: see for example [14, 15].

It is a very non-trivial test of AdS/CFT that these results can be verified in explicit detail on the gravity side of the duality. The five-dimensional bulk geometry dual to a renormalization group flow is of the form

$$ds^2 = e^{2A(r)}(-dt^2 + d\vec{x}^2) + dr^2\,. \tag{4.3}$$

This form is unique (up to reparametrizations of r) once we demand 3+1-dimensional Poincaré invariance—that is, once we say that we want to study vacua. If $A(r) = r/L$, then (4.3) describes anti-de Sitter space. In general we would take $A(r)$ to be some monotonic function of r, but with $A(r) \sim r/L$ for large r so that the geometry still is asymptotically anti-de Sitter. You could check this by setting $h = 1$ in (2.1) and taking the near horizon limit—only the r in (2.1) is a reparametrization of the one in (4.3). It's worth developing one more piece of intuition: events that take place at definite values of r, say $r = r_1$ and $r = r_2$, have their energies redshifted by a factor of $e^{A(r_1)}$ and $e^{A(r_2)}$, at least as perceived by an observer at some fiducial value of r where we set $A = 0$. For this reason, the large A, large r region of the geometry pertains to ultraviolet physics in the dual quantum field theory, whereas the small (or perhaps negative) r region pertains to infrared physics. In short, *the radial coordinate is energy scale.*

[2] A fine point is that the $U(1)$ factor decouples from the $SU(N)$. This $U(1)$ pertains to the center of mass motion of the D3-branes, and this is in some sense projected out in a near-horizon limit.

With this intuition in place, it should make sense that a renormalization group flow from a UV fixed point (in this case, $\mathcal{N} = 4$ super-Yang-Mills) to an IR fixed point (the special $\mathcal{N} = 1$ fixed we described earlier) should be reflected in the bulk geometry by a solution which is asymptotically AdS_5 both at large positive and large negative r. That is, $A(r) \sim r/L_{UV}$ as $r \to \infty$, and $A(r) \sim r/L_{IR}$ as $r \to -\infty$. A simple, general argument (based on the so-called "null energy" condition in the bulk theory) shows that $A''(r) \leq 0$. So $L_{IR} < L_{UV}$. It can further be shown that $c \sim L^3$, both in the UV and the IR. So $c_{IR} < c_{UV}$, and there is a monotonic function of energy scale, namely $1/A'(r)^3$, which interpolates between ultraviolet and infrared values of the central charge. This amounts to a generalization of Zamolodchikov's c-theorem. Precise calculations on the gravity side reproduce the factor 27/32 by which c decreases in the particular RG flow under consideration, as well as the finite anomalous dimensions at the $\mathcal{N} = 1$ fixed point.

5. Confinement

There are various configurations in AdS/CFT which exhibit confinement. A few of them are completely smooth in the bulk—notably the ones in [7, 16, 17]. But here I will give a discussion that relies more on generic features and does not presuppose a completely well-controlled supergravity geometry. I believe this to be worthwhile, because although some confining theories may indeed admit non-singular AdS duals, I doubt this is generic. This part of my talk, I suspect, would have been of primary interest to Prof. Gribov, and I regret now that I never had the chance to meet him and talk physics.

Most deformations of the lagrangian of $\mathcal{N} = 4$ super-Yang-Mills theory will result in confinement.[3] Let us add masses to all the fermions and scalars in the field theory. The beta function then becomes negative, and generically there is confinement in the infrared. The mass terms for the scalars and the fermions are dual to specific deformations of anti-de Sitter space, which however leave the asymptotics the same, corresponding to the fact that these are relevant deformations of the gauge theory. Generically supersymmetry will be broken by these terms, with a characterstic scale M_{SUSY}. In the dual geometry, deviations from AdS_5 will become appreciable at a radius $r/L \sim \log M_{SUSY}$. At smaller radii, the geometry distorts considerably, and in most situations becomes singular at finite r, at least in its five-dimensional description. A higher-dimensional description might be non-singular, or if not, string theory may resolve the singularities. There are few general truths regarding such singularities, but I believe I am safe in saying that there are many examples which must become well-defined in a full stringy treatment, some of which we understand already.

Two point functions may still be computed using (3.2) and (3.3), and a spectral expansion reveals a mass gap of order M_{SUSY}. There is some subtlety in using a supergravity approximation when there is a singularity in the geometry: one practical way to proceed is to check that the wave-function of the supergravity field one is interested in remains finite as the singularity is approached. The point here is not that one can understand every

[3] It takes very special deformations, like the one discussed in the previous section, or ones preserving $\mathcal{N} = 2$ supersymmetry, to avoid confinement in the IR.

feature of asymptotically AdS duals of confining gauge theories (at least, not yet); but rather that we can understand essential qualitative features that verify that confinement has indeed occurred. Someday we hope to do better, once strings in Ramond-Ramond backgrounds are better understood.

In advance of this understanding, we can nevertheless make a meaningful test of the area law for Wilson loops. For the theories we are considering, an area law really should pertain for massive external quarks, because there are no dynamical fields in the gauge theory which carry a fundamental or anti-fundamental charge under the $SU(N)$ gauge group. These are the charges that string ends carry—a fact that should be clear from our original setup, where strings running between one brane and another carry charges of fundamental $\otimes$ anti-fundamental. The strategy for adding external quarks is to dangle a string into AdS from the boundary. Where the string ends on the boundary, there we have a quark and an anti-quark. The energy of the configuration may be computed from the string's world-volume action. The essential feature of the bulk geometry that must pertain in order to see an area law is that the string should be repelled from the singularity. This turns out to be quite natural if the string coupling blows up at the singularity; and that is reasonable since confinement is understood as the effects of a superficially divergent coupling. Naively, the energy of the string is its tension times its length; but in fact one must be careful to include a red-shift factor $\sqrt{G_{tt}}$ to account for the fact that the bulk geometry is a curved spacetime. Here G_{tt} is the time component of the metric the string feels. For singularities where the coupling blows up, this string metric generally has a minimum for $\sqrt{G_{tt}}$ in a non-singular part of the geometry: let us call this maximum redshift factor $\sqrt{G^*_{tt}}$. For configurations where the separation between the string ends is a distance R much greater than the anti-de Sitter space radius L, the minimal energy configuration of the string is where it traverses most of the distance at the location of maximum redshift. The effective tension will then be

$$\frac{1}{\alpha'_{QCD}} = \frac{\sqrt{G^*_{tt}}}{\alpha'} \sim \sqrt{g^2_{YM} N} M^2_{gap} \,, \tag{5.1}$$

where in the last expression we have given the result that almost invariably arises from computation in specific models. The factor of $\sqrt{g^2_{YM} N}$ is universal, and in the case of unperturbed $\mathcal{N} = 4$ super-Yang-Mills theory it is understood in terms of a summation of rainbow graphs. Thus, in sum, we can say that the QCD string in this setting is after all a fundamental string, just at large redshift.

6. Conclusions

AdS/CFT is exciting because it is much more than a duality between anti-de Sitter space and conformal field theories. It is a general correspondence between theories with gravity and without, which elucidates properties of both. The quantitative statement of the correspondence, (3.2) and (3.3), is an analog of the S-matrix. This analogy is apt because like the S-matrix, the string partition function that appears in (3.2) depends only on asymptotic data, but it is nevertheless believed to be a guideline to defining all physical processes in the bulk in terms of what happens at the boundary.

AdS/CFT tells us how to describe microstates of certain black branes in terms of manifestly unitary quantum field theories; and it tells how to compute at strong coupling in these quantum field theories using the near-horizon geometry of the black brane. The encoding of quantum field theory physics in the geometry of branes provides new insights into renormalization group flow, and into confinement.

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References

[1] A. Strominger and C. Vafa, *Phys. Lett.* **B379** (1996) 99–104, hep-th/9601029.

[2] S. S. Gubser, I. R. Klebanov, and A. W. Peet, *Phys. Rev.* **D54** (1996) 3915–3919, hep-th/9602135.

[3] I. R. Klebanov, *Nucl. Phys.* **B496** (1997) 231–242, hep-th/9702076.

[4] S. S. Gubser and I. R. Klebanov, *Phys. Lett.* **B413** (1997) 41–48, hep-th/9708005.

[5] J. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998) 231–252, hep-th/9711200.

[6] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, *Phys. Lett.* **B428** (1998) 105, hep-th/9802109.

[7] E. Witten, *Adv. Theor. Math. Phys.* **2** (1998) 253–291, hep-th/9802150.

[8] E. Witten, *Adv. Theor. Math. Phys.* **2** (1998) 505, hep-th/9803131.

[9] D. Z. Freedman, S. S. Gubser, K. Pilch, and N. P. Warner, hep-th/9904017.

[10] S. S. Gubser, hep-th/9902155.

[11] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, *Phys. Rept.* **323** (2000) 183, hep-th/9905111.

[12] J. Polchinski, *Phys. Rev. Lett.* **75** (1995) 4724–4727, hep-th/9510017.

[13] P. Breitenlohner and D. Z. Freedman, *Phys. Lett.* **B115** (1982) 197.

[14] R. G. Leigh and M. J. Strassler, *Nucl. Phys.* **B447** (1995) 95–136, hep-th/9503121.

[15] D. Anselmi, D. Z. Freedman, M. T. Grisaru, and A. A. Johansen, *Nucl. Phys.* **B526** (1998) 543–571, hep-th/9708042.

[16] I. R. Klebanov and M. J. Strassler, *JHEP* **08** (2000) 052, hep-th/0007191.

[17] J. M. Maldacena and C. Nunez, hep-th/0008001.