CP Violation

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ABSTRACT: Several pieces of direct and indirect evidence now suggest that the Kobayashi-Maskawa mechanism plays a distinguished role for CP violation at the electroweak scale. This talk provides a general overview of CP violation in its various contexts, emphasizing CP violation in flavour-violating interactions, such as due to the Kobayashi-Maskawa mechanism. I then review a few recent theoretical developments relevant to the interpretation of CP violation.[†]

1. Introduction

The breaking of CP symmetry ("CP violation"), the composition of parity and charge conjugation, is an interesting phenomenon for several reasons:

a. CP violation together with CPT symmetry implies non-invariance of the microscopic equations of motion under time-reversal. CP violation rather than C violation implies different physical properties of matter and antimatter. These two facts are of fundamental importance for our understanding of the laws of Nature, and they were perceived as revolutionary, when CP violation was discovered in 1964 [3]. They were also important in the development of the fundamental theory of particles, since the observation of CP violation motivated some early extensions of the Standard Model as it was known at the time (1973), either by extending the Higgs sector [4] or by adding a third generation of quarks and leptons [5]. Nature has opted for the second possibility for certain, and the Kobayashi-Maskawa mechanism of CP violation has become part of today's Standard Model. From today's perspective time-reversal non-invariance and the distinction of matter and antimatter, though fundamental, appear no longer surprising and even "natural". What remains

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[*]Speaker.

[†]CP violation is inseparably linked to flavour physics in the Standard Model. Three plenary talks have been devoted to this subject at this conference. The talk by Ligeti [1] reports on recent results in flavour physics excluding CP violation. The talk by Hamel de Monchenault [2] summarizes experimental results on CP violation in B decays. CP violation is currently a rather phenomenological subject and some overlap of this talk with [2] is unavoidable. In addition to discussing theoretical aspects of CP violation, this talk also covers recent experimental results on CP violation in the kaon system.

surprising, however, is the peculiar way in which CP violation occurs or rather does not occur in the Standard Model and its possible extensions.

b. Assuming that the evolution of the universe began from a matter-antimatter symmetric state, CP violation is necessary [6] to generate the matter-antimatter asymmetric state of the universe that one observes today. At the electroweak phase transition the Standard Model satisfies all the other necessary criteria for baryogenesis (baryon number violation, departure from thermal equilibrium) [7], but CP violation in the Standard Model is too weak to explain the observed baryon-to-photon ratio. With the above assumption on the initial condition of the cosmic evolution, our own existence provides evidence for a source of CP violation beyond the Standard Model.

Electroweak baryogenesis has the attractive feature that it couples the required new mechanisms of CP violation to the electroweak scale, therefore making them testable also in particle collider experiments. Nevertheless it now appears more likely that the matter-antimatter asymmetry is not related to the sources of CP violation that one may observe at colliders. Two facts have contributed to this change of perspective: first, the lower limit on the masses of Higgs bosons has been increasing. A heavier Higgs boson implies a weaker (first-order) electroweak phase transition. As a consequence electroweak baryogenesis is already too weak over most of the parameter space of even the minimal supersymmetric extension of the Standard Model. Second, the observation of small neutrino masses through neutrino oscillations is explained most naturally by invoking the seesaw mechanism, which in turn is most naturally realized by postulating massive neutrinos, which are singlets under the Standard Model gauge group. All three necessary conditions for the generation of lepton number are naturally realized in the decay of the massive neutrino(s). Lepton number is then partially converted into baryon number via B+L-violating (but B–L conserving) sphaleron transitions [8]. While the leptogenesis scenario is very appealing, the new sources of CP violation related to the Yukawa couplings of the heavy neutrinos occur at scales of order of the heavy neutrino mass, $M_R \sim (10^{12} - 10^{16})$ GeV (needed to explain the small left-handed neutrino masses), and are not directly testable with collider experiments in the near future. For this reason, CP violation in the context of baryogenesis will not be discussed further in this talk.

c. CP violation in the Standard Model is essentially an electroweak phenomenon originating from the Yukawa couplings of the quarks to the Higgs boson. This implies that probes of CP violation are indirect probes of the electroweak scale or TeV scale, complementary to direct probes such as the observation of Higgs bosons. This is probably the most important reason for the current interest in CP symmetry breaking: in addition to testing the Kobayashi-Maskawa mechanism of CP violation in the Standard Model, experiments directed at CP violation limit the construction of extensions of the Standard Model at the TeV scale. There is an analogy between CP symmetry and electroweak symmetry breaking. Both occur at the electroweak scale and for both the Standard Model provides a simple mechanism. However, neither of the two symmetry breaking mechanisms has been sufficiently tested up to now.

d. Leaving aside the matter-antimatter asymmetry in the universe as evidence for CP violation since this depends on a further assumption, CP violation has now been observed

in the weak interactions of quarks in three different ways: in the mixing of the neutral kaon flavour eigenstates (ϵ, 1964) [3]; in the decay amplitudes of neutral kaons (ϵ'/ϵ, 1999) [9, 10]; in the mixing of the neutral B_d meson flavour eigenstates ($\sin(2\beta)$, 2001) [11, 12]. It will be seen below that these pieces of data together with others not directly related to CP violation suggest that the Kobayashi-Maskawa mechanism of CP violation is most likely the dominant source of CP violation at the electroweak scale. The latest piece of evidence also rules out that CP symmetry is an approximate symmetry. As a consequence generic extensions of the Standard Model at the TeV scale needed to explain the stability of the electroweak scale suffer from a CP fine-tuning problem since any such extension implies the existence of many new CP-violating parameters which have no generic reason to be small, but which put these theories into conflict with experiment, if they are large. Despite the apparent success of the standard theory of CP violation, the problem of CP and flavor violation therefore remains as mysterious as before.

2. CP violation in the Standard Model

CP violation can occur in the Standard Model in three different ways:

2.1 The θ term

The strong interactions could be CP-violating [13, 14, 15]. The topology of gauge fields implies that the correct vacuum is given by a superposition $|\theta\rangle = \sum_n e^{in\theta}|n\rangle$ of the degenerate vacua $|n\rangle$ in which pure gauge fields have winding number n. Correlation functions in the θ-vacuum can be computed by adding to the Lagrangian the term

$$\mathcal{L}_\theta = \theta \cdot \frac{g_s^2}{32\pi^2}\, G_{\mu\nu}^A \tilde{G}^{A,\mu\nu}, \tag{2.1}$$

where θ now represents a parameter of the theory. Physical observables can depend on θ only through the combination $e^{i\theta}\det\mathcal{M}$, where $\mathcal{M}$ is the quark mass matrix. A non-zero value of $\tilde{\theta} = \theta + \arg\det\mathcal{M}$ violates CP symmetry. It also implies an electric dipole moment of the neutron of order $10^{-16}\,\tilde{\theta}\,e\,$cm [16]. The non-observation of any such electric dipole moment constrains $\tilde{\theta} < 10^{-10}$ and causes what is known as the strong CP problem, since the Standard Model provides no mechanism that would require $\tilde{\theta}$ to vanish naturally. The strong CP problem has become more severe with the observation of large CP violation in B meson decays, since one now knows with more confidence that the quark mass matrix has no reason to be real a priori.

There exist mechanisms that render $\tilde{\theta}$ exactly zero or very small through renormalization effects. None of these mechanisms is convincing enough to provide a default solution to the problem. What makes the strong CP problem so difficult to solve is that one does not have a clue at what energy scale the solution should be sought. Strong CP violation is not discussed further in this talk (see the discussion in [17]).

2.2 The neutrino mass matrix

The Standard Model is an effective theory defined by its gauge symmetries and its particle content. CP violation appears in the lepton sector if neutrinos are massive. The leading

operator in the effective Lagrangian is [18]

$$\frac{f_{ij}}{\Lambda} \cdot [(L^T \epsilon)_i i\sigma^2 H][H^T i\sigma^2 L_j], \tag{2.2}$$

where L_i and H denote the lepton and Higgs doublets, respectively. After electroweak symmetry breaking this generates a Majorana neutrino mass matrix with three CP-violating phases. One of these phases could be observed in neutrino oscillations, the other two phases only in observables sensitive to the Majorana nature of neutrinos.

Unless the couplings f_{ij} are extremely small, the scale Λ must be large to account for small neutrinos masses, which suggests that leptonic CP violation is related to very large scales. For example, the standard see-saw mechanism makes the f_{ij} dependent on the CP-violating phases in the heavy gauge-singlet neutrino mass matrix. As a consequence one may have interesting model-dependent relations between leptogenesis, CP violation in lepton-flavour violating processes and neutrino physics, but since the observations are all indirect through low-energy experiments, one may at best hope for accumulating enough evidence to make a particular model particularly plausible. Such experiments seem to be possible, but not in the near future, and for this reason leptonic CP violation is not discussed further here. It should be noted that there is in general no connection between CP violation in the quark and lepton sector except in grand unification models, where the two relevant Yukawa matrices are related. Even then, further assumptions are necessary for a quantitative relation.

2.3 The CKM matrix

CP violation can appear in the quark sector of the Standard Model at the level of renormalizable interactions [5]. The quark Yukawa interactions read

$$\mathcal{L}_Y = -y_{ij}^d \bar{Q}_i' H d_{Rj}' - y_{ij}^u \bar{Q}_i' \epsilon H^* u_{Rj}' + \text{h.c.}, \tag{2.3}$$

with Q' the left-handed quark SU(2)-doublets, u_R' and d_R' the right-handed SU(2)-singlets and $i, j = 1, 2, 3$ generation indeces. The complex mass matrices that arise after electroweak symmetry breaking are diagonalized by separate unitary transformations $U_{L,R}^{u,d}$ of the left- and right-handed up- and down-type fields. Only the combination

$$V_{\text{CKM}} = U_L^{u\dagger} U_L^d = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \tag{2.4}$$

referred to as the CKM matrix, is observable, since the charged current interactions now read

$$-\frac{e}{\sqrt{2}\sin\theta_W} \bar{u}_{Li} \gamma^\mu [V_{\text{CKM}}]_{ij} d_{Lj} W_\mu^+ + \text{h.c.}. \tag{2.5}$$

At tree level flavour and CP violation in the quark sector can occur in the Standard Model only through charged current interactions (assuming $\tilde{\theta} = 0$). With three generations of quarks, the CKM matrix contains one physical CP-violating phase. Any CP-violating

observable in flavour-violating processes must be related to this single phase. The verification or, perhaps rather, falsification of this highly constrained scenario is the primary goal of many current B- and K-physics experiments. This type of CP violation is therefore discussed in some detail in later sections.

For reasons not understood the CKM matrix has a hierarchical structure as regards transitions between generations. It is therefore often represented in the approximate form [19]

$$
V^{\mathrm{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4), \qquad (2.6)
$$

where $\lambda \approx 0.224$ and A, ρ, η are counted as order unity. The unitarity of the CKM matrix leads to a number of relations between rows and columns of the matrix. The one which is most useful for B-physics is obtained by multiplying the first column by the complex conjugate of the third:

$$
V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \qquad (2.7)
$$

If $\eta \neq 0$ (which implies CP violation) this relation can be represented as a triangle in the complex plane, called the unitarity triangle. See Figure 1, which also introduces some notation for the angles of the triangle that will be referred to later on.

The hierarchy of the CKM matrix implies that CP violation is a small effect in the Standard Model even when the CP-violating phase is large. More precisely, CP-violating observables are either small numbers, or else they are constructed out of small numbers such as small branching fractions of rare decays. The hierarchy of quark masses and mixing angles represents a puzzle, sometimes called the flavour problem, which will also not be discussed further in this talk.
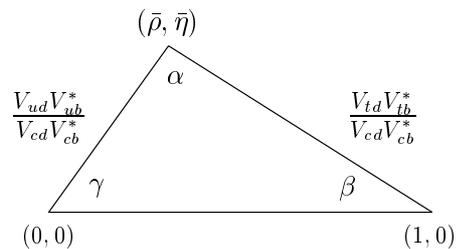

Figure 1: The unitarity triangle.

3. Kaon decays

Rare kaon decays have been very important for the construction of the Standard Model and for CP violation in particular. Kaons continue to be among the most sensitive probes of flavour-changing interactions. This is illustrated by the suppression of $|\Delta S| = 2$ transitions in the Standard Model due to weak coupling, the GIM mechanism and small CKM matrix elements. None of these suppressions needs to hold in extensions of the Standard Model.

3.1 CP violation in mixing (indirect)

Due to CP violation in $K\bar{K}$ mixing, the neutral kaon mass eigenstates are superpositions of CP-even and CP-odd components. The long-lived kaon state, $K_L \approx K_2 + \bar{\epsilon}K_1$, is predominantly CP-odd, but decays into two pions through its small CP-even component

K_1. The decay $K_L \to \pi\pi$ constituted the first observation of CP violation ever [3]. The quantity $|\epsilon| = 2.27 \cdot 10^{-3}$ (equal to $|\bar\epsilon|$ in the standard phase convention to very good accuracy) has now been measured in many different ways. There are two new results related to $\bar\epsilon$:

1. The charge asymmetry in $K_L \to \pi e \nu$ decay has been measured very precisely by KTeV [20]:

$$2\,\mathrm{Re}\,\bar\epsilon \approx \frac{\Gamma(\pi^- e^+ \nu) - \Gamma(\pi^+ e^- \bar\nu)}{\Gamma(\pi^- e^+ \nu) + \Gamma(\pi^+ e^- \bar\nu)} = (3.322 \pm 0.074) \cdot 10^{-3}. \tag{3.1}$$

2. The rare decay $K_L \to \pi^+\pi^- e^+ e^-$ can proceed through the CP-violating $K_L \to \pi^+\pi^-$ decay with subsequent radiation of a virtual photon which converts into an $e^+ e^-$-pair. The amplitude for this decay is proportional to $\bar\epsilon$. It can also proceed through a CP-conserving direct amplitude $K_L \to \pi^+\pi^-\gamma^*$. Since this amplitude is dynamically suppressed, there exists the possibility of a large CP-violating asymmetry. KTeV and NA48 investigated the angular distribution $d\Gamma/d\phi$, where ϕ is the angle between the $\pi^+\pi^-$ and the $e^+ e^-$ decay planes. The decay plane asymmetry is CP-violating and has been found to be [21, 22]

$$A \equiv \left| \frac{1}{\Gamma} \int d\phi \, \frac{d\Gamma}{d\phi} \, \mathrm{sign}(\sin(2\phi)) \right| = \begin{cases} (13.9 \pm 2.7 \pm 2.0)\% & \text{NA48} \\ (13.6 \pm 2.5 \pm 1.2)\% & \text{KTeV} \end{cases}, \tag{3.2}$$

in agreement with theoretical expectations [23].

3.2 CP violation in decay (direct)

CP-violating effects in kaon decays can also occur due to the interference of two decay amplitudes with different CP-violating phases independent of CP violation in $K\bar K$ mixing. In the Standard Model both manifestations of CP violation are related, but one can construct models in which CP violation arises only in $|\Delta S| = 2$ interactions and not in the decay amplitude.

There are a number of searches for direct CP violation with no result expected in the Standard Model at the current sensitivities of the experiments. The HyperCP experiment searches for direct CP violation in hyperon decays and in charged kaon decays to three pions. The CP-violating asymmetries in hyperon decay are compatible with zero with an error of about $2 \cdot 10^{-3}$ [24, 25] compared to a Standard Model expectation of $10^{-(4-5)}$. Another example is the transverse muon polarization

$$P_\perp = \langle \vec{S}_\mu \cdot \frac{(\vec{p}_\pi \times \vec{p}_\mu)}{|\vec{p}_\pi \times \vec{p}_\mu|} \rangle \tag{3.3}$$

in $K^+ \to \pi^0 \mu^+ \nu$ decay. This observable is odd under naive time-reversal (no exchange of initial and final state) and can therefore also be generated by (electromagnetic) final state interactions. With a neutral pion in the final state this effect is very small [26] resulting

in $P_\perp \sim 10^{-(6-7)}$ in the Standard Model. CP-violating interactions in extensions of the Standard Model could yield a transverse polarization of order 10^{-3} through interference with the dominant charged current amplitude. The current result $P_\perp = -(3.3 \pm 3.7 \pm 0.9) \cdot 10^{-3}$ by the KEK-E246 experiment [27] therefore begins to reach an interesting level of sensitivity.

Much experimental effort has been invested into the search for direct CP violation in kaon decays to two pions. The double ratio

$$\frac{\Gamma(K_L \to \pi^0\pi^0)\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^0\pi^0)\Gamma(K_L \to \pi^+\pi^-)} \approx 1 - 6\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right), \tag{3.4}$$

if different from unity, implies such an effect, since both ratios would equal $|\epsilon|$, if CP violation occurred only in mixing. The existence of this effect has been conclusively demonstrated by two experiments in 1999, following the first hints of a non-vanishing ϵ'/ϵ in 1992 [28]. The new results of 2001 have further clarified the situation, which is now summarized by [29, 30]

$$\frac{\epsilon'}{\epsilon} = \begin{cases} (15.3 \pm 2.6) \cdot 10^{-4} & \text{NA48 (97-99)} \\ (20.7 \pm 2.8) \cdot 10^{-4} & \text{KTeV (96/97).} \end{cases} \tag{3.5}$$

The theory of ϵ'/ϵ is rather complicated. The short-distance contributions have been worked out to next-to-leading order [31, 32] and do not constitute a major source of uncertainty, but the remaining hadronic matrix elements continue to prevent an accurate computation of ϵ'/ϵ. The following approximate representation of the result [33],

$$\frac{\epsilon'}{\epsilon} = 16 \cdot 10^{-4} \left[\frac{\mathrm{Im}\, V_{ts}^* V_{td}}{1.2 \cdot 10^{-4}}\right] \left(\frac{110\,\mathrm{MeV}}{m_s(2\,\mathrm{GeV})}\right)^2 \tag{3.6}$$

$$\times \left\{\underbrace{B_6^{(1/2)}(1 - \Omega_{IB})}_{\text{QCD penguin}} - \underbrace{0.4\left(\frac{\bar{m}_t}{165\,\mathrm{GeV}}\right)^{2.5} B_8^{(3/2)}}_{\text{EW penguin}}\right\},$$

illustrates the difficulty that arises from a cancellation between strong and electroweak penguin contributions and the need to know the hadronic matrix elements $B_i \propto \langle \pi\pi | O_i | K \rangle$, which involve a two-pion final state, accurately. Before 1999 it was commonly, though not universally, assumed that $B_{6,8} \approx 1$ near their vacuum saturation value, and with the isospin breaking factor $\Omega_{IB} \approx 0.25$, this gives only about $6 \cdot 10^{-4}$. The experimental result has triggered a large theoretical activity over the past two years directed towards better understanding the hadronic matrix elements. Alternatively, extensions of the Standard Model have been invoked to explain a supposed enhancement of ϵ'/ϵ.

I am not in the position to review this activity in detail, a fraction of which has been represented at this conference [34, 35, 36, 37, 38]. Different approaches continue to disagree by large factors, but it appears now certain that serious hadronic matrix element calculations must account for final state interactions of the two pions. Since ϵ'/ϵ is now accurately known experimentally and could be used to constrain η, the CP-violating parameter of

the Standard Model, or new CP-violating interactions, it will be important in the future that theoretical calculations have control over the approximations involved. Chiral perturbation theory combined with a large-N_c matching of the operators in the non-leptonic Hamiltonian can probably go furthest towards this goal with analytic methods. The calculation reported in [39] finds $B_6^{(1/2)}$ enhanced by a factor 1.55 through $\pi\pi$ rescattering in the isoscalar channel, while $B_8^{(3/2)}$ remains close to 1. When this is combined with a re-evaluation of isospin breaking which gives Ω_{IB} smaller than before [40, 41, 42], chiral perturbation theory supplemented by a resummation of rescattering effects may account for the experimental result within theoretical uncertainties. However, the same approach does not explain the enhancement of the real part of the $\Delta I = 1/2$-amplitude known as the $\Delta I = 1/2$ rule. As a matter of principle, lattice QCD can settle the matrix element calculation definitively, since $K \to \pi\pi$ matrix elements computed in a lattice of finite volume, can be matched to continuum, infinite-volume matrix elements including all information on rescattering [43, 44]. Due to the potential cancellations the matrix elements will, however, be needed with high precision.

Despite the fact that ϵ'/ϵ cannot be computed with great precision at present and converted into a stringent test of the CKM structure, the fact that direct CP violation is large is important for model building. Also, in the Standard Model, ϵ'/ϵ provides further evidence that η, i.e. the CKM phase is sizeable.

3.3 Very rare kaon decays

There exist several proposals (E949 and KOPIO at Brookhaven; CKM and KAMI at Fermilab) to measure the very rare decays $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ with expected branching fractions of about $(7.5 \pm 3) \times 10^{-11}$ and $(2.6 \pm 1.2) \times 10^{-11}$, respectively. The first of these decays is CP-conserving and constrains (ρ, η) to lie on a certain ellipse in the (ρ, η)-plane. The second decay is CP-violating and determines η. The branching fractions are predicted theoretically with high precision: the short-distance corrections are known to next-to-leading order [45] and the hadronic matrix elements can be obtained from semi-leptonic kaon decays. The $K^+ \to \pi^+ \nu\bar{\nu}$ decay is predicted less accurately than $K_L \to \pi^0 \nu\bar{\nu}$ due to the presence of a charm contribution. However, potential non-perturbative $1/m_c^2$-corrections due to this charm contribution have been estimated to be small [46]. Therefore these two kaon modes alone, given precise measurements of these experimentally rather challenging decays, can fix the shape of the unitarity triangle precisely, or uncover inconsistencies with other constraints. In fact, the $K \to \pi\nu\bar{\nu}$ modes are themselves sensitive probes of modifications of the effective flavour-changing $\bar{s}dZ$ coupling. Two[1] $K^+ \to \pi^+ \nu\bar{\nu}$ events have been observed by the BNL E787 experiment [47] resulting in a branching fraction $\text{Br}(K^+ \to \pi^+ \nu\bar{\nu}) = (16^{+18}_{-8}) \times 10^{-11}$, somewhat larger than expected but consistent with expectations within the experimental error.

4. Constraints on the unitarity triangle

The CP-violating quantity ϵ in $K\bar{K}$ mixing, $|V_{ub}/V_{cb}|$, and the mass differences of the

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[1]The second event was published after the conference.

neutral B mesons, ΔM_{B_d}, ΔM_{B_s}, are used to constrain $(\bar{\rho}, \bar{\eta})$, the apex of the unitarity triangle.[2] I take here the point of view that this constrains the quantity $\sin(2\beta)$ indirectly. The range obtained is then compared with the direct measurement of $\sin(2\beta)$ (discussed in the next section). Alternatively, one could include the direct measurement of $\sin(2\beta)$ as a fifth observable into the fit. It is not the purpose of this talk to go into the details of the theoretical calculations that contribute to these constraints, since this would lead away from the topic of CP violation. Recent summaries of lattice calculations of the relevant hadronic parameters can be found in [49, 50, 51]. The status of the determination of $|V_{cb}|$ and $|V_{ub}|$ is reviewed in the talk by Ligeti [1].

The following equations summarize the four constraints in compact form. With $\lambda = 0.224$, $|V_{cb}| = 0.041 \pm 0.002$, $\epsilon = (2.280 \pm 0.019) \times 10^{-3}$ and $\Delta M_{B_d} = (0.487 \pm 0.014)\,\mathrm{ps}^{-1}$, and neglecting small errors, one obtains:

Observable	Constraint	Dominant error
$\left\lvert \dfrac{V_{ub}}{V_{cb}} \right\rvert$	$\sqrt{\bar{\rho}^2 + \bar{\eta}^2} = 0.37 \times \dfrac{\lvert V_{ub}/V_{cb}\rvert}{0.085}$	$\pm 20\%\ (\lvert V_{ub}\rvert)$
ΔM_{B_d}	$\sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} = 0.83 \times \dfrac{f_{B_d} B_{B_d}^{1/2}}{230\,\mathrm{MeV}}$	$\pm 15\%\ (f_{B_d} B_{B_d}^{1/2})$
$\dfrac{\Delta M_{B_d}}{\Delta M_{B_s}}$	$\sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} = 0.87 \times \dfrac{\xi}{1.16} \times \sqrt{17.5\,\mathrm{ps}^{-1}/\Delta M_{B_s}}$	$\pm 6\%\ (\xi)$
ϵ	$\bar{\eta}\,(1.31 \pm 0.05 - \bar{\rho}) = 0.35 \times 0.87/\hat{B}_K$	$\pm 15\%\ (\hat{B}_K)$

One notes that the dominant uncertainties are theoretical except, perhaps, for $|V_{ub}|$, for which the relative size of experimental and theoretical errors depends on the method of determination. Also the constraint from ΔM_{B_s} currently gives only an upper bound on one of the sides of the unitarity triangle, since only a lower bound on ΔM_{B_s} is measured.

It is in principle straightforward to fit $(\bar{\rho}, \bar{\eta})$ to the four observables listed above except for the fact that the dominant errors are theoretical and therefore do not (usually) admit a statistical interpretation. Different statistical procedures are being used ("frequentist", "Bayesian", "scanning", "Gaussian", etc.). Figure 2 shows two representatives of such global fits, one using a variant of the scanning approach [52] (upper panel), the other using the Bayesian (or inferential) approach [53] (lower panel).

The four quantities are seen to be in remarkable agreement and reveal no sign of inconsistency of the Kobayashi-Maskawa mechanism for CP violation. Also the various statistical procedures appear to give similar results when the same inputs are used (which is not strictly the case in the Figure above). The combined fit results in $\bar{\rho} = 0.21 \pm 0.17$, $\bar{\eta} = 0.35 \pm 0.14$ and an indirect determination of the angles of the unitarity triangle,

$$\sin(2\alpha) = -0.24 \pm 0.72, \quad \sin(2\beta) = 0.68 \pm 0.21, \quad \gamma = (58 \pm 24)^\circ, \qquad (4.1)$$

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[2]The definitions $\bar{\rho} = \rho\,(1 - \lambda^2/2)$, $\bar{\eta} = \eta\,(1 - \lambda^2/2)$ render the location of the apex accurate to order λ^5 [48] and will be used in the following.
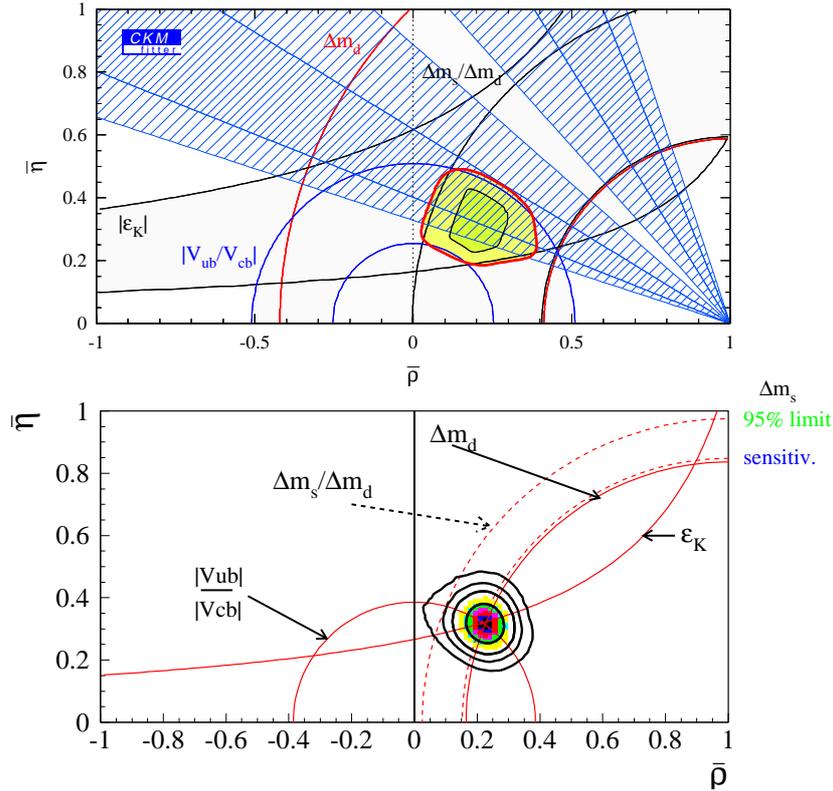
Figure 2: Two summaries of unitarity triangle constraints (excluding the direct measurement of $\sin(2\beta)$ which is overlaid in the upper panel) [52, 53].

where the errors should imply a 95% confidence level [52].

In the future one can expect improvements in this fit from a better measurement of $|V_{ub}|$ and from progress in lattice QCD on the relevant quantities. If ΔM_{B_s} is indeed in the range predicted by the Standard Model, it should be measured soon at the Tevatron collider, which then determines the top-quark side of the unitarity triangle much more accurately. One also expects that the direct measurement of $\sin(2\beta)$ will soon become one of the most stringent constraints on $(\bar{\rho}, \bar{\eta})$. Further information will come from non-leptonic B decays. These two topics are discussed below.

If inconsistencies between the various quantities should arise, it will be important to identify the culprit. The determinations of $|V_{ub}|$ from semileptonic decays and the phase $-\gamma$ of V_{ub} from non-leptonic decays with interference of $b \to c\bar{u}D$ (no weak phase) and $b \to u\bar{c}D$ (weak phase γ) tree transitions and their conjugates ($D = d, s$) are unlikely to be modified strongly by new flavour-changing interactions and hence determine $(\bar{\rho}, \bar{\eta})$ even in the presence of "New Physics". In contrast, most of our current knowledge on $(\bar{\rho}, \bar{\eta})$ derives from meson mixing (ϵ, $\Delta M_{B_{d,s}}$ and the direct measurement of $\sin(2\beta)$), which being second order in weak interactions is expected to be more sensitive to New Physics than non-leptonic decays. The current consistency of the quantities related to mixing is therefore even more auspicious. As already mentioned, by the end of this decade the unitarity triangle could be accurately determined from rare kaon decays alone, and also from B decays. Altogether,

the Kobayashi-Maskawa mechanism will be decisively and precisely tested.

5. Interpretation of sin(2β)

In 2001 CP violation has been observed for the first time in B meson decays, more precisely in the interference of mixing and decay. Assume that both, B^0 and $\bar{B}^0$, can decay into a CP eigenstate f, call the amplitude of the former decay A, the latter $\bar{A}$ and define $\lambda = e^{-i\phi_d}\bar{A}/A$ with $\phi_d = 2\beta$ the phase of the $B\bar{B}$ mixing amplitude in the Standard Model (standard phase convention). A B meson identified as B^0 at time $t = 0$ can decay into f at a later time t either directly or indirectly through its $\bar{B}^0$ component acquired by mixing. If there is CP violation, the amplitude for the CP conjugate process will be different, resulting in a time-dependent asymmetry

$$A_{\mathrm{CP}}(t) = \frac{\Gamma(\bar{B}^0(t) \to f) - \Gamma(B^0(t) \to f)}{\Gamma(\bar{B}^0(t) \to f) + \Gamma(B^0(t) \to f)}.$$
$$= \frac{2\mathrm{Im}\lambda}{1 + |\lambda|^2}\sin(\Delta M_B t) - \frac{1 - |\lambda|^2}{1 + |\lambda|^2}\cos(\Delta M_B t) \qquad (5.1)$$

In deriving this result it is assumed that the lifetime difference of the B meson eigenstates is negligible, which is a very good approximation for B_d mesons. One also assumes that CP violation in mixing (the corresponding ϵ-parameter for B mesons) is negligible. This has been verified at the percent level through the charge asymmetry in semi-leptonic decays. Alternatively, ϵ_B can be obtained from the time-dependent totally inclusive decay asymmetry [54]. When A is dominated by a single weak phase, $A = |A|e^{i\delta_W}$ (so that $|\lambda| = 1$), the time-dependent asymmetry (5.1) is proportional to $\mp\sin 2(\beta + \delta_W)$, the sign depending on the CP eigenvalue of f.

The final state $J/\psi K_S$ (and related ones) is unique in this respect, since the second term in the decay amplitude

$$A(\bar{B} \to J/\psi K) = V_{cb}V_{cs}^*(T - P) + V_{ub}V_{us}^*(T_u - P) \qquad (5.2)$$

is only of the order of a percent, since it is suppressed by λ^2 due to a small CKM factor and further suppressed by a penguin loop or $u\bar{u} \to c\bar{c}$ rescattering. Furthermore $\delta_W \approx 0$ for $b \to c\bar{c}s$. Hence the mixing-induced CP asymmetry in $B \to J/\psi K$ decay determines the $B\bar{B}$ mixing phase (relative to $b \to c\bar{c}s$), i.e. $\sin(2\beta)$ in the Standard Model, with little theoretical uncertainty [55, 56]. It determines the $B\bar{B}$ mixing phase also beyond the Standard Model, since it is unlikely that the CKM-favoured $b \to c\bar{c}s$ transition acquires a large CP-violating phase from new flavour-changing interactions. The assumption that the amplitude has only one dominant term can be partially checked by fitting a $\cos(\Delta M_B t)$-term to the time-dependent asymmetry and by searching for a CP-violating asymmetry in the charged $B^+ \to J/\psi K^+$ decay and its CP-conjugate.

The asymmetry is now precisely measured by the two B factories. The central values reported by both experiments have been increasing over the past year as the statistics of

the experiments improved and now reads [11, 12]

$$\sin(2\beta) = \begin{cases} 0.59 \pm 0.15 & \text{BaBar} \\ 0.99 \pm 0.15 & \text{Belle,} \end{cases} \tag{5.3}$$

yielding the world average[3] $\sin(2\beta) = 0.79 \pm 0.10$. The fact that this asymmetry is large and in agreement with the indirect determination of the angle β in (4.1) leads to two important conclusions on the nature of CP violation:

- CP is not an approximate symmetry of Nature (as could have been if CP violation in kaon decays were caused by some non-standard interaction).

- the Kobayashi-Maskawa mechanism of CP violation is most likely the dominant source of CP violation at the electroweak scale.

The $B\bar{B}$ mixing matrix element is given by $M_{12} - \Gamma_{12}/2$, where Γ_{12} is related to the lifetime difference and can be neglected and M_{12}, including a potential contribution from new flavour-changing interactions, can be written as

$$M_{12} = (V_{td}^* V_{tb})^2 \hat{M}_{12} + M_{\text{NP}}. \tag{5.4}$$

The non-standard contribution to $B\bar{B}$ mixing is constrained by the requirement that no conflict arises with the measurement of ΔM_{B_d} related to the modulus of M_{12}. There are two generic options for the non-standard contribution. The first is to assume that new flavour-violating interactions are still proportional to the CKM matrix so that M_{NP} is proportional to $(V_{td}^* V_{tb})^2$. In this case the time-dependent CP asymmetry in $B_d \to J/\psi K$ decay continues to determine $\sin(2\beta)$. The corresponding class of models is often referred to as "minimal flavour violation models" [57]. If one further assumes that the effective $|\Delta B| = 2$ interaction continues to be lefthanded, there is a strong correlation between modifications of $B\bar{B}$ mixing and $K\bar{K}$ mixing. One then finds that only small modifications of the $B\bar{B}$ mixing phase are possible given the constraints on $K\bar{K}$ mixing and $\Delta M_{B_{d,s}}$, in particular $\sin(2\beta) > 0.42$ [58, 59], and a preferred range from 0.5 to 0.8 [60]. An extension of this analysis which allows a modification of $\Delta M_{B_d}/\Delta M_{B_s}$ has been considered in [61, 62]. The second option relaxes the assumption that M_{NP} is proportional to $(V_{td}^* V_{tb})^2$, so that the time-dependent CP asymmetry in $B_d \to J/\psi K$ decay is no longer directly related to $\sin(2\beta)$. In general the new flavour-violating interactions then contain new CP phases and it is possible to arrange them such that the time-dependent CP asymmetry in $B_d \to J/\psi K$ decay can take any value. There has been much interest prior to this conference in exploring models that allow the asymmetry to be small, motivated in particular by the BaBar measurement $\sin(2\beta) = 0.12 \pm 0.38$ as of summer 2000, which has now been superseded by the result quoted above. Generic features of models with new CP- and flavour-violating interactions will be briefly described later, but the current status of the measurement does no longer mandate a detailed discussion of the "small-$\sin(2\beta)$" scenario.

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[3]This result has been updated by the new Belle data published shortly after the conference. At the time of the conference the Belle result was $\sin(2\beta) = 0.58^{+0.32+0.09}_{-0.34-0.10}$ resulting in the world average $\sin(2\beta) = 0.61 \pm 0.13$.

6. CP violation in B meson decays

The Kobayashi-Maskawa mechanism predicts large CP-violating effects only in B decays and very rare K decays. The primary focus of the coming years will be to verify the many relations between different observables predicted in the Kobayashi-Maskawa scenario, since the CKM matrix contains only a single phase.

An example of this type is the decay $\bar{B}_d \to \phi K$ due to the penguin $b \to s\bar{s}s$ transition at the quark level. In the Standard Model the time-dependent CP asymmetry of this decay is also proportional to $\sin(2\beta)$ to reasonable (though not as good) precision. However, new interactions are more likely to affect the loop-induced penguin transition than the tree decay $b \to c\bar{c}s$ and may be revealed if the time-dependent asymmetry in $\bar{B}_d \to \phi K$ turns out to be different from that in $\bar{B}_d \to J/\psi K$. However, if the difference is small, its interpretation requires that one controls the strong interaction effects connected with the presence of a small up-quark penguin amplitude with a different weak phase. This difficulty is of a very general nature in B decays.

6.1 CP violation in decay

The need to control strong interaction effects is closely related to the possibility of observing CP violation in the decay amplitude ("direct CP violation"). The decay amplitude has to have at least two components with different CP-violating ("weak") phases,

$$A(B \to f) = A_1 e^{i\delta_{S1}} e^{i\delta_{W1}} + A_2 e^{i\delta_{S2}} e^{i\delta_{W2}}. \tag{6.1}$$

If the CP-conserving ("strong interaction") phases are also different, the partial width of the decay differs from that of its CP-conjugate, $\Gamma(B \to f) \neq \Gamma(\bar{B} \to \bar{f})$. Many rare B decays are expected to exhibit CP violation in decay, because a given final state can often be reached by different operators $\mathcal{O}_i$ with different CKM factors from the weak effective Hamiltonian ("tree", "QCD, electroweak, and magnetic penguins") leading to the interference of a tree and a sizeable or even dominant penguin amplitude. The weak phase difference can only be determined, however, if the strong interaction amplitudes are known. This is also necessary for mixing-induced CP asymmetries, if the decay amplitude is not dominated by a single term. On the technical level, the matrix elements $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$ have to be known for a two-body final state. The problem is analogous to the computation of ϵ'/ϵ in kaon decay, except that now the initial state is heavy.

There exist two complementary approaches to obtain the strong interaction amplitudes, $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$. The first, "traditional" approach employs a general parameterization of the decay amplitudes of a set of related decays, implementing SU(2)-isospin relations. The remaining strong interaction parameters are then determined from data (often also using SU(3) flavour symmetry and "little" further assumptions on the magnitudes of some amplitudes). Very often this requires difficult measurements. The second approach attempts to calculate the strong interaction amplitudes directly from QCD with factorization methods also used in high-energy strong interaction processes. A systematic formulation of this approach has been given only recently [63] and makes essential use of the fact that the b quark mass is large. There is currently no theoretical framework that also covers

$1/m_b$-corrections systematically, so there is an intrinsic limitation to the accuracy that one can expect from this approach. In the following the "QCD factorization approach" to non-leptonic B decays is briefly described.

6.2 QCD factorization

In the heavy quark limit the b quark decays into very energetic quarks (and gluons), which must recombine to form two mesons. Using methods from the heavy quark expansion and soft-collinear factorization ("colour-transparency") one can argue that the amplitude of a decay into two light mesons assumes a factorized form [63, 64]. Schematically,

$$A(\bar{B} \to M_1 M_2) = F^{B \to M_1}(0) \int_0^1 du \, T^I(u) \Phi_{M_2}(u)$$
$$+ \int d\xi du dv \, T^{II}(\xi, u, v) \Phi_B(\xi) \Phi_{M_1}(v) \Phi_{M_2}(u), \tag{6.2}$$

where $F^{B \to M_1}$ is a form factor, Φ_X denote light-cone distribution amplitudes and $T^{I,II}$ are hard-scattering kernels, which can be computed in perturbation theory. (M_1 is the meson that picks up the spectator quark from the B meson.) This result extends the Brodsky-Lepage approach to exclusive hard processes [65], because it shows factorization also in the presence of soft interactions in the $B \to M_1$ form factor. (There exist a number of calculations of B decays based on the (controversial) assumption that the form factor is calculable in the Brodsky-Lepage approach [66, 67, 68, 69].) The formula above implies:

- There is no long-distance interaction between the constituents of the meson M_2 and the (BM_1) system. This is the precise meaning of factorization.

- The second line represents a hard-gluon interaction with the spectator quark and appears only at order α_s. At lowest order $T^I(u)$ is a constant proportional to the decay constant of M_2, so that $A(\bar{B} \to M_1 M_2) \propto i f_{M_2} F^{B \to M_1}(0)$ at lowest order. This reproduces "naive factorization" [70], but (6.2) implies that radiative corrections to this result can be computed.

- Final state rescattering is included in the hard-scattering kernels and therefore computable in the heavy-quark limit. In the heavy quark limit inelastic rescattering dominates and the sum of all rescatterings is dual to the partonic calculation.

The factorized form of (6.2) is valid up to $1/m_b$ corrections, some of which can be large. The extent to which the QCD factorization formalism can be of quantitative use is not yet fully known. It has been applied so far to a number of charmless two-body final states of pseudoscalar [71, 72, 73] and vector mesons [74, 75] as discussed in part at this conference [76, 77]. The QCD factorization approach has been successful in explaining the universality of strong-interaction effects in class-I $B \to D$ + light meson decays and in understanding the non-universality in the corresponding class-II decays [64, 78].[4] It also appears to account naturally for the magnitude of the πK branching fractions, sometimes

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[4]Non-leptonic final states with D mesons are reviewed at this conference in [1].

considered as unexpectedly large, but there is currently no test that would allow one to conclude that the computation of strong interaction phases which are either of order α_s or $1/m_b$ is reliable in the case of penguin-dominated final states [71]. Such tests will be possible soon. The non-observation of direct CP violation at the current level of sensitivity [2] supports the idea that strong rescattering effects are suppressed.

6.3 The angle α

The angle $\alpha = \pi - \beta - \gamma$ can be obtained from direct CP violation in decays with interference of $b \to u\bar{u}d$ (tree, γ) and $b \to d\bar{q}q$ (penguin, β) or from the mixing-induced asymmetry in decays based on $b \to u\bar{u}d$. If one takes the point of view that the $B\bar{B}$ mixing phase is determined experimentally by the mixing-induced CP asymmetry in $B_d \to J/\psi K$ decay irrespective of whether it is correctly described by the Standard Model, then the second method actually determines γ.

The time-dependent CP asymmetry in $B \to \pi^+\pi^-$ decay is an example of this type and determines α (or γ, depending on the point of view), but only if the penguin amplitude is neglected. This is now known not to be a good approximation. Neglecting only electroweak penguin amplitudes, the isospin amplitude system for the three, charged and neutral, $\pi\pi$ final states contains five real strong interaction parameters, just as many as there are independent branching fractions under the same assumption. Adding the time-dependent asymmetry gives a sixth observable that allows one to determine α up to discrete ambiguities [79]. Since this method requires a measurement of the small $B \to \pi^0\pi^0$ branching fractions, it has practical difficulties. Already bounds on the CP-averaged $\pi^0\pi^0$ branching fraction can be useful to constrain the amplitude system [80, 81, 82]. If $\mathrm{Br}(\pi^0\pi^0)$ is small, the strong phase of the penguin-to-tree ratio cannot be large. In fact $\mathrm{Br}(\pi^0\pi^0) = 0$ implies $\mathrm{Br}(\pi^+\pi^-) = 2\,\mathrm{Br}(\pi^\pm\pi^0)$. Conversely, a deviation from the last relation implies that $\mathrm{Br}(\pi^0\pi^0)$ cannot be too small. Further constraints on the $\pi\pi$ modes can be obtained only by assuming also SU(3) or U-spin symmetry. This relates, for example, $B_d \to \pi^+\pi^-$ to $B_s \to K^+K^-$. The inverted CKM hierarchy of penguin and tree amplitude in the second decay can in principle be used to determine γ from a combined measurement of the time-dependent and direct CP asymmetries in both decays [83].

Other methods have been devised that allow one to eliminate all hadronic parameters by measurements without the need to measure the $\pi^0\pi^0$ mode. One method uses the interference of CP-violating phases with CP-conserving phases from the resonant ρ-meson propagator in the decays $B_d \to \{\rho^+\pi^-, \rho^0\pi^0, \rho^-\pi^+\} \to \pi^+\pi^-\pi^0$ [84, 85]. The disadvantage of this method is that the analysis of the Dalitz plot of the three-particle final state requires comparatively large statistics. In addition, theoretical difficulties due to non-resonant pion production have not yet been fully removed.

The angle α can be determined from the time-dependent CP asymmetry in $B \to \pi^+\pi^-$ decay alone, if the relative magnitude of the penguin amplitude, P/T, can be computed. The asymmetry (5.1) is given by

$$A_{\mathrm{CP}}[\pi\pi](t) = S_{\pi\pi} \sin(\Delta M_{B_d}\, t) + A_{\mathrm{CP}}^{\mathrm{dir}}[\pi\pi] \cos(\Delta M_{B_d}\, t), \qquad (6.3)$$
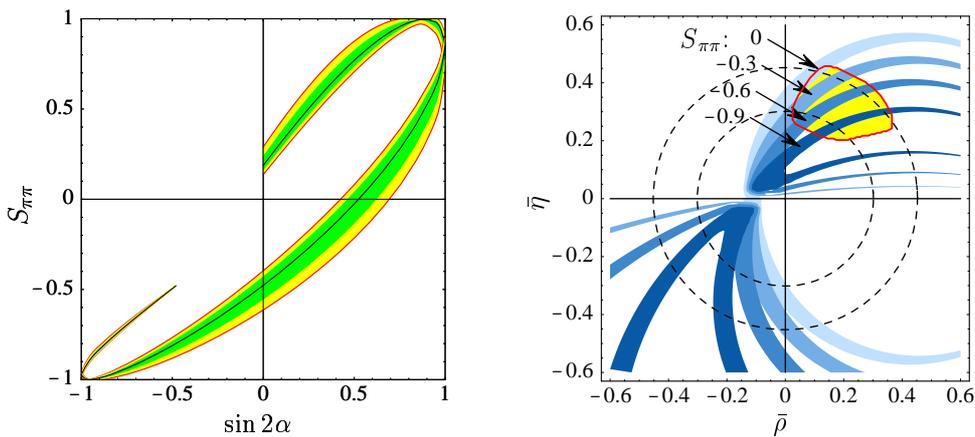
Figure 3: Correlation between the CP-asymmetry and $\sin(2\alpha)$ (left) and the corresponding constraint in the $(\bar{\rho}, \bar{\eta})$ plane (right) [71].

where $S_{\pi\pi} = \sin(2\alpha)$, if $P/T = 0$, in which case the direct CP asymmetry $A_{\rm CP}^{\rm dir}[\pi\pi]$ vanishes. The direct CP asymmetry is proportional to the sine of the strong phase of P/T and can be used as a phenomenological check of the computation of P/T. Figure 3 displays the correlation between the CP-asymmetry $S_{\pi\pi}$ and $\sin(2\alpha)$ (left) and the corresponding constraint in the $(\bar{\rho}, \bar{\eta})$ plane (right), when P/T is computed in the QCD factorization approach [71]. The Figure illustrates that even if theoretical (or experimental) uncertainties prevent an accurate determination of $\sin(2\alpha)$ in this way, the inaccurate result on $\sin(2\alpha)$ still translates into a useful constraint in the $(\bar{\rho}, \bar{\eta})$ plane. This reflects the fact that other observables do not constrain $\sin(2\alpha)$ very well as seen from (4.1).

6.4 The angle γ

The preferred methods to determine directly the angle γ, the phase of V_{ub}^*, rely on decays with interference of $b \to c\bar{u}D$ (no phase) and $b \to u\bar{c}D$ (phase γ) transitions and their conjugates ($D = d, s$). These decays receive no penguin contributions and are arguably insensitive to new flavour-changing interactions. γ can be extracted from either of the following decay classes, $B_d(t) \to D^\pm \pi^\mp$ [86] (or more recent variants [87]), $B_d(t) \to DK_S$ [88], $B^\pm \to K^\pm D_{\rm CP}$ [89], $B_s(t) \to D_s^\pm K^\mp$ [90], since every one of them provides sufficiently many observables to eliminate all strong interaction parameters. None of these strategies is simple to carry out experimentally, however, since they involve either small CP asymmetries, or small branching fractions, or disparate amplitudes, or rapid B_s oscillations.

The possibility to determine γ from decays with interference of $b \to u\bar{u}D$ (tree, phase γ) and $b \to Dq\bar{q}$ (penguin, phase 0 ($D = s$), β ($D = d$)) transitions has therefore been thoroughly investigated recently, in particular the decays $B \to \pi K$. The branching fractions for these modes are of order 10^{-5} and have already been measured with an error of $\pm(10 - 20)\%$ [2], including first measurements of direct CP asymmetries (all compatible with zero). The drawback of these and related modes is that the amplitudes contain more strong interaction parameters than there are observables. SU(3) symmetry and the

structure of the weak effective Hamiltonian allow one to construct a number of interesting bounds on γ [91, 92], but a full understanding of these modes requires a calculation of the penguin-to-tree amplitude ratio including its strong rescattering phase.

The $B \to \pi K$ decays are penguin-dominated, because the tree amplitudes are CKM suppressed. The final states $\pi^0 K^\pm$ and $\pi^0 K^0$ have significant electroweak penguin contributions. The amplitude system contains 11 real strong interaction parameters. Flavour symmetry is useful to constrain some of the amplitude parameters:

- Isospin symmetry implies [93]

$$\mathrm{Br}(\pi^0 \bar{K}^0) = \frac{\mathrm{Br}(\pi^+ K^-)\mathrm{Br}(\pi^- \bar{K}^0)}{4\mathrm{Br}(\pi^0 K^-)} \times \left\{ 1 + O(\epsilon^2) \right\}, \tag{6.4}$$

where $\epsilon \sim 0.3$ is related to the tree-to-penguin ratio.[5] Unless the correction term is unexpectedly large this relation suggests a $\pi^0 K^0$ branching fraction of order 6×10^{-6}, about a factor $1.5 - 2$ smaller than the current measurements.

- SU(3) or U-spin symmetry imply:

   a. The dominant electroweak penguin amplitude is determined [92].

   b. The magnitude of the tree amplitude for $I = 3/2$ final states is related to $\mathrm{Br}(\pi^\pm \pi^0)$.

   c. Rescattering and annihilation contributions to the (otherwise) pure penguin decay $B^+ \to \pi^+ K^0$ are constrained by $\mathrm{Br}(K^+ K^0)$, where they are CKM enhanced relative to the penguin amplitude [94].

SU(3) flavour symmetry together with a few further dynamical assumptions (detailed below) suffice to derive bounds on γ from CP-averaged branching fractions alone. The inequality [91]

$$\sin^2 \gamma \leq \frac{\tau(B^+)}{\tau(B_d)} \frac{\mathrm{Br}(\pi^+ K^-)}{\mathrm{Br}(\pi^- \bar{K}^0)} \equiv R \tag{6.5}$$

excludes γ near $90°$ if $R < 1$ and is derived upon assuming that the rescattering contribution mentioned above and a colour-suppressed electroweak penguin amplitude are negligible. Current data give $R = 1.06 \pm 0.18$. The ratio of charged decay modes satisfies [92] (neglecting again the rescattering contribution to $B^+ \to \pi^+ K^0$ which appears here suppressed by a factor $\bar{\epsilon}_{3/2}$)

$$2 \cdot \frac{\mathrm{Br}(\pi^0 K^-)}{\mathrm{Br}(\pi^- \bar{K}^0)} \equiv R_*^{-1} \leq \left(1 + \bar{\epsilon}_{3/2} \left| q - \cos\gamma \right| \right)^2 + \bar{\epsilon}_{3/2}^2 \sin^2 \gamma, \tag{6.6}$$

where q and $\bar{\epsilon}_{3/2}$ are determined according to a. and b. above, respectively. This bound is particularly interesting, since, if $R_*^{-1} > 1$, it excludes a region in γ around $55°$, which is favoured by the indirect unitarity triangle constraints (4.1). Current data give $R_*^{-1} =$

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[5]Here and in the remainder of this section "Br" always refers to branching fractions averaged over a decay mode and its CP-conjugate.
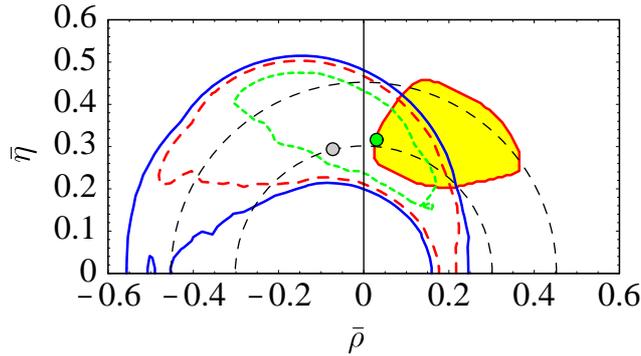
Figure 4: 95% (solid), 90% (dashed) and 68% (short-dashed) confidence level contours in the $(\bar{\rho}, \bar{\eta})$ plane obtained from a global fit to the CP averaged $B \to \pi K, \pi\pi$ branching fractions, using the scanning method as described in [52]. The right dot shows the overall best fit, whereas the left dot indicates the best fit for the default hadronic parameter set. The light-shaded region indicates the region preferred by the standard global fit, excluding the direct measurement of $\sin(2\beta)$.

1.40 ± 0.23. This prefers $\gamma > 90°$, but the error is still too large to speculate about the implications of this statement. A similar reasoning applies to the final states $\pi^+ K^-$, $\pi^0 K^0$ and their CP-conjugates in the decay of the neutral B_d and $\bar{B}_d$ mesons [95], for which the time-dependent CP-asymmetry $B_d(t) \to \pi^0 K_S$ provides an additional observable that could be used to constrain the system of hadronic quantities. Eq. (6.6) can be turned into a determination of γ if one assumes that the strong phase of the tree amplitude relative to the penguin amplitude is not too large [71]. This assumption is justified by theoretical calculations as discussed next, but will eventually be verified experimentally by the observation of small direct CP asymmetries.

The possibility to compute strong interaction effects in non-leptonic $B \to \pi K, \pi\pi$ decays with the QCD factorization method and to determine γ has been investigated in detail [71]. Figure 4 shows the result of a global fit of $(\bar{\rho}, \bar{\eta})$ to CP-averaged $B \to \pi K, \pi\pi$ branching fractions. The result is consistent with the standard fit (overlaid light-shaded region) based on meson mixing and $|V_{ub}|$, but shows a preference for larger γ or smaller $|V_{ub}|$. If the estimate of the theory uncertainty (included in the curves in the Figure) is correct, non-leptonic decays together with $|V_{ub}|$ from semileptonic decays already imply the existence of a CP-violating phase of V_{ub} at the 2-3 σ level. Similar conclusions have been obtained in [96, 97] with different theory inputs and no attempt to quantify the theoretical error. On the other hand the analysis in [98] sacrifices some theoretical input and enlarges the fit by the corresponding hadronic parameter to conclude that no determination of γ is possible from $B \to \pi K, \pi\pi$ decays.

7. Beyond B and K decays

CP violation may occur outside the K and B meson systems, but the pattern of flavour-changing interactions implied by the CKM matrix leads to the conclusion that only null effects are expected in the Standard Model at the current levels of experimental sensitivity.

CP-violating observables in D meson decays and flavour-conserving reactions could therefore provide unambiguous evidence for extensions of the Standard Model which contain new flavour-changing interactions and new sources of CP violation. Here is a brief list of topics which had to be omitted in this talk:[6]

Charmed mesons. $D\bar{D}$ mixing is strongly suppressed by the GIM mechanism. CP-violating phenomena are further suppressed by small CKM factors, so that CP violation is small in mixing and direct CP asymmetries in charm decays are expected to be at most at the permille level. From a theoretical point of view the properties of charm mesons are especially difficult to compute reliably, since neither chiral perturbation theory nor the heavy quark limit are useful. The charm system is good for order-of-magnitude effects in extensions of the Standard Model and the experimental bounds are beginning to approach an interesting region for such effects.

Flavour-conserving CP violation. CP violation without flavour violation is closely related to the strong CP problem in the Standard Model. Once $\tilde{\theta}$ is set to zero by *fiat*, flavour-conserving CP violation induced by the CKM matrix is unobservably small. The situation is very different in extensions of the Standard Model, which can contain flavour-conserving CP-violating interactions at tree level, for instance in the scalar potential. For this reason bounds on electric dipole moments of the neutron and the leptons put very important constraints on such extensions. At future colliders one can search for CP violation in top quark and Higgs interactions in high energy collisions. Interesting signals are only expected in extensions of the Standard Model. These topics are reviewed in [99].

CPT violation. CPT has been assumed to be a good symmetry in this talk and no distinction between CP and T violation has been made. As is well-known, CPT symmetry is a general consequence of locality and relativistic invariance in quantum field theories. Insisting on locality of the effective field theory, CPT violation is most naturally discussed as a consequence of broken Poincaré invariance. At this conference consequences of CPT non-invariance in the evolution and decay of entangled meson-antimeson states [100] and the possibility of an anomalous CPT symmetry in theories with chiral fermions [101] have been presented.

8. CP violation in extensions of the Standard Model

The emerging success of the Kobayashi-Maskawa mechanism of CP violation is sometimes accompanied by a sentiment of disappointment that the Standard Model has not finally given way to a more fundamental theory. The implications of this success are, perhaps, more appreciated, when it is viewed from the perspective of the year 1973, when the mechanism was conceived. After all, the Kobayashi-Maskawa mechanism predicted a new generation of particles on the basis of the tiny and obscure effect of CP violation in $K\bar{K}$ mixing. It then predicted relations between CP-violating quantities in K, D, B-physics which *a priori* might be very different. The fact that it has taken nearly 30 years to assemble the experimental tools to test this framework does not diminish the spectacular fact that once again Nature has realized a structure that originated from pure reasoning.

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[6]The experimental and theoretical status of $D\bar{D}$ mixing has been reviewed in Ligeti's talk [1].

Nevertheless several arguments make it plausible that the Kobayashi-Maskawa mechanism is not the final word on CP violation. The strong and cosmological CP problem (baryogenesis) continue to call for an explanation, probably related to high energy scales. There may be an aesthetic appeal to realizing the full Poincaré group as a symmetry of the Lagrangian, in which case CP and P symmetry breaking must be spontaneous. One of the strongest arguments is, however, that the electroweak hierarchy problem seems to require an extension of the Standard Model at the TeV scale. Generic extensions have more sources of CP violation than the CKM matrix. These have not (yet) been seen, suggesting that there is some unknown principle that singles out the CKM matrix as the dominant source of flavour and CP violation. In the following I give a rather colloquial overview of CP violation in generic extensions of the Standard Model at the TeV scale. This is perhaps an academic catalogue, but it illustrates how restrictive the Kobayashi-Maskawa framework is.

8.1 Extended Higgs sector

Extending the Higgs sector by just a second doublet opens many new possibilities. The Higgs potential may now contain complex couplings, leading to Higgs bosons without definite CP parity, to CP violation in charged Higgs interactions, flavour-changing neutral currents, and CP violation in flavour-conserving interactions such as $t\bar{t}H$ and electric dipole moments. The Lagrangian could also be CP-conserving with CP violation occurring spontaneously through a relative phase of the two Higgs vacuum expectations values [4].

Without further restrictions both scenarios already cause too much CP violation and flavour-changing neutral currents, so that either the Higgs bosons must be very heavy or some special structure imposed. For example, discrete symmetries may imply that up-type and down-type quarks couple to only one Higgs doublet, a restriction known as "natural flavour conservation" [102, 103], since it forbids flavour-changing neutral currents (and also makes spontaneous CP violation impossible with only two doublets). With flavour conservation imposed, CP and flavour violation occur through the CKM matrix, but in addition to the usual charged currents also in charged Higgs interactions. This is usually considered in the context of supersymmetry, since extended Higgs models suffer from the same hierarchy problem as the Standard Model.

8.2 Extended gauge sector (left-right symmetry)

Left-right-symmetric theories with gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ are attractive [104], because parity and CP symmetry can be broken spontaneously. The minimal model requires already an elaborate Higgs sector (with triplets in addition to doublets) and suffers from the hierarchy problem. CP violation in the quark sector now occurs through left- and right-handed charged currents with their respective CKM matrices. But since all CP violation arises through a single phase in a Higgs vacuum expectation value, there is now a conflict between suppressing flavour-changing currents to a phenomenologically acceptable level and the need to make the effects of this phase large enough to generate the CP violating phenomena already observed. The minimal left-right symmetric model is therefore no longer viable [105, 106].

Spontaneous CP symmetry breaking is excluded in the minimal supersymmetric extension of the Standard Model, but it is an option, when the model is augmented by an extra gauge singlet superfield (next-to-minimal supersymmetric standard model) or by the choice of an enlarged Higgs sector, as discussed at this conference [107, 108].

8.3 Extended fermion sector

The Standard model can be extended by an extra d-type quark with electric charge $-1/3$ [109]. This quark should be a weak singlet in order not to conflict electroweak precision tests. After electroweak symmetry breaking, the down-quark mass matrix must be diagonalized by a unitary 4×4 matrix. The motivation for such an extension of the Standard Model may be less clear, in particular as there is no symmetry principle that would make the extra singlet-quark naturally light. However, this theory provides an example in which the unitarity "triangle" does no longer close to a triangle, but is extended to a quadrangle:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + \underbrace{V_{td}V_{tb}^*}_{\approx 8 \cdot 10^{-3}} + \underbrace{U_{db}}_{< 10^{-3}} = 0 \qquad (8.1)$$

The unitarity triangle "deficit" U_{db} also determines the strength of tree-level flavour-changing Z boson couplings and is currently constrained by $B\bar{B}$ mixing and rare decays to about a tenth of the length of a side of the triangle. (The corresponding coupling U_{ds} is constrained much more tightly in the kaon system by the non-observation of $K \to \mu^+\mu^-$ decay.) This model could in principle still give large modifications of $B\bar{B}$ mixing and non-leptonic B decays, including CP asymmetries [110].

8.4 Supersymmetry

The minimal supersymmetric standard model [111, 112] is arguably the most natural solution to the electroweak hierarchy problem, but it is not particularly natural in its most general form from the point of view of CP violation. The Lagrangian including the most general renormalizable operators that break supersymmetry softly contains 44 CP-violating constants of nature. (This does not yet include a neutrino mass matrix and also assumes that R-parity is conserved.) One of them is the usual CKM phase which appears in charged current and chargino interactions. Three phases appear in flavour-conserving CP observables, 27 in flavour- and CP-violating quark-squark-gluino interactions (squark mass matrices and A-terms) and 13 in the (s)lepton sector.

The flavour-conserving phases must be small to comply with the non-observation of electric dipole moments. An intriguing feature of supersymmetry is the existence of CP and flavour violation in strong interactions, that is the possibility of a flavour-changing, CP-violating quark-squark-gluino vertex. These interactions can be much stronger than the Standard Model weak interactions and to suppress them to a phenomenologically acceptable level, one has to assume that either (some of) the masses of superparticles are rather large, or that the squark mass matrices are diagonal in the same basis that also diagonalizes the quark mass matrices (alignment) or that the squarks have degenerate masses, in which case a generalization of the GIM mechanism suppresses flavour-changing couplings

[113, 114]. Since almost all CP-violating phases of the minimal supersymmetric standard model originate from supersymmetry breaking terms, one must understand supersymmetry breaking to answer the question why CP and flavour violation are so strongly suppressed. There exist mechanisms which can naturally realize one or the other of the conditions listed above (for example supersymmetry breaking through gauge interactions [115]), but none of the mechanisms is somehow singled out.

There is currently much activity aiming at constraining the flavour- and CP-violating couplings from the many pieces of data that become now available. In fact these couplings are so many-fold that the CP-violating effects observed in kaon and B meson decays can all be ascribed to them (allowing, in particular, the CKM phase to be small) at the price of making the consistency of the Kobayashi-Maskawa mechanism appear accidental. For example, several mechanisms, making use of flavour-changing strong interactions, have been proposed that could enlarge ϵ'/ϵ [116, 117, 118] without conflicting other data, and the impact of these interactions on B decays has been discussed in two talks at this conference [119, 120]. The hope could be that eventually some pattern of restrictions on these small couplings will be seen that could give a hint on the origin of supersymmetry breaking. It is also plausible to assume that strong flavour and CP violation is absent (or too small to observe) in supersymmetry for one or the other yet unknown reason. Neglecting also the flavour-conserving CP-violating effects, the CKM matrix is then the only effect of interest. The presence of additional particles with CKM couplings still implies modifications of meson mixing and rare decays, but these modifications are now much smaller and, in general (but excepting rare radiative or leptonic decays), precise theoretical results are needed to disentangle them from hadronic uncertainties.

Whatever the outcome of the search for new CP violation may be, it will restrict the options for model building severely. The current data point towards a privileged standing of the CKM matrix. However, a theoretical rationale for this privileged standing is yet to be discovered.

9. Conclusions

I. The (expected) observation of large CP violation in B decays together with ϵ'/ϵ and the consistency of indirect determinations of the unitarity triangle imply that:

- CP is not an approximate symmetry of Nature – rather CP violation is rare in the Standard Model because of small flavour mixing.

- the Kobayashi-Maskawa mechanism of CP violation in charged currents is *probably* the *dominant* source of CP violation *at the electroweak scale*.

II. CP and electroweak symmetry breaking provide complementary motivations to search for extensions of the Standard Model, but:

- on the one hand, there exists no favoured candidate model for CP violation beyond the Standard Model – rather there is a CP problem in many conventional extensions.

  - on the other hand, baryogenesis requires CP violation beyond the Standard Model, probably decoupled from CP violation observable at accelerators.

III. The study of CP violation is at a turning point with many new experimental capabilities and new theoretical methods to interpret non-leptonic decay data. Perhaps the most important result of the near future, however, will be to find (or not find) the B_s mass difference $\Delta M_{B_s} \approx 17.5\,\text{ps}^{-1}$, confirming once more the Standard Model paradigm (or to put it into serious difficulty).

Acknowledgement

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