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Time evolution of confined quantum systems - a non perturbative approach

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Abstract: Using the non-perturbative method of *dressed* states introduced in previous publications, we study effects of the electromagnetic field on an atom, in the case the field is described by an ensemble of non interacting harmonic oscillators. This method allows to separate the whole system into the *dressed* atom and the *dressed* field, in terms of which a non-perturbative approach is possible. When applied to study atomic behaviours in cavities, the method accounts for experimentally observed inhibition of atomic decay in small cavities.

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1. Introduction

In spite of its wide applicability, there are situations where the use of perturbation theory is not possible, as in the low energy domain of Quantum Chromodynamics where confinement of quarks and gluons takes place, or are of little usefulness, as for instance in Atomic physics, in resonant effects associated with the coupling of atoms with strong radiofrequency fields. These situations have led since a long time ago to attempts to by-pass the limitations of perturbation theory, in particular in situations where strong effective couplings are involved. In some non perturbative approaches in statistical physics and constructive field theory, general theorems can be derived using cluster-like expansions and other related methods [1]. In some cases, these methods lead to the rigorous construction of field theoretical models (see for instance [2] and other references therein), but, in spite of the rigor and in some cases the beauty of demonstrations, they are not of great usefulness in calculations of a predictive character.

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In fact, from a phenomenological point of view there are situations even in the scope of QED, where perturbation methods are of little usefulness, for instance, resonant effects associated with the coupling of atoms with strong radiofrequency fields [3]. As remarked in [4], the theoretical understanding of these effects using perturbative methods requires the calculation of very high-order terms in perturbation theory, what makes the standard Feynman diagrams technique practically unreliable. The trials of treating non-perturbativelly systems of this type, are at the source of the idea of "dressed atom", introduced originally in refs [5] and [6]. Since then this concept has been used to investigate several situations involving the interaction of atoms and electromagnetic fields, as for instance, atoms embedded in a strong radiofrequency field background [7, 8], and atoms in intense resonant laser beams [9]. In order to give a precise mathematical definition and a clear physical meaning to the idea of a dressed atom, a crucial aspect is the non-linear character of the problem involved in realistic situations, which in general does not allow that a rigorous definition of a "dressed atom" could be given. Calculations in the situations described above involve very high perturbative orders, which make the usual Feynman diagrams technique practically unreliable. A way to by-pass these mathematical difficulties, is to assume that under certain conditions the coupled atom-electromagnetic field system may be approximated by the system composed of a harmonic oscillator coupled *linearly* to the field by means of some effective coupling constant g. This is the case in the context of the general QED linear response theory, where the electric dipole interaction gives the leading contribution to the radiation process ([10], [12]). Also, in a slightly different context, recently a significative number of works has been spared to the study of cavity QED, in particular to the theoretical investigation of higher-generation Schrodinger cat-states in high-Q cavities, as has been done for instance in [13]. Linear approximations of this type have been applied along the last years in condensed matter physics for studies of Brownian motion and in quantum optics to study decoherence, by assuming a linear coupling between a cavity harmonic mode and a thermal bath of oscillators at zero temperature, as it has been done in [14] and [15].

In recent publications [20, 21] a method (*dressed* coordinates and *dressed* states) has been introduced that allows a non-perturbative approach to situations of the type described above, provided that the interaction between the parts of the system can be approximated by a linear coupling. More precisely, the method applies for all systems that can be described by a Hamiltonian of the form,

$$H = \frac{1}{2}\left[p_0^2 + \omega_0^2 q_0^2 + \sum_{k=1}^{N}(p_k^2 + \omega_k^2 q_k^2)\right] - q_0 \sum_{k=1}^{N} c_k q_k, \tag{1.1}$$

where the subscript 0 refers to the atom and $k = 1, 2, ...N$ refer to the harmonic environment modes. The limit $N \to \infty$ in Eq.(1.1) is understood. In the case of the coupled atom field system, this formalism recovers the experimental observation that excited states of atoms in sufficiently small cavities are stable. It allows to give formulas for the probability of an atom to remain excited for an infinitely long time, provided it is placed in a cavity of appropriate size [21]. For an emission frequency in the visible red, the size of such cavity

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is in agreement with experimental observations [22, 23].

2. The eigenfrequencies spectrum

We consider for a moment, as in [20], the problem of a harmonic oscillator q_0 of (bare) frequency ω_0 coupled to N other oscillators q_i of frequencies ω_i, $i = 1, 2, ...N$. In the limit $N \to \infty$ we recover our original situation of the coupling particle-bath after redefinition of divergent quantities, in a manner analogous to naive mass renormalization in field theories. The bilinear Hamiltonian (1.1) can be turned to principal axis by means of a point transformation,

$$q_\mu = t^r_\mu Q_r\,, \;\; p_\mu = t^r_\mu P_r; \;\; \mu = (0, \{k\}), \; k = 1, 2, ..., N; \;\; r = 0, ...N, \tag{2.1}$$

performed by an orthonormal matrix $T = (t^r_\mu)$. The subscript $\mu = 0$ and $\mu = k$ refer respectively to the particle and the harmonic modes of the bath and r refers to the normal modes. In terms of normal momenta and coordinates, the transformed Hamiltonian in principal axis reads,

$$H = \frac{1}{2}\sum_{r=0}^{N}(P_r^2 + \Omega_r^2 Q_r^2), \tag{2.2}$$

where the Ω_r's are the normal frequencies corresponding to the possible collective stable oscillation modes of the coupled system. The matrix elements t^r_μ are given by [20]

$$t^r_k = \frac{c_k}{(\omega_k^2 - \Omega_r^2)} t^r_0\,, \;\; t^r_0 = \left[1 + \sum_{k=1}^{N}\frac{c_k^2}{(\omega_k^2 - \Omega_r^2)^2}\right]^{-\frac{1}{2}} \tag{2.3}$$

with the condition,

$$\omega_0^2 - \Omega_r^2 = \sum_{k=1}^{N}\frac{c_k^2}{\omega_k^2 - \Omega_r^2}. \tag{2.4}$$

We take $c_k = \eta(\omega_k)^n$, where $\eta = \sqrt{2g\Delta\omega}$, g being a coupling constant and $\Delta\omega$ the (regular) spacing between two field frequencies. In this case the environment is classified according to $n > 1$, $n = 1$, or $n < 1$, respectively as *supraohmic*, *ohmic* or *subohmic*. For a subohmic environment the sum in Eq.(2.4) is convergent and the frequency ω_0 is well defined. For ohmic and supraohmic environments the sum in the right hand side of Eq.(2.4) diverges what makes the equation meaningless as it stands, a renormalization procedure being needed. In this case, as a first step we add and subtract the quantity $\Omega_r^2(\omega_k^2)^{n-1}$ to the numerator of the right-hand side of Eq.(2.4). After changing the term corresponding to the subtraction of $\Omega_r^2(\omega_k^2)^{n-1}$ to the left-hand side, Eq.(2.4) can be rewritten in the form,

$$\omega_0^2 - \eta^2\sum_{k=1}^{N}\frac{(\omega_k^2)^n - \Omega_r^2(\omega_k^2)^{n-1}}{\omega_k^2 - \Omega_r^2} - \Omega_r^2 = \sum_{k=1}^{N}\frac{\Omega_r^2(\omega_k^2)^{n-1}}{\omega_k^2 - \Omega_r^2}. \tag{2.5}$$

If $n = 1$ (*ohmic* system), this step is sufficient, the right-hand side of Eq.(2.5) is convergent and we define from the left-hand side of Eq.(2.5) the renormalized frequency by Eq. (2.13)

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below. If $n \geq 1$ further steps are necessary: we add and subtract to the numerator of the right-hand side of Eq.(2.5) the quantity $(\Omega_r^2)^2(\omega_k^2)^{n-2}$ and we change the term corresponding to its subtraction to the left-hand side. The process is continued until the series in right-hand member of the resulting equation is convergent. This is attained after a number of steps, and the result can be rewritten in the form,

$$\omega_0^2 - \delta\omega^2 - \Omega_r^2 = \eta^2 \Omega_r^{2[\![n]\!]} \sum_{k=1}^{N} \frac{1}{\omega_k^2 - \Omega_r^2}, \tag{2.6}$$

where we have defined the counterterm,

$$\delta\omega^2 = \frac{\eta^2}{4} \sum_{k=1}^{N} \sum_{\alpha=1}^{[\![n]\!]} \Omega_r^{2\alpha} \omega_k^{2(n-\alpha)}, \tag{2.7}$$

with the notation $[\![n]\!]$ standing for the smallest integer containing n. Note that all the k-dependence characteristic of the numerator of the right-hand side of Eq.(2.4) has moved to the counterterm (2.7). From an analysis of Eq.(2.6) it can be seen that if $\omega_0^2 > \delta\omega^2$ Eq.(2.6) yields only positive solutions for Ω^2, while if $\omega_0^2 < \delta\omega^2$, Eq.(2.6) has a negative solution Ω_-^2. This means that in this case there is a damped collective normal mode that does not allows stationary configurations. Nevertheless it should be remarked that in a different context, it is precisely this runaway solution that is related to the existence of a bound state in the Lee-Friedrechs model [24]. This solution is considered in Ref. [25] in the framework of a model to describe qualitatively the existence of bound states in particle physics. We will not consider this situation. We consider the situation in which all normal modes are harmonic which corresponds to take $\omega_0^2 > \delta\omega^2$ and define the *renormalized* frequency

$$\bar{\omega}^2 = \omega_0^2 - \delta\omega^2, \tag{2.8}$$

in terms of which Eq.(2.6) becomes,

$$\bar{\omega}^2 - \Omega_r^2 = \eta^2 \sum_{k=1}^{N} \frac{\Omega_r^{2[\![n]\!]}}{\omega_k^2 - \Omega_r^2}. \tag{2.9}$$

We see that in the limit $N \to \infty$ the above procedure is exactly the analogous of naive mass renormalization in Quantum Field Theory: the addition of a counterterm $-\delta\omega^2 q_0^2$ allows to compensate the infinity of ω_0^2 in such a way as to leave a finite, physically meaninful renormalized frequency $\bar{\omega}$. This simple renormalization scheme has been originally introduced in ref.[26].

To proceed, we take the constant η as $\eta = \sqrt{2g\Delta\omega}$, $\Delta\omega$ being the interval between two neighbouring bath frequencies (supposed uniform) and where g is some constant [with dimension of $(frequency)^{2-\eta}$]. For reasons that will become apparent later, we restrict ouselves to the physical situations in which the environment frequencies ω_k can be writen in the form

$$\omega_k = 2k\pi/L, \quad k = 1, 2, \ldots . \tag{2.10}$$

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Then using the formula[27],

$$\sum_{k=1}^{N}\frac{1}{(k^2-u^2)}=\left[\frac{1}{2u^2}-\frac{\pi}{u}\cot(\pi u)\right], \tag{2.11}$$

Eq.(2.9) can be written in closed form,

$$\cot(\frac{L\Omega}{2c})=\frac{\Omega^3}{\pi g\Omega^{2[\![n]\!]}}+\frac{c}{L\Omega}(1-\frac{\bar{\omega}^2L\Omega^2}{\pi gc\Omega^{2[\![n]\!]}}). \tag{2.12}$$

For an ohmic environment we have $c_k=\eta\omega_k$ and $\delta\omega^2=N\eta^2$. Taking in Eq. (2.6) $\omega_0^2>N\eta^2$, the *renormalized* oscillator frequency $\bar{\omega}$ is given by,

$$\bar{\omega}=\sqrt{\omega_0^2-N\eta^2}, \tag{2.13}$$

and the eigenfrequencies spectrum for an *ohmic* environment is given by the equation,

$$\cot(\frac{L\Omega}{2c})=\frac{\Omega}{\pi g}+\frac{c}{L\Omega}(1-\frac{\bar{\omega}^2L}{\pi gc}). \tag{2.14}$$

The solutions of Eq.(2.14) or Eq.(2.12) with respect to Ω give the spectrum of eigenfrequencies Ω_r corresponding to the collective normal modes.
The transformation matrix elements turning the material body-bath system to principal axis is obtained in terms of the physically meaningful quantities Ω_r, $\bar{\omega}$, after some rather long but straightforward manipulations analogous to what has been done in [20]. They read,

$$t_0^r=\frac{\eta\Omega_r}{\sqrt{(\Omega_r^2-\bar{\omega}^2)^2+\frac{\eta^2}{2}(3\Omega_r^2-\bar{\omega}^2)+\pi^2g^2\Omega_r^2}}\ ,\quad t_k^r=\frac{\eta\omega_k}{\omega_k^2-\Omega_r^2}t_0^r. \tag{2.15}$$

3. The dressed states

To fix our framework and to give precise applications of our formalism, we study in this paper an *ohmic* environment. The normalized eigenstates of our system (eigenstates of the Hamiltonian in principal axis) can be written in terms of normal coordinates,

$$\langle Q|n_0,n_1,...\rangle\equiv\phi_{n_0n_1n_2...}(Q,t)=\prod_s\left[\sqrt{\frac{2^{n_s}}{n_s!}}H_{n_s}(\sqrt{\frac{\Omega_s}{\hbar}}Q_s)\right]\Gamma_0(Q)\mathrm{e}^{-i\sum_s n_s\Omega_s t}, \tag{3.1}$$

where H_{n_s} stands for the n_s-th Hermite polynomial and Γ_0 is the normalized vacuum eigenfunction.

Next we intend to divide the system into the *dressed* particle and the *dressed* environment by means of some conveniently chosen *dressed* coordinates, q_0' and q_j' associated respectively to the *dressed* particle and to the *dressed* oscillators composing the environment. These coordinates will allow a natural division of the system into the dressed (physically observed) particle and into the *dressed* environment. The dressed particle will contain automatically all the effects of the environment on it. Clearly, these dressed coordinates

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should not be introduced arbitrarily. Since our problem is linear, we will require a linear transformation between the normal and *dressed* coordinates (notice that this is not the same transformation (2.1) linking the normal to the bare coordinates). Also, we demand the physical condition of vacuum stability. We assume that at some given time ($t = 0$) the system is described by *dressed* states, whose wavefunctions are defined by,

$$\psi_{\kappa_0\kappa_1...}(q') = \prod_{\mu}\left[(2^{-\kappa_\mu}\kappa_\mu!)^{-\frac{1}{2}}H_{\kappa_\mu}(\sqrt{\frac{\bar{\omega}_\mu}{\hbar}}q'_\mu)\right]\Gamma_0(q')\ , \tag{3.2}$$

where $\kappa_\mu\ , \mu = 0\ , \{i\}$ are non-negative integers, $q'_\mu = q'_0, q'_i$, $\bar{\omega}_\mu = (\bar{\omega}, \omega_i)$ and Γ_0 is the invariant ground state eigenfunction introduced in Eq.(3.1). Note that the above wavefunctions will evolve in time in a more complicated form than the unitary evolution of the eigenstates (3.1), since these wavefunctions are not eigenstates of the diagonal Hamiltonian (1.1). It is precisely the non unitary evolution of these wavefunctions that will allow (see below) a non-perturbative study of the radiation and dissipation processes of the particle. To satisfy the physical condition of vacuum stability (invariance under a tranformation from normal to *dressed* coordinates) we remember that the the ground state eigenfunction of the system has the form,

$$\Gamma_0(Q) \propto \mathrm{e}^{-\frac{1}{2\hbar}\sum_{r=0}^{N}\Omega_r Q_r^2}\ , \tag{3.3}$$

and we require that the ground state in terms of the *dressed* coordinates should have the form

$$\Gamma_0(q') \propto \mathrm{e}^{-\frac{1}{2\hbar}\sum_{\mu=0}^{N}\bar{\omega}_\mu (q'_\mu)^2}\ . \tag{3.4}$$

From Eqs.(3.3) and (3.4) it can be seen that the vacuum invariance requirement is satisfied if we define *dressed* coordinates by,

$$\sqrt{\bar{\omega}_\mu}q'_\mu = \sum_{r=0}^{N} t_\mu^r\sqrt{\Omega_r}Q_r\ . \tag{3.5}$$

These *dressed* coordinates are new *collective* coordinates, different from the bare coordinates $q_0\ ,\ \{q_i\}$ describing the bare particle and the free field modes, and also from the normal (collective) coordinates $\{Q_r\}$. Indeed these *dressed* coordinates are related to the bare coordinates by [20],

$$q'_\mu = \sum_\nu \alpha_{\mu\nu}q_\nu\ ;\qquad \alpha_{\mu\nu} = \frac{1}{\sqrt{\bar{\omega}_\mu}}\sum_r t_\mu^r t_\nu^r\sqrt{\Omega_r}. \tag{3.6}$$

As we have already mentioned above our *dressed* states, given by Eq.(3.2), are *collective* but *non stable* states, linear combinations of the (stable) eigensatates (3.1) defined in terms of the normal modes. Moreover our dressed states have the interesting property of distributing the energy initially in a particular dressed state, among itself and all other dressed states with precise and well defined probability amplitudes. For instance, let us consider the *dressed* state $\mid 0, 0, ...1(\mu), 0...\rangle$, represented by the wavefunction $\psi_{00...1(\mu)0...}(q')$. It describes the configuration in which only the dressed oscillator q'_μ is in the first excited level. Then it is shown in [20] the following expression for the time evolution of the

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first-level excited dressed oscillator q'_μ,

$$| 0,0,...1(\mu),0...\rangle\,(t) = \sum_\nu f^{\mu\nu}(t)|\,0,0,...1(\nu),0...\rangle\,(0)$$
$$f^{\mu\nu}(t) = \sum_s t^s_\mu t^s_\nu e^{-i\Omega_s t}. \tag{3.7}$$

We *choose* the dressed states as physically meaningful and we test successfully this hypothesis by studying the radiation process by an atom in a cavity. In both cases, of a very large or a very small cavity, our results are in agreement with experimental observations. Having introduced *dressed* coordinates and *dressed states*, in the next section we will apply these concepts to study the time evolution of excited atomic states.

4. The radiation process

In this section we study the radiation process of the *dressed* particle when it is prepared in such a way that initially it is in its first excited state. We shall consider two situations, the particle in free space and the particle confined in a cavity of diameter L.

4.1 The particle in free space

Consider that the atom is initially in its first excited level. In this case the spectrum of the given frequencies ω_k has a continuous distribution [20], and the probability that the dressed atom still remain in its first excited state at a time $t \gg 1/\bar{\omega}$, has the following expression,

$$|f^{00}(t)|^2 = \left(1+\frac{\pi^2 g^2}{4\bar{\omega}^2}\right)e^{-\pi g t} - e^{-\frac{\pi g t}{2}}\left[\frac{8g}{\bar{\omega}^4 t^3}\left(\sin\kappa t + \frac{\pi g}{2\kappa}\cos\kappa t\right)\right] + \frac{16 g^2}{\bar{\omega}^8 t^6}. \tag{4.1}$$

We see from Eq. (4.1) that asymptotically the probability that the dressed particle be still excited at a very large time t, obeys a power law, $|f^{00}(t)|^2|_{t\to\infty} \approx \frac{16g^2}{\bar{\omega}^8 t^6}$. However, in the weak coupling regime we are considering here, and for some values of g and $\bar{\omega}$, we can see that this behaviour is dominant only for extremely large values of t, where the probability $|f^{00}(t)|^2$ is vanishingly small. For lower values of t (but satisfying the condition $t \gg 1/\bar{\omega}$), the leading behaviour is given by the exponential law. For instance let us take $\bar{\omega} = 4.0 \times 10^{14}/s$, $g = \bar{\omega}/137$. In this case the condition $t \gg 1/\bar{\omega}$ corresponds to $t \gg 2.5 \times 10^{-15} s$. A numerical analysis of Eq.(4.1) with these data shows that for t in the interval $10^{-13}s < t < 10^{-12}s$, the curve describing the function in Eq.(4.1) is practically indistinguishable from the pure exponential $exp(-\pi g t)$. For those values of g an $\bar{\omega}$ the contribution from the monomial term in Eq.(4.1) is negligible. However, we could find other values of g and $\bar{\omega}$ for which the behaviour of the probability $|f^{00}(t)|^2$ would depart from an exponential law.

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4.2 Behaviour of the confined system

Let us now consider the *ohmic* system in which the particle is placed in the center of a cavity of diameter L, in the case of a very small L, *i.e.* that satisfies the condition of being much smaller than the coherence lenght, $L << 2c/g$. We note that from a physical point of view, L stands for either the diameter of a spherical cavity or the spacing between infinite paralell mirrors. To obtain the eigenfrequencies spectrum, we remark that from a graphical analysis of Eq.(2.14) it can be seen that in the case of small values of L, its solutions are very near the frequency values corresponding to the asymptots of the curve $\cot(\frac{L\Omega}{2c})$, which correspond to the environment modes $\omega_i = i2\pi c/L$, except from the smallest eigenfrequency Ω_0. As we take larger and larger solutions for the eigenfrequencies Ω_k , $k = 1, 2, ...$, they are nearer and nearer to the asymptots corresponding to the field modes. For instance, for a value of L of the order of $2 \times 10^{-2} m$ and $\bar{\omega} \sim 10^{10}/s$, only the lowest eigenfrequency Ω_0 is significantly different from the field frequency corresponding to the first asymptot, all the other eigenfrequencies Ω_k, $k = 1, 2, ...$ being very close to the field modes $k2\pi c/L$. For higher values of $\bar{\omega}$ (and lower values of L) the differences between the eigenfrequencies and the field modes frequencies are still smaller. Thus to solve Eq.(2.14) for the larger eigenfrequencies we expand the function $\cot(\frac{L\Omega}{2c})$ around the values corresponding to the asymptots. We write,

$$\Omega_k = \frac{2\pi c}{L}(k + \epsilon_k), \quad k = 1, 2, .. \tag{4.2}$$

with $0 < \epsilon_k < 1$, satisfying the equation,

$$\cot(\pi\epsilon_k) = \frac{2c}{gL}(k + \epsilon_k) + \frac{1}{(k + \epsilon_k)}(1 - \frac{\bar{\omega}^2 L}{2\pi gc}). \tag{4.3}$$

But since for a small value of L every ϵ_k is much smaller than 1, Eq.(4.3) can be linearized in ϵ_k, giving,

$$\epsilon_k = \frac{4\pi gcLk}{2(4\pi^2 c^2 k^2 - \bar{\omega}^2 L^2}. \tag{4.4}$$

Eqs.(4.2) and (4.4) give approximate solutions to the eigenfrequencies Ω_k, $k = 1, 2....$

To solve Eq.(2.14) with respect to the lowest eigenfrequency Ω_0, let us assume that it satisfies the condition $\Omega_0 L/2c << 1$ (we will see below that this condition is compatible with the condition of a small L as defined above). Inserting the condition $\Omega_0 L/2c << 1$ in Eq.(2.14) and keeping up to quadratic terms in Ω we obtain the solution for the lowest eigenfrequency Ω_0,

$$\Omega_0 = \frac{\bar{\omega}}{\sqrt{1 + \frac{\pi gL}{2c}}}. \tag{4.5}$$

Consistency between Eq.(4.5) and the condition $\Omega_0 L/2c << 1$ gives a condition on L,

$$L \ll \frac{2c}{g} f \; ; \quad f = \frac{\pi}{2}\left(\frac{g}{\bar{\omega}}\right)^2 \left(1 + \sqrt{1 + \frac{4}{\pi^2}\left(\frac{\bar{\omega}}{g}\right)^2}\right). \tag{4.6}$$

Let us consider as in the preceding section, the situation of *weak* coupling, and let us consider the situation where the dressed atom is initially in its first excited level. Then

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from Eq.(3.7) we obtain the probability that it will still be excited after a ellapsed time t,

$$|f^{00}(t)|^2 = (t_0^0)^4 + 2\sum_{k=1}^{\infty}(t_0^0)^2(t_0^k)^2\cos(\Omega_k - \Omega_0)t + \sum_{k,l=1}^{\infty}(t_0^k)^2(t_0^l)^2\cos(\Omega_k - \Omega_l)t. \tag{4.7}$$

In the case of *weak* coupling a physically interesting situation is when interactions of electromagnetic type are involved. In this case, we take $g = \alpha\bar{\omega}$, where $\alpha = 1/137$ is the fine structure constant. Then the factor f multiplying $2c/g$ in Eq.(4.6) is ~ 0.07 and the condition $L \ll 2c/g$ is replaced by the more restrictive one, $L \ll 0.07(2c/g)$. For a typical infrared frequency, for instance $\bar{\omega} \sim 2,0 \times 10^{11}/s$, our calculations are valid for a value of L, $L \ll 10^{-3}m$.

From Eqs.(2.15) and using the above expressions for the eigenfrequencies for small L, we obtain the matrix elements,

$$(t_0^0)^2 \approx 1 - \frac{\pi g L}{2c}; \;\; (t_0^k)^2 \approx \frac{gL}{\pi c k^2}. \tag{4.8}$$

To obtain the above equations we have neglected the corrective term ϵ_k, from the expressions for the eigenfrequencies Ω_k. Nevertheless, corrections in ϵ_k should be included in the expressions for the matrix elements t_k^k, in order to avoid spurious singularities due to our approximation. This can be clearly seen from Eq.(2.15), due to the vanishing of the denominator $\omega_k^2 - \Omega_r^2$, for $r = k$, if we neglect terms depending on ϵ_k.

Using Eqs.(4.8) in Eq.(4.7), we obtain

$$|f^{00}(t)|^2 \approx 1 - \pi\delta + 4(\frac{\delta}{\pi} - \delta^2)\sum_{k=1}^{\infty}\frac{1}{k^2}\cos(\Omega_k - \Omega_0)t + \pi^2\delta^2 + \frac{4}{\pi^2}\delta^2\sum_{k,l=1}^{\infty}\frac{1}{k^2 l^2}\cos(\Omega_k - \Omega_l)t, \tag{4.9}$$

where we have introduced the dimensionless parameter $\delta = Lg/2c \;\ll 1$, corresponding to a small value of L and we remember that the eigenfrequencies are given by Eqs.(4.2) and (4.4). As time goes on, the probability that the material body be excited attains periodically a minimum value which has a lower bound given by,

$$\mathrm{Min}(|f^{00}(t)|^2) = 1 - \frac{5\pi}{3}\delta + \frac{14\pi^2}{9}\delta^2. \tag{4.10}$$

For a frequency $\bar{\omega}$ of the order $\bar{\omega} \sim 4.00 \times 10^{14}/s$ (in the red visible), which corresponds to $\delta \sim 0.005$ and $L \sim 1.0 \times 10^{-6}m$, we see from Eq.(4.10) that the probability that the material body be at any time excited will never fall below a value ~ 0.97, or a decay probability that is never higher that a value ~ 0.03. It is interesting to compare this result with experimental observations in [22, 23], where stability is found for atoms emitting in the

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visible range placed between two parallel mirrors a distance $L = 1.1 \times 10^{-6} m$ apart from one another. For lower frequencies the value of the spacing L ensuring quasi-stability of the same order as above, for the excited particle may be considerably larger. For instance, for $\bar{\omega}$ in a typical microwave value, $\bar{\omega} \sim 2,00 \times 10^{10}/s$ and taking also $\delta \sim 0.005$, the probability that the material body remain in the first excited level at any time would be larger than a value of the order of 97%, for a value of L, $L \sim 2.0 \times 10^{-2} m$. The probability that the material body remain excited as time goes on, oscillates with time between a maximum and a minimum values and never departs significantly from the situation of stability in the excited state.

5. Concluding Remarks

We proposed a non-perturbative treatement of a quantum system consisting of an atom coupled to a field described by non-interacting oscillators. We have used *dressed* states which allow to divide the system into the *dressed* particle and the *dressed* field by means of some conveniently chosen *dressed* coordinates, q'_0 and q'_j associated respectively to the *dressed* atom and to the *dressed* oscillators composing the environment. In terms of these coordinates a division of the system into the dressed (physically observed) atom and the *dressed* field arises naturally. The dressed particle will contain automatically all the effects of the environment on it. This formalism allows a non-perturbative approach to the time evolution of a system that may be approximated by a particle coupled linearly to its environment, in rather different situations as confinement of atoms in cavities or the Brownian motion. In other words, underlying our dressed states formalism there is an unified way to study two physically different situations, the radiation process and the Brownian motion. We have approached one of these situations using the *dressed* states, and we have obtained results in good agreement with experimental observations or with expected behaviours. For atomic systems we recover with our formalism the experimental observation that excited states of atoms in sufficiently small cavities are stable. We are able to give formulas for the probability of an atom to remain excited for an extremely long time, provided it is placed in a cavity of appropriate size. For an emission frequency in the visible red, the size of such cavity is in agreement with experimental observations [21]. The generalization of the work presented in this paper to the case of a generic (supraohmic or subohmic) environment and finite temperature is in progress and will be presented elsewhere.

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