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Workshop on Integrable Theories, Solitons and Duality

BPS Z_k strings, string tensions and confinement in non-Abelian theories

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Abstract: In this talk we review some generalizations of 't Hooft and Mandelstam ideas on confinement for theories with non-Abelian unbroken gauge groups. In order to do that, we consider N=2 super Yang-Mills with one flavor and a mass breaking term. One of the spontaneous symmetry breaking is accomplished by a scalar that can be in particular in the representation of the diquark condensate. We analyze the phases of the theory. In the superconducting phase, we show the existence of BPS Z_k-strings and calculate exactly their string tension in a straightforward way. We also find that magnetic fluxes of the monopole and Z_k-strings are proportional to one another allowing for monopole confinement in a phase transition. We further show that some of the resulting confining theories can be obtained by adding a deformation term to $N = 2$ or $N = 4$ superconformal theories.

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1. Introduction

One of the oldest open problems in particle physics is the quark confinement. It is believed that it could be explained as being a phenomena dual to a non-Abelian generalization of Meissner effect, as was proposed by 't Hooft and Mandelstam many years ago [1]. Some progress has been made by Seiberg and Witten [2], who starting from an $N = 2$ $SU(2)$ supersymmetric theory obtained an effective $N = 2$ super QED with an $N = 2$ mass

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breaking term. In this theory, the $U(1)$ is broken to a discrete group and as it happens the theory develops string solutions and confinement of (Abelian) electric charges occurs. Since then, many vary interesting works appeared[3]. However, usually it is considered that the gauge group is completely broken to $U(1)^{\text{rank}(G)}$ and then to its discrete center by Higgs mechanism. Therefore these theories don't have $SU(3) \times U(1)_{\text{em}}$ as subgroup of the unbroken gauge group and the monopoles belong to $U(1)$ representations and not to representations of non-Abelian groups.

In this talk, we shall review [4] and [5] where we generalize some of the 't Hooft and Mandelstam ideas to non-Abelian theories but avoiding the mentioned problems. In order to do that, we consider $N = 2$ super Yang-Mills with a breaking mass term and with arbitrary simple gauge group which is broken to non-Abelian residual gauge group. One of the spontaneous symmetry breaking is produced by a complex scalar ϕ that could be for example in the symmetric part of the tensor product of k fundamental representations. In particular if $k = 2$, this scalar is in the representation of a diquark condensate and therefore it can be thought as being itself the condensate. We therefore could consider this theory as being an effective theory. The non-vanishing expectation value of ϕ gives rise to a monopole confinement, like in the Abelian-Higgs theory. We shall show that, by varying a mass parameter m, we can pass from an unbroken phase to a phase with free monopoles and then to a superconducting phase with Z_k-strings and confined monopoles. In the free-monopole phase, there exist (solitonic) monopole solutions which are expected to fill irreducible representations of the dual unbroken gauge group[6]. In this phase we recover $N = 2$ supersymmetry and show that some of these theories are conformal invariant. In the superconducting phase we shall prove the existence of BPS Z_k-string solutions and calculate exactly their string tensions. We also show that the fluxes of the magnetic monopoles and strings are proportional to one another and therefore the monopoles can get confined. From the values of the magnetic fluxes we calculate the threshold length for the string breaking, producing a new monopole-antimonopole pair. In our theory the bare mass μ of ϕ is not required to satisfy $\mu^2 < 0$ in order to have spontaneous symmetry breaking. Therefore in the dual formulation, where one could interpret ϕ as the monopole condensate, when $k = 2$, we don't need to have a monopole mass satisfying the problematic condition $M_{\text{mon}}^2 < 0$ mentioned by 't Hooft[7].

2. Confinement in Abelian-Higgs theory

Due to the broad audience in this conference, let us review the BPS string solutions in Abelian-Higgs theory and the basic ideas of 't Hooft and Mandelstam on confinement[1].

As is well known, a superconductor is described by the BCS theory. In this theory it happens a condensation of electron pairs. This condensate is associated to a complex scalar field whose dynamics is governed by the Abelian-Higgs Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\phi^* D^\mu\phi - V(\phi)\,, \qquad (2.1)$$

[1]For a review in those subjects see [8] and [7].

where $D_\mu\phi = \partial_\mu\phi + iq_\phi A_\mu\phi$ and

$$V(\phi) = \frac{\lambda}{8}\left(|\phi|^2 - a^2\right) .$$

The constant q_ϕ is the electric charge of ϕ. In particular if $\phi^\dagger$ is a condensate of electron pairs, then $q_\phi = 2e$. In this case, when $a^2 > 0$, the $U(1)$ gauge group is broken to Z_2. In that phase, Abelian-Higgs is the effective theory which describes normal superconductors. Since in this phase $\Pi_1(U(1)/Z_2)$ is nontrivial we can have string solutions. In order to obtain these solutions we shall look for a static configuration, with cylindrical symmetry around the z-axis and the only non-vanishing component of the field strength is $B_3 \equiv -F_{12}$. In order that the string have a finite string tension T (i.e. energy per unit length), when the radial coordinate $\rho \to \infty$, the string solution must satisfy the vacuum equations

$$\begin{aligned} D_\mu\phi &= 0\,, \\ V(\phi) &= 0\,, \\ F_{\mu\nu} &= 0\,. \end{aligned} \tag{2.2}$$

Then, one can obtain the string tension lower bound [9] (for a review see [8])

$$T \geq \frac{1}{2} q_\phi a^2 \left|\Phi_{\rm st}\right| , \tag{2.3}$$

where

$$\Phi_{\rm st} \equiv \int d^2x\, B_3 = -\oint dl_I\, A_I\,, \quad I = 1,2\,,$$

is the string magnetic flux. We shall adopt the convention that capital Latin indices always denote the coordinates $I = 1,2$. From the boundary conditions (2.2), it follows that at $\rho \to \infty$

$$\begin{aligned} |\phi| = a \;\to\; \phi(\varphi) &= a e^{i\beta(\varphi)}\,, \\ D_I\phi = 0 \;\to\; A_I(\varphi) &= \frac{i}{q_\phi}\phi^{-1}\partial_I\phi = \frac{\epsilon_{IJ}x^J}{q_\phi\rho^2}\partial_\varphi\beta\,, \end{aligned} \tag{2.4}$$

where $\beta(\varphi)$ can be a multi-valued function. But since $\phi(\varphi)$ is single valued

$$\beta(\varphi + 2\pi) - \beta(\varphi) = 2\pi n\ , \ \text{ where } n \in Z\,.$$

Then,

$$\Phi_{\rm st} = \frac{2\pi}{q_\phi} n\,, \tag{2.5}$$

and it results that

$$T \geq a^2\pi\,|n|\ . \tag{2.6}$$

The bound is saturated when[9]

$$\begin{aligned} D_0\phi \;=\; D_3\phi &= 0\,, \\ D_\pm\phi &= 0\,, \\ B_3 \pm \frac{q_\phi}{2}\left(\phi^*\phi - a^2\right) &= 0\,, \\ V(\phi) &= \frac{q_\phi^2}{8}\left(|\phi|^2 - a^2\right)^2 \end{aligned} \tag{2.7}$$

with "+" if $n > 0$ and "−" if $n < 0$ and where $D_\pm \equiv D_1 \pm iD_2$. The last relation implies the constraint on the couplings of the theory $\lambda = q_\phi^2$. This constraint appear in $N = 2$ super-Maxwell. In order for this Lagrangian to be $N = 2$ super-Maxwell we just need to introduce some extra fields (note that since ϕ has a non-vanishing electric charge it should belong to the hypermultiplet and not to the $N = 2$ vector supermultiplet). One can show that the solutions of these equations satisfy automatically the equations of motion.

The string ansatz is constructed by multiplying the asymptotic configuration (2.5) by arbitrary functions of ρ[10],

$$\begin{aligned} \phi(\varphi, \rho) &= f(\rho) a e^{in\varphi} , \\ A_I(\varphi, \rho) &= g(\rho) \frac{n}{q_\phi \rho^2} \epsilon_{IJ} x^J , \end{aligned}$$

and, in order to recover the asymptotic configuration at $\rho \to \infty$, we consider the boundary condition

$$g(\infty) = 1 = f(\infty) .$$

On the other hand, in other to kill the singularities at the origin, we consider the boundary condition

$$g(0) = 0 \text{ and } f(0) = 0.$$

Putting this ansatz in the BPS conditions, it results the first order differential equations

$$\begin{aligned} g'(\rho) &= \mp \frac{q_\phi^2 a^2 \rho}{2n} \left(|f(\rho)|^2 - 1 \right) , \\ f'(\rho) &= \pm \frac{n}{\rho} \left(1 - g(\rho)\right) f(\rho) . \end{aligned} \tag{2.8}$$

Although they don't have an analytic solution, Taubes has proven [11] the existence of a solution to these differential equations with the above boundary conditions. Moreover, he showed that all solutions to the full static equations are solutions to BPS equations, if $\lambda = q_\phi^2$. Since that BPS string solution has the lowest value of the string tension in a given topological sector, it is automatically stable.

't Hooft and Mandelstam [1] had the idea that if one puts a (Dirac) monopole and antimonopole in a superconductor, their magnetic lines could not spread over space but must rather form a string which gives rise to a confining potential between the monopoles. This idea only makes sense since the (Dirac) monopole magnetic flux is

$$\Phi_{\text{mon}} = g = 2\pi/e ,$$

which is consistent with the string's magnetic flux quantization condition (2.5), allowing one to attach to the monopole two strings with $n = 1$, when $q_\phi = 2e$. Then, using the electromagnetic duality of Maxwell theory one could map this monopole confining system to an electric charge confining system.

3. The BPS conditions for strings in non-Abelian theories

Let us now generalize some of these ideas to a non-Abelian theory[4][5]. For simplicity, let us consider an arbitrary gauge group G which is simple, connected and simply-connected. We shall consider a Yang-Mills theory with a complex scalar S in the adjoint representation and another complex scalar ϕ. We consider a scalar S in the adjoint representation because in a spontaneous symmetry breaking it produces an exact symmetry group G_S with a $U(1)$ factor, which allows the existence of monopole solutions. Additionally, another motivation for having a scalar in the adjoint representation is because with it, we can form an $N = 2$ vector supermultiplet and, like in the Abelian-Higgs theory, the BPS string solutions appear naturally in a theory with $N = 2$ supersymmetry. Moreover, in a theory with particle content of $N = 2$ supersymmetry, the monopole spin is consistent with the quark-monopole duality[12] which is another important ingredient in 't Hooft and Mandelstam's ideas. However, with S in the adjoint, we can not in general produce a spontaneous symmetry breaking which has a non-trivial first homotopy group of the vacuum manifold, which is a necessary condition for the existence of a string. One way to produce a spontaneous symmetry breaking satisfying this condition is to introduce a complex scalar ϕ in a representation which contains the weight state $|k\lambda_\phi\rangle$ [13], where k is an integer greater or equal to two, and λ_ϕ a fundamental weight. We can have at least three possibilities: one is to consider ϕ in the representation with $k\lambda_\phi$ as highest weight, which we shall denote $R_{k\lambda_\phi}$. We can also consider ϕ to be in the direct product of k fundamental representations with fundamental weight λ_ϕ, which we shall denote $R^{\otimes}_{k\lambda_\phi}$. Finally a third possibility would be to consider ϕ in the symmetric part of $R^{\otimes}_{k\lambda_\phi}$, called $R^{\text{sym}}_{k\lambda_\phi}$, which always contains $R_{k\lambda_\phi}$. This last possibility has an extra physical motivation that if $k = 2$, it corresponds to the representation of a condensate of two fermions (which we naively will call quarks) in the fundamental representation with fundamental weight λ_ϕ, and we can interpret ϕ as being this diquark condensate, similarly to the Abelian theory. In this case, when ϕ takes a non-trivial expectation value, it also gives rise to a mass term for these quarks. We shall see that ϕ will be responsible for the monopole confinement. Therefore, in this case we shall have that a nonzero vacuum expectation value of the diquark condensate gives rise to monopole confinement in a non-Abelian theory. If we are considering the dual theory, ϕ could be interpreted as a monopole condensate and it would give rise to a quark confinement.

In order to have $N = 2$ supersymmetry, we should need another complex scalar to be in the same hypermultiplet as ϕ. For simplicity's sake, however, we shall ignore it setting it to zero.

Let us then consider the Lagrangian

$$L = -\frac{1}{4}G_a^{\mu\nu}G_{a\mu\nu} + \frac{1}{2}\left(D_\mu S\right)_a^*\left(D^\mu S\right)_a + \frac{1}{2}\left(D_\mu\phi^\dagger\right)\left(D^\mu\phi\right) - V(S,\phi) \qquad (3.1)$$

with potential given by

$$V(S,\phi) = \frac{1}{2}\left(Y_aY_a + F^\dagger F\right)$$

where

$$\begin{aligned} Y_a &= \frac{e}{2}\left(\phi^\dagger T_a \phi + S_b^* i f_{abc} S_c - m\left(\frac{S_a + S_a^*}{2}\right)\right), \\ F &= e\left(S^\dagger - \frac{\mu}{e}\right)\phi\,. \end{aligned}$$

This potential is the bosonic part of $N=2$ super Yang-Mills with one flavor (when one of the aforementioned scalars of the hypermultiplet is put equal to zero). The parameter μ gives a bare mass to ϕ and m gives a bare mass to the real part of S which softly breaks $N=2$ SUSY. The parameter m also is responsible for spontaneous gauge symmetry breaking and, as for the mass parameter a in the Abelian case, one can consider it as a function of temperature. In [4], we started with a generic potential and have shown that in order to obtain the string BPS conditions, the potential is constrained to have this form with $N=2$ SUSY like in the Abelian case.

In order that the string tension T be finite, the string configuration at $\rho \to \infty$ must satisfy the vacuum equations

$$\begin{aligned} D_\mu S = D_\mu \phi &= 0\,, \\ V(S,\phi) &= 0\,, \\ G_{\mu\nu} &= 0\,. \end{aligned} \tag{3.2}$$

Let $S = M + iN$, where M and N are real scalar fields and $B^a \equiv B_3^a = -G_{12}^a/2$. Then one can show [4] that the string tension in this theory satisfy the inequality

$$T \geq \frac{me}{2}\left|\int d^2x\,\{M_a B_a\}\right| \tag{3.3}$$

and the bound is saturated if and only if

$$\begin{aligned} D_0\phi &= D_3\phi = D_0 S = D_3 S = 0 \\ D_\pm \phi &= 0\,, \\ D_\mp S &= 0\,, \\ B_a \pm Y_a &= 0\,, \\ V(S,\phi) - \frac{1}{2}Y_a^2 &= 0\,, \end{aligned} \tag{3.4}$$

which are BPS conditions for the string. Since it is a static configuration with axial symmetry, the first conditions imply that $W_0 = 0 = W_3$. The last BPS condition implies that $F=0$. One can check that 1/4 of the $N=2$ supersymmetry transformations vanish for field configurations satisfying the string BPS conditions in the limit $m \to 0$.

Differently from the Abelian case, these equations are only consistent with the equations of motion when m vanishes [4]. However this condition must be understood in the limiting case $m \to 0$, as we shall discuss later on. Therefore, it is only in this limit that we can have BPS strings satisfying (3.4). The explanation for this fact is the following: remember that a static solution of the equations of motion correspond to an extreme of the Hamiltonian, for solutions where $W_0 = 0$. On the other hand, a solution of the BPS conditions saturate the bound of the string tension in a given topological sector. But that does not necessarily guarantee that it is an extreme.

Now we shall analyze if the theory have a vacuum which produces a spontaneous symmetry consistent with the existence of string solution.

4. Phases of the theory

The vacuum equation $V(S, \phi) = 0$ is equivalent to

$$Y_a = 0 = F\,. \tag{4.1}$$

In order to the topological string solutions to exist, we look for vacuum solutions of the form

$$\begin{aligned} \phi^{\text{vac}} &= a|k\lambda_\phi >\,, \\ S^{\text{vac}} &= b\lambda_\phi \cdot H\,, \\ W_\mu^{\text{vac}} &= 0\,, \end{aligned} \tag{4.2}$$

where a and b are complex constants, k is a integer greater or equal to two and λ_ϕ is an arbitrary fundamental weight. If $a \neq 0$, this configuration breaks $G \to G_\phi$ in such a way that[13] $\Pi_1(G/G_\phi) = Z_k$, which is a necessary condition for the existence of Z_k-strings. Let us consider that $\mu > 0$. Following [4], from the vacuum conditions $Y_a = 0 = F$, one can conclude that

$$\begin{aligned} |a|^2 &= \frac{mb}{k}\,, \\ \left(kb\lambda_\phi^2 - \frac{\mu}{e}\right) a &= 0\,. \end{aligned}$$

There are three possibilities:

(i) If $m < 0 \;\Rightarrow\; a = 0 = b$ and the gauge group G remains unbroken.

(ii) If $m = 0 \;\Rightarrow\; a = 0$ and b can be any constant. In this case, S^{vac} breaks[13]

$$G \to G_S \equiv (K \times U(1))\,/Z_l\,, \tag{4.3}$$

where K is the subgroup of G associated to the algebra whose Dynkin diagram is given by removing the dot corresponding to λ_ϕ from that of G. The $U(1)$ factor is generated by $\lambda_\phi \cdot H$ and Z_l is a discrete subgroup of $U(1)$ and K. The $N = 2$ supersymmetry is restored in this case.

(iii) If $m > 0 \Rightarrow$

$$|a|^2 = \frac{m\mu}{k^2 e\lambda_\phi^2}\,, \tag{4.4}$$

$$b = \frac{\mu}{ke\lambda_\phi^2}\,, \tag{4.5}$$

and G is further broken to[13]

$$G \to G_\phi \equiv (K \times Z_{kl})/Z_l \supset G_S\,. \tag{4.6}$$

In particular for $k = 2$ we have for example,

$$\begin{aligned} \mathrm{Spin}(10) &\to (SU(5) \times Z_{10})/Z_5\,,\\ SU(3) &\to (SU(2) \times Z_4)/Z_2\,. \end{aligned}$$

Therefore by continuously changing the value of the parameter m we can produce a symmetry breaking pattern $G \to G_S \to G_\phi$. It is interesting to note that, unlike the Abelian-Higgs theory, in our theory the bare mass μ of ϕ is not required to satisfy $\mu^2 < 0$ in order to have spontaneous symmetry breaking. Therefore in the dual formulation, where one could interpret ϕ as the monopole condensate when $\phi \in R^{\mathrm{sym}}_{2\lambda_\phi}$, we don't need to have a monopole mass satisfying the problematic condition $M^2_{\mathrm{mon}} < 0$ mentioned by 't Hooft[7].

Let us analyze in more detail the last two phases[5].

5. The $m = 0$ or free-monopole phase

Since b is an arbitrary non-vanishing constant, we shall consider it to be given by (4.5), in order to have the same value as the case when $m < 0$. The non-vanishing expectation value of S^{vac} defines the $U(1)$ direction in G_S, (4.3), and one can define corresponding $U(1)$ charge as [14]

$$Q \equiv e\frac{S^{\mathrm{vac}}}{|S^{\mathrm{vac}}|} = e\frac{\lambda_\phi \cdot H}{|\lambda_\phi|}\,. \tag{5.1}$$

Since

$$Q\phi^{\mathrm{vac}} = ek|\lambda_\phi|\,\phi^{\mathrm{vac}}\,,$$

the electric charge of ϕ^{vac} is

$$q_\phi = ek|\lambda_\phi|\,. \tag{5.2}$$

Since in this phase $\Pi_2(G/G_S) = Z$, it can exist Z-magnetic monopoles. Indeed, for each root α such that $2\alpha^{\mathrm{V}} \cdot \lambda_\phi \neq 0$ (where $\alpha^{\mathrm{V}} \equiv \alpha/\alpha^2$), one can construct Z-monopoles [15]. The U(1) magnetic charge of these monopoles are [15]

$$g \equiv \frac{1}{|v|}\int dS_i M^a B^a_i = \frac{4\pi}{e}\frac{v\cdot\alpha^{\mathrm{V}}}{|v|} \tag{5.3}$$

where $v \equiv b\lambda_\phi$ and $B^a_i \equiv -\epsilon_{ijk}G^a_{jk}/2$ are the non-Abelian magnetic fields. These monopoles fill supermultiplets of $N = 2$ supersymmetry [16] and satisfy the mass formula

$$m_{\mathrm{mon}} = |v||g|\,. \tag{5.4}$$

Not all of these monopoles are stable. The stable or fundamental BPS monopoles are those lowest magnetic charge associated to the roots α which satisfy $2\alpha^{\mathrm{V}} \cdot \lambda_\phi = \pm 1$[17]. From now on, we shall only consider these fundamental monopoles, which are believed to fill representations of the gauge subgroup K[6]. Their magnetic charge (5.3) can be written as

$$g = \frac{2\pi k}{q_\phi} \tag{5.5}$$

Differently from the case in which the gauge group is broken to $U(1)^{\text{rank}(G)}$, it is very important to note that when G is broken to $U(1) \times K/Z_l$, the existence of monopole solutions can explain the 1/3 factor in the electric charge quantization condition for the colored particles like quarks and gluons when $K = SU(3)$ [18].

It is interesting to note that for the particular case where the gauge group is $G = SU(2)$ and scalar ϕ belongs to symmetric part of the direct product of the fundamental with itself, which corresponds to the adjoint representation, the supersymmetry of the theory is enhanced to $N = 4$. In this case, the theory has vanishing beta function.

There are other examples of vanishing β functions when $m = 0$. The β function of $N = 2$ super Yang-Mills with a hypermultiplet is given by

$$\beta(e) = \frac{-e^3}{(4\pi)^2}\left[h^{\text{V}} - x_\phi\right]$$

where h^{V} is the dual Coxeter number of G and x_ϕ is the Dynkin index of ϕ's representation. If ϕ belongs to the direct product of 2 fundamental representations, $R^{\otimes}_{2\lambda_\phi}$,

$$x_\phi = 2 d_{\lambda_\phi} x_{\lambda_\phi}\ ,$$

where x_{λ_ϕ} and d_{λ_ϕ} are, respectively, the Dynkin index and the dimension of the representation associated to the fundamental weight λ_ϕ.

Therefore for $SU(N)$ (which has $h^{\text{V}} = N$), if ϕ is in the tensor product of the fundamental representation of dimension $d_{\lambda_{N-1}} = N$ with itself (which has Dynkin index $x_{\lambda_{N-1}} = 1/2$), $x_\phi = N$ and the β function vanishes. Therefore the theory is superconformal (if we take $\mu = 0$). In this phase, $SU(N)$ is broken to $U(N-1) \sim \left(SU(N-1) \otimes U(1)\right)/Z_{N-1}$.

6. The $m > 0$ or superconducting phase

In the "$m > 0$" phase, the U(1) factor is broken and, like in the Abelian-Higgs theory, the corresponding force lines cannot spread over space. Since G is broken in such a way that $\Pi_1(G/G_\phi) = Z_k$, these force lines may form topological Z_k-strings. We shall show the existence of BPS Z_k-strings in this phase and obtain their string tensions. We shall also show that, as in the Abelian Higgs theory, the U(1) magnetic flux Φ_{mon} of the above monopoles is proportional to the BPS Z_k-string magnetic flux Φ_{st}, and therefore these $U(1)$ flux lines coming out of the monopole can be squeezed into Z_k-strings, which can give rise to a confining potential.

6.1 The BPS Z_k-string solutions

At $\rho \to \infty$, the string must tend to vacuum solutions in any angular direction φ. Let us denote $\phi(\varphi) = \phi(\varphi, \rho \to \infty)$, $S(\varphi) = S(\varphi, \rho \to \infty)$, etc. Then, the vacuum equations (3.2) imply that this asymptotic field configuration must be related by gauge transformations from a vacuum configuration, which we shall consider (4.2), i.e.

$$\begin{aligned} W_I(\varphi) &= \frac{-1}{ie} \left(\partial_I g(\varphi)\right) g(\varphi)^{-1} \,, \quad I = 1,2\,, \\ \phi(\varphi) &= g(\varphi)\phi^{\text{vac}}\,, \\ S(\varphi) &= g(\varphi) S^{\text{vac}} g(\varphi)^{-1}\,, \end{aligned}$$

for some $g(\varphi) \in G$. Then, in order for the field configurations to be single-valued, $g(2\pi)g(0) \in G_\phi$. Without lost of generality we shall consider $g(0) = 1$. Since G is simply connected (which can always be done by going to the universal covering group), a necessary condition for the existence of strings is that $g(2\pi)$ belongs to a non-connected component of G_ϕ. Let $g(\varphi) = \exp i\varphi L$. Then, at $\rho \to \infty$

$$\begin{aligned} \phi(\varphi) &= a e^{i\varphi L} | k\lambda_\phi > \,, \\ mS(\varphi) &= ka^2 e^{i\varphi L} \lambda_\phi \cdot H e^{-i\varphi L}\,, \\ W_I(\varphi) &= \frac{\epsilon_{IJ} x^J}{e\rho^2} L\,, \quad I,J = 1,2\,. \end{aligned} \tag{6.1}$$

Some possible choices for the Lie algebra element L are

$$L_n = \frac{n}{k} \frac{\lambda_\phi \cdot H}{\lambda_\phi^2}\,, \tag{6.2}$$

with n being a non-vanishing integer defined modulo k. With these choices it is possible to show[4] that $g(2\pi) \in G_\phi$.

We can construct the string ansatz by multiplying by arbitrary functions of ρ, the asymptotic configuration (6.1) with L_n given by (6.2),

$$\begin{aligned} \phi(\varphi,\rho) &= f(\rho) e^{in\varphi} a | k\lambda_\phi > \,, \\ mS(\varphi,\rho) &= h(\rho) k a^2 \lambda_\phi \cdot H\,, \\ W_I(\varphi,\rho) &= g(\rho) L_n \frac{\epsilon_{IJ} x^J}{e\rho^2} \quad \to \quad B_3(\varphi,\rho) = \frac{L_n}{e\rho} g'(\rho)\,, \\ W_0(\varphi,\rho) &= W_3(\varphi,\rho) = 0\,, \end{aligned} \tag{6.3}$$

with the boundary conditions

$$f(\infty) = g(\infty) = h(\infty) = 1\,,$$

in order to recover the asymptotic configuration at $\rho \to \infty$ and

$$f(0) = g(0) = 0$$

in order to eliminate singularities at $\rho = 0$.

Putting this ansatz in the BPS conditions it results that:

$$\begin{aligned} h(\rho) &= \text{const} \ = 1 \\ f'(\rho) &= \pm \frac{n}{\rho} \left[1 - g(\rho)\right] f(\rho) \\ g'(\rho) &= \mp \frac{q_\phi^2 a^2 \rho}{2n} \left[|f(\rho)|^2 - 1 \right] \end{aligned}$$

which are exactly the same differential equations with same boundary conditions which appear in the $U(1)$ case (2.8) (with q_ϕ given by (5.2)), whose existence of solution has been proven by Taubes.

As we mentioned before, the BPS conditions are compatible with the equations of motion when m vanishes. However, if we do this, $a = 0$ and there is no symmetry breaking, which is necessary in order for string solutions to exist. This result is very similar to what happens for the BPS monopole (see for instance [14]). In that case, one of the BPS conditions is $V(\phi) = \lambda(\phi^2 - a^2)^2/4 = 0$, which implies the vanishing of the coupling λ. However, that condition must be understood in the Prasad-Sommerfield limiting case $\lambda \to 0$ [19] in order to retain the boundary condition $|\phi| \to a$ as $r \to \infty$, and to have symmetry breaking. In our case, we have the same situation with a small difference: if one considers $m \to 0$, then $a \to 0$. We can avoid this problem by allowing $\mu \to \infty$ such that $m\mu$, or equivalently a, remains constant, implying that the field ϕ becomes infinitely heavy.

6.2 The Z_k-string magnetic flux and the string tension

Since the scalar M^a defines the $U(1)$ direction inside G, we define a gauge invariant $U(1)$ string magnetic flux

$$\Phi_{\text{st}} \equiv \frac{1}{|v|} \int d^2x \, M^a B_3^a \tag{6.4}$$

which is similar to the $U(1)$ monopole magnetic flux (5.3), but with surface integral taken over the plane perpendicular to the string. Using the BPS string ansatz we obtain that

$$\Phi_{\text{st}} = \oint dl_I A_I = \frac{2\pi n}{q_\phi} \quad , \qquad n \in Z_k \, , \tag{6.5}$$

where $A_I \equiv W_I^a M^a/|v|$, $I = 1, 2$ and q_ϕ given by (5.2). This flux quantization condition is also very similar to the Abelian result[2] (2.5) and generalizes, for example, the string magnetic flux for $SU(2)$[20] and for $SO(10)$[21](up to a $\sqrt{2}$ factor). In [22], it is also calculated fluxes for the $SU(n)$ theory, but with the gauge group completely broken to its center and a different definition of string flux which is not gauge invariant. Note that we can rewrite the above result as

$$\Phi_{\text{st}} q_\phi = 2\pi n \ , \quad n \in Z_k \, .$$

We can conclude that for the fundamental (anti)monopoles, the $U(1)$ magnetic flux $\Phi_{\text{mon}} = g$, which is given in (5.5), is consistent with Φ_{st}, if $n = k$. This can be interpreted that for one fundamental monopole we could attach k Z_k-strings with $n = 1$. That is consistent with the fact that k Z_k-strings with $n = 1$ have trivial first homotopy, as do the monopoles.

With the above definition of string flux, the string tension bound (3.3) can be written as

$$T \geq \frac{1}{2} q_\phi |a|^2 |\Phi_{\text{st}}| = \pi |a|^2 |n| \, , \ \ n \in Z_k \, ,$$

which generalizes the $U(1)$ results (2.3) and (2.6). The bound hold for the BPS string. Since the tension is constant, it may cause a confining potential between monopoles increasing linearly with their distance, which may produce quark confinement in a dual theory.

[2] It is interesting to note that (6.5) corresponds to the Abelian flux (2.5) divided by $|\lambda_\phi|$ when $k = 2$.

6.3 The monopole confinement

In the $m < 0$ phase, by topological arguments [23][24] one would expect that the monopoles produced in the $m = 0$ phase develop a flux line or string and get confined. We can see this more concretely in the following way: as usual, in order to obtain the asymptotic scalar configuration of a (spherically symmetric) monopole, starting from the vacuum configuration (4.2) one performs a spherically symmetric gauge transformation. Then for the case of a fundamental monopole and considering the $k = 2$ we can show[5] that asymptotically

$$\phi(\theta,\varphi) = a\left\{\cos^2\frac{\theta}{2}|2\lambda_\phi > -\frac{\sqrt{2}}{2}\sin\theta e^{-i\varphi}|2\lambda_\phi - \alpha > + \sin^2\frac{\theta}{2}e^{-2i\varphi}|2\lambda_\phi - 2\alpha >\right\}.$$

Therefore at $\theta = \pi$,

$$\phi(\pi,\varphi) = ae^{-2i\varphi}|2\lambda_\phi - 2\alpha >$$

which is singular. This generalizes Nambu's result [25] for the $SU(2)\times U(1)$ case. In order to cancel the singularity we should attach a string in the $z < 0$ axis with a zero in the core, as in our string ansatz (6.3). One could construct an ansatz for $\phi(r,\theta,\phi)$ by multiplying the above asymptotic configuration by a function $F(r,\theta)$ such that $F(r,\pi) = 0$.

The string tension for k strings with $n = 1$ must satisfy

$$T \geq k\pi|a|^2 .$$

Then, the threshold length $d^{\rm th}$ for the string to break producing a new monopole-antimonopole pair, with masses (5.4), is derived form the relation

$$\frac{4\pi}{e}\frac{k|a|^2|\lambda_\phi|}{m} = E^{\rm th} = Td^{\rm th} \geq k\pi|a|^2 d^{\rm th} ,$$

which results in

$$d^{\rm th} \leq \frac{4|\lambda_\phi|}{me} .$$

The monopole-antimonopole pair tends to deconfine when $m \to 0_+$, as one would expect, when $T \to 0$ and $d^{\rm th} \to \infty$.

7. Summary and conclusions

In this talk, we have presented some generalizations of the ideas of 't Hooft and Mandelstam to non-Abelian theories. Like in the Abelian-Higgs theory, we have seen that our non-Abelian theory presents some different phases, including a superconducting phase, where we have proven the existence of BPS Z_k-string solutions and have calculated exactly their string tension. We also showed that the fluxes of the magnetic monopoles and strings are proportional and therefore the monopoles can get confined. But differently from the Abelian case, or also from the theories in which are used Abelian projection, our monopoles are not Dirac monopoles, and so we can associate naturally mass to them. Due to that, we could calculate the threshold length for the string to break in a new monopole-antimonopole pair. Also in our theory the unbroken gauge group is non-Abelian and our monopoles should fill some representation of this unbroken gauge group. In many cases, the unbroken group can contain $SU(3) \otimes U(1)_{\rm em}$ as subgroup. Moreover we have seen that we could consider the scalar ϕ as a diquark condensate and unlike the Abelian theory, in our theory the bare mass μ of ϕ is not required to satisfy $\mu^2 < 0$ in order to have spontaneous symmetry breaking. Therefore in the dual formulation, where one could interpret ϕ as being the monopole condensate, we don't need to have a monopole mass satisfying the problematic condition $M_{\rm mon}^2 < 0$ mentioned by 't Hooft.

It is expected that a confining theory obtained by a deformation of superconformal gauge theory in 4 dimensions should satisfy a gauge/string correspondence [26], which would be a kind of deformation of the CFT/AdS correspondence [27]. In the gauge/string correspondences it is usually considered confining gauge theories with $SU(N)$ or $U(N)$ completely broken to its center. We have seen that some of our confining theories are obtaining by adding a deformation to superconformal theories and which breaks $U(N-1)$ to $SU(N-1) \otimes Z_2$ (up to a discrete factor). It would be interesting to know if these theories also satisfy a gauge/string correspondence.

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