PROCEEDINGS

Workshop on Integrable Theories, Solitons and Duality

Some remarks on a deformed quantum field theory

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ABSTRACT: We review and extend some results obtained in the context of a deformed quantum field theory which is interpreted as a phenomenological quantum theory describing the scattering of spin-0 composite particles. We discuss in a more detailed way the generalization of Wick's expansion for this case and present the computation of the scattering $1+2 \to 1'+2'$ up to second order in the coupling constant. The result we obtained shows that the structure of a composite particle, described here phenomenologically by the deformed algebraic structure, modify in a simple but non-trivial way the perturbation expansion for the process under consideration.

KEYWORDS: Heisenberg algebra; quantum field theory; perturbative computation .

1. Introduction

One of the essential tools in the construction of a quantum field theory (QFT) is the harmonic oscillator algebra known as Heisenberg algebra. This is originated from the interpretation of the generators of this algebra as creating or annihilating particle states.

On the other hand, in order to interpret the generators, $A^\dagger$ and A, of a q-deformed Heisenberg algebra [1] as creating or annihilating particle states, respectively one should

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explain what kind of physical particle has the non additive property present in q-oscillators systems, since for q-oscillators the energy of n particles is not equal to n times the energy of one particle.

The first step in this direction was given in [2]. In that paper it was shown that creation and annihilation operators of correlated fermion pairs, in simple many body systems, satisfy a deformed Heisenberg algebra that can be approximated by q-oscillators. Since the combined pairs of fermions can be viewed as a composite system it seems reasonable to explore the consequences of using q-oscillators as an approximated way to describe composite particles in the context of the formalism of second quantization.

We present here a QFT [3] that creates at any point of the space-time, particles described by a q-deformed Heisenberg algebra interpreting it as a phenomenological theory describing the interaction of composite particles. We construct the propagator, defined as the Dyson-Wick contraction of two fields, for the deformed free theory, present a generalization of Wick's expansion and the results of the scattering $1+2 \to 1'+2'$ up to second order in the coupling constant [4].

The generalization of Wick's expansion is necessary in order to compute perturbatively this scattering process since the propagator is not a c-number and as a consequence some nonstandard results come out. In order to illustrate these, a typical term is considered and calculated in detail.

The result obtained concerning the scattering process shows that the structure of a composite particle, described here phenomenologically by the algebraic structure, modify in a simple but non-trivial way the perturbation expansion for the process under consideration.

In section 2, we realize q-oscillators in terms of physical variables and discuss its interpretation in terms of a phenomenological description of composite particles in the context of the formalism of second quantization. In section 3, we generalize Wick's expansion and compute the scattering process under consideration to second order in the coupling constant. In section 4, we end up with some remarks and discussions on the possibility of constructing a consistent QFT based on a more general algebraic structure known as Generalized Heisenberg Algebra (GHA) [5],[6] and also on the application of this deformed approach to investigate the pion-nucleon interaction in the framework of the linear sigma model.

2. Lattice realization of q-oscillators

The algebra generated by b, $b^\dagger$, and N, described by the relations

$$\begin{aligned} &b\,b^\dagger - q^2\, b^\dagger\, b = 1 \ , \\ &[N, b] = -b \ , \ \left[N, b^\dagger\right] = b^\dagger \ , \end{aligned} \tag{2.1}$$

is known as q-oscillator algebra [1]. Defining $b^\dagger = A^\dagger/(\pm N_0)$, $b = A/(\pm N_0)$, where $N_0^2 = 1 + \alpha_0\,(q^2-1)$, q and α_0 are real numbers, and

$$J_0 = q^{2N}\,\alpha_0 + [N]_{q^2} \ , \tag{2.2}$$

one can see [4] that A, $A^\dagger$ and J_0 satisfy

$$\left[J_0, A^\dagger\right]_{q^2} = A^\dagger, \tag{2.3}$$

$$[J_0, A]_{q^{-2}} = -\frac{1}{q^2} A, \tag{2.4}$$

$$\left[A^\dagger, A\right] = (1-q^2)\, J_0 - 1, \tag{2.5}$$

where $[a,b]_r \equiv a\, b - r\, b\, a$ is the r-deformed commutation of two operators a and b. The above relations correspond to the linear case of the GHA [5],[6] which is an algebraic structure having as generators the creation and annihilation operators of a quantum system, and an operator J_0 which, as shown in [7], is the Hamiltonian of the quantum system under consideration.

Let us consider a one-dimensional lattice in momentum space. The two possible definitions of discrete derivatives on this lattice are

$$(\partial_p f)(p) = \frac{1}{a}\left[f(p+a) - f(p)\right], \tag{2.6}$$

$$(\bar{\partial}_p f)(p) = \frac{1}{a}\left[f(p) - f(p-a)\right], \tag{2.7}$$

where a is the lattice spacing. With these derivatives it is possible to introduce the momentum shift operators

$$T = 1 + a\, \partial_p\,, \tag{2.8}$$

$$\bar{T} = 1 - a\, \bar{\partial}_p\,, \tag{2.9}$$

that move the momentum value by a

$$(T f)(p) = f(p+a)\,, \tag{2.10}$$

$$(\bar{T} f)(p) = f(p-a) \tag{2.11}$$

and satisfy

$$T\bar{T} = \bar{T}T = \hat{1}\,, \tag{2.12}$$

where $\hat{1}$ means the identity on the algebra of functions of p. Finally, we also introduce the momentum operator P [8]

$$(Pf)(p) = p\, f(p)\,. \tag{2.13}$$

In order to present the realization of the deformed Heisenberg algebra eqs. (2.3)-(2.5) in terms of physical operators, we can associate to this one-parameter deformed Heisenberg algebra the one-dimensional lattice we have just presented. Observe that we can write J_0 in this case as

$$J_0 = q^{2P/a}\, \alpha_0 + [P/a]_{q^2}\,, \tag{2.14}$$

with P given by eq. (2.13). The application of the operator P to the vector state $|m\rangle$ gives [7]

$$P\,|m\rangle = m\, a\, |m\rangle\,,\, m = 0, 1, \cdots\,, \tag{2.15}$$

which can be written as $N = P/a$ with $N|m\rangle = m|m\rangle$. Moreover,

$$\bar{T}\,|m\rangle = |m+1\rangle \ , m = 0, 1, \cdots \ , \tag{2.16}$$

where $\bar{T}$ and $T = \bar{T}^\dagger$ are defined in eqs. (2.8-2.9).

Let us now define

$$A^\dagger = S(P)\,\bar{T} \ , \tag{2.17}$$
$$A = T\,S(P) \ , \tag{2.18}$$

where,

$$S(P)^2 = J_0 - \alpha_0 \ , \tag{2.19}$$

α_0 being the lowest J_0 eigenvalue. It was proven in [3] that the realization given in eqs. (2.14) and (2.17)-(2.18) really satisfies eqs. (2.3)-(2.5).

In what follows we discuss an interpretation of the deformed Heisenberg algebra that will be used in the next section. It is well known that Heisenberg algebra is an essential tool in the second quantization formalism because their generators create and annihilate point particles. As in the generalized case the energy difference of any two successive levels is not equal, one can still consider that the ladder operators of the deformed Heisenberg algebra create and annihilate particles with the difference that the total energy of n particles is not equal to n times the energy of each particle. The next question to be answered is what kind of free physical particle can have this non-additive energy.

In [2] it was shown that the algebra of fermion pairs of zero angular momentum can be approximated by the q-oscillator algebra, eq. (2.1). Moreover, the pairing Hamiltonian has the above mentioned non-additivity property. Let us briefly focus on the shell model of nuclear collective motion. Fermion pairs of angular momentum $J = 0$ in the theory of pairing in a single-j shell are created by the pair-creation operator

$$B^\dagger = \frac{1}{\sqrt{\Omega}} \sum_{m>0} (-1)^{j+m}\, f^\dagger_{j,m} f^\dagger_{j,-m} \ , \tag{2.20}$$

with $-j \leq m \leq j$, where $f^\dagger_{j,m}$ are fermion creation operators and $2\Omega = 2j+1$ is the degeneracy of the shell. The pair creation operator just defined and the annihilation operator satisfy a deformed Heisenberg algebra given by

$$\left[B, B^\dagger\right] = 1 - \frac{N_F}{\Omega} \ , \tag{2.21}$$

with $N_F = \sum_{m>0}(f^\dagger_{j,m} f_{j,m} + f^\dagger_{j,-m} f_{j,-m})$, the fermion number operator while the pairing Hamiltonian is $H = -G\Omega B^\dagger B$. In [2] it was shown that the deformed algebra of composite operators given in eq. (2.21) can be approximated by the q-oscillator algebra given in eq. (2.1) with $q = \exp(-1/\Omega)$ and the pairing Hamiltonian being approximated by the q-oscillator Hamiltonian $H = -G\Omega[N]_{q^2}$.

From the fact that the combined pairs of fermions created by the operator $B^\dagger$ can be viewed as a composite system that is approximated by the q-oscillator algebra, eq. (2.1) with $q = \exp(-1/\Omega)$, it seems reasonable to explore the consequences of using q-oscillators as an approximated way to describe composite particles in the context of the formalism of second quantization.

3. First and second order computation and Wick's expansion

In this section we discuss a QFT having as excitations objects described by the one-parameter deformed algebra given in eqs. (2.3)-(2.5). In this QFT the mass spectrum consists of only one particle with mass m. In this case the energy of n particles is not equal to n times the energy of one particle and therefore the energy does not obey the additivity rule. This non-additivity comes from the fact that q-oscillators are seen as an approximated way to describe composite particles in the context of the formalism of second quantization.

3.1 First order analysis

In [3], following similar steps to those used to construct a standard spin-0 quantum field theory [9] we analyzed a deformed QFT to first order in the coupling constant. The initial observation is that the analog of the Heisenberg algebra obeyed by the quantum excitations of a standard QFT is in this case

$$[\chi, P] = -iaQ \;, \tag{3.1}$$
$$[P, Q] = -ia\chi \;, \tag{3.2}$$
$$[\chi, Q] = -2iS(P)\left(S(P+a) - S(P-a)\right), \tag{3.3}$$

where

$$\chi \equiv i\left(S(P)(1 - a\bar{\partial}_p) - (1 + a\partial_p)S(P)\right) = -i(A - A^\dagger) \;, \tag{3.4}$$
$$Q \equiv S(P)(1 - a\bar{\partial}_p) + (1 + a\partial_p)S(P) = A + A^\dagger \;, \tag{3.5}$$

P is defined in eq. (2.13) and ∂_p and $\bar{\partial}_p$ are the left and right discrete derivatives defined in eqs. (2.6) and (2.7), respectively.

Using eqs. (3.4) and (3.5) for χ and Q it is possible to define

$$\phi(\vec{r}, t) = \sum_{\vec{k}} \frac{1}{\sqrt{2\Omega\omega(\vec{k})}} \left(A^\dagger_{\vec{k}}\, e^{-i\vec{k}.\vec{r}} + A_{\vec{k}}\, e^{i\vec{k}.\vec{r}}\right), \tag{3.6}$$

$$\Pi(\vec{r}, t) = \sum_{\vec{k}} \frac{i\omega(\vec{k})}{\sqrt{2\Omega\omega(\vec{k})}} \left(A^\dagger_{\vec{k}}\, e^{-i\vec{k}.\vec{r}} - A_{\vec{k}}\, e^{i\vec{k}.\vec{r}}\right), \tag{3.7}$$

where $\omega(\vec{k}) = \sqrt{\vec{k}^2 + m^2}$, m is a real parameter and Ω is the volume of a rectangular box and

$$\wp(\vec{r}, t) = \sum_{\vec{k}} \sqrt{\frac{\omega(\vec{k})}{2\Omega}}\; S_{\vec{k}}\, e^{i\vec{k}.\vec{r}} . \tag{3.8}$$

We stress that an independent copy of the one-dimensional momentum lattice defined in the previous section was introduced at each point of this $\vec{k}$-lattice so that $P^\dagger_{\vec{k}} = P_{\vec{k}}$ and $T_{\vec{k}}$, $\bar{T}_{\vec{k}}$ and $S_{\vec{k}}$ have the same definitions given previously, eqs. (2.8)-(2.9) and (2.19), through the substitution $P \to P_{\vec{k}}$. Moreover,

$$A^\dagger_{\vec{k}} = S_{\vec{k}}\, \bar{T}_{\vec{k}}\,, \tag{3.9}$$

$$A_{\vec{k}} = T_{\vec{k}}\, S_{\vec{k}}\,, \tag{3.10}$$

$$J_0(\vec{k}) = q^{2P_{\vec{k}}/a}\, \alpha_0 + \left[P_{\vec{k}}/a\right]_{q^2} \ , \tag{3.11}$$

satisfy the same algebra given in eqs. (2.3)-(2.5) for each point of this $\vec{k}$-lattice and the operators $A^\dagger_{\vec{k}}$, $A_{\vec{k}}$ and $J_0(\vec{k})$ commute among them for different points of this $\vec{k}$-lattice.

By a straightforward computation, the Hamiltonian

$$H = \frac{1}{2} \int d^3r \left(\Pi(\vec{r},t)^2 + u|\wp(\vec{r},t)|^2 + \phi(\vec{r},t)(-\vec{\nabla}^2 + m^2)\phi(\vec{r},t) \right) \ , \tag{3.12}$$

where u is an arbitrary number, can be written as

$$H = \frac{1}{2} \sum_{\vec{k}} \omega(\vec{k}) \left(S_{\vec{k}}(N+1)^2 + (1+u)\, S_{\vec{k}}(N)^2 - (q^2-1)\alpha_0 - 1 \right) \ . \tag{3.13}$$

Note that in the limit $q \to 1$ ($u \to 0$), the above Hamiltonian is proportional to the number operator.

The eigenvectors of H form a complete set and span the Hilbert space of this system. They are

$$|0\rangle, \ A^\dagger_{\vec{k}}|0\rangle, \ A^\dagger_{\vec{k}} A^\dagger_{\vec{k}'}|0\rangle \text{ for } \vec{k} \neq \vec{k}', \ (A^\dagger_{\vec{k}})^2|0\rangle, \ \cdots \ , \tag{3.14}$$

where the state $|0\rangle$ satisfies as usual $A_{\vec{k}}|0\rangle = 0$ for all $\vec{k}$ and $A_{\vec{k}}$, $A^\dagger_{\vec{k}}$ for each $\vec{k}$ satisfy the q-deformed Heisenberg algebra given by eqs. (2.3)-(2.5).

Let us define $E^{(n)}(\vec{k})$ as the energy eigenvalue of the state $(A^\dagger_{\vec{k}})^n|0\rangle$. Note that for the Hamiltonian in eq. (3.13) we have $E^{(2)}(\vec{k}) \neq 2E^{(1)}(\vec{k})$ which is a property of non-additivity similar to that we have commented for the composite system made with fermions pairs.

The time evolution of the fields can be studied by means of Heisenberg's equation for $A^\dagger_{\vec{k}}$, $A_{\vec{k}}$ and $S_{\vec{k}}$. Now, let us define

$$\begin{aligned} h(N_{\vec{k}}) &\equiv \frac{1}{2}(1+u+q^2)\left(S^2(N_{\vec{k}}+1) - S^2(N_{\vec{k}}) \right) \\ &\equiv \frac{1}{2}(1+u+q^2)\Delta E(N_{\vec{k}}) \ . \end{aligned} \tag{3.15}$$

Thus, using eqs. (3.13) and (2.3)-(2.5) we obtain

$$\left[H, A^\dagger_{\vec{k}} \right] = \omega(\vec{k})\, A^\dagger_{\vec{k}}\ h(N_{\vec{k}}) \ . \tag{3.16}$$

The Heisenberg equation can be solved and the Fourier transformation shown in eq. (3.6) can thus be written as

$$\phi(\vec{r},t) = \alpha(\vec{r},t) + \alpha(\vec{r},t)^\dagger \ , \tag{3.17}$$

where

$$\alpha(\vec{r},t) = \sum_{\vec{k}} \frac{1}{\sqrt{2\Omega\omega(\vec{k})}}\ A_{\vec{k}}\, e^{i\vec{k}.\vec{r} - iq^{-2}\omega(\vec{k})\, h(N_{\vec{k}})\, t} \ , \tag{3.18}$$

$A_{\vec{k}}$ given in eq. (3.18) is time-independent and $\alpha(\vec{r},t)^\dagger$ is the Hermitian conjugate of $\alpha(\vec{r},t)$.

The Feynman propagator $D_F^N(x_1,x_2)$ defined, as usual, as the Dyson-Wick contraction between [1] $\phi(x_1)$ and $\phi(x_2)$, can be computed using eqs. (2.3)-(2.5) and (3.17)-(3.18) and is given in the integral representation as

$$D_F^N(x) = \frac{-i}{(2\pi)^4}\int \frac{S(N_{\vec{k}}+1)^2 e^{i\vec{k}.\vec{r}-ik_0\,h(N_{\vec{k}})\,t}\ d^4k}{k^2+m^2} - (N \to N-1)\,, \tag{3.19}$$

where the second part of the right hand side of the above equation can be obtained just doing $N \to N-1$. Note that when $q \to 1$, $h(N_{\vec{k}}) \to 1$ and $S_{\vec{k}}(N+1)^2 - S_{\vec{k}}(N)^2 \to 1$, the standard result for the propagator is recovered. It is interesting to point out that this propagator is not a simple c-number since it depends on the number operator N.

We shall now present the result of the first order scattering process $1+2 \to 1^{'}+2^{'}$ for $p_1 \neq p_2 \neq p_1' \neq p_2'$ with the initial state

$$|1,2\rangle \equiv \frac{1}{N_0^2} A_{p_1}^\dagger\, A_{p_2}^\dagger |0\rangle\ , \tag{3.20}$$

and the final state

$$|1^{'},2^{'}\rangle \equiv \frac{1}{N_0^2} A_{p_1^{'}}^\dagger\, A_{p_2^{'}}^\dagger |0\rangle\ , \tag{3.21}$$

where A_{p_i} and $A_{p_i}^\dagger$ satisfy the algebraic relations in eqs. (2.3)-(2.5).These particles are supposed to be described by the Hamiltonian given in eq. (3.12) with an interaction given by $\lambda\int\, :\, \phi(\vec{r},t)^4\, :\, d^3r$. To the lowest order in λ, we have (now S means the standard S-matrix)

$$\langle 1^{'},2^{'}|S|1,2\rangle = -i\lambda\int d^4x\langle 1^{'},2^{'}| : \phi^4(x) : |1,2\rangle\ . \tag{3.22}$$

In [3] we computed the first order scattering process and we obtained the following result

$$\langle 1^{'},2^{'}|S|1,2\rangle = \frac{-6(2\pi)^4 i{N_0}^2\lambda}{Q\Omega^2\sqrt{\omega_{\vec{p}_1}\omega_{\vec{p}_2}\omega_{\vec{p'}_1}\omega_{\vec{p'}_2}}}\delta^4(P_1+P_2-P_1'-P_2')\ , \tag{3.23}$$

where

$$P_i = (\vec{p}_i,\omega_{\vec{p}_i}),\ P_i' = (\vec{p'}_i,\omega_{\vec{p'}_i}) \tag{3.24}$$

and $Q=(1+u+q^2)/2$. Note that when $q \to 1$ we have $N_0 \to 1$, $u=0$, $Q \to 1$ and eq. (3.23) becomes the standard undeformed result [9].

[1] Our notation means that $x_i \equiv (\vec{r_i},t_i)$

3.2 Wick's expansion

Now let us present a generalization of Wick's expansion which is an essential tool in order to compute high order scattering processes in the coupling constant. The propagator in the present case, see eq. (3.19), is not a simple c-number since it depends on the number operator N. This fact induces modifications in the standard Wick expansion.

The consequences of the propagator being not a c-number can already be seen in the Wick's expansion for three fields. After standard calculations we obtain

$$T(\phi(x_1)\phi(x_2)\phi(x_3)) =: \phi(x_1)\phi(x_2)\phi(x_3) : + : \underbracket{\phi(x_1)\phi}(x_2)\phi(x_3) :$$

$$+ : \phi(x_1)\underbracket{\phi(x_2)\phi}(x_3) : + : \underbracket{\phi(x_1)\phi(x_2)\phi}(x_3) : , \tag{3.25}$$

where $: \phi(x_1)\phi(x_2)\phi(x_3) :$ is the standard normal order of the product of three fields and

$$: \underbracket{\phi(x_1)\phi}(x_2)\phi(x_3) := D_F^N(x_1, x_2)\, \phi(x_3) , \tag{3.26}$$

$$: \phi(x_1)\underbracket{\phi(x_2)\phi}(x_3) := \phi(x_1) D_F^N(x_2, x_3) , \tag{3.27}$$

$$: \underbracket{\phi(x_1)\phi(x_2)\phi}(x_3) := D_F^N(x_1, x_3)\alpha(x_2) + \alpha^\dagger(x_2) D_F^N(x_1, x_3) . \tag{3.28}$$

Note that

$$\phi(x_2) D_F^N(x_1, x_3) \neq D_F^N(x_1, x_3)\phi(x_2) \neq D_F^N(x_1, x_3)\alpha(x_2) + \alpha^\dagger(x_2) D_F^N(x_1, x_3) , \tag{3.29}$$

since the propagator depends on the number operator.

In order to establish our notations, we define the field $\phi(x_i)$ as given in eqs. (3.17)-(3.18) with $\vec{k} \to \vec{k}_i$, and $\vec{k}$ being the momentum to be integrated in the propagator. In the case of four fields, or more, another typical difference appears. For instance, for four fields the process of normal ordering the term $\alpha(x_1)\alpha(x_2)\ \alpha^\dagger(x_3)\alpha^\dagger(x_4)$ generates the term $\alpha(x_1) D_N(x_2, x_3)\alpha^\dagger(x_4)$ that is not yet normal ordered. When normal ordering it one sees that since the propagator depends on the number operator one obtains

$$\alpha(x_1) D_N(x_2, x_3)\alpha^\dagger(x_4) = D_{N+\delta_{\vec{k}_1,\vec{k}}}(x_2, x_3) \left[\alpha^\dagger(x_4)\alpha(x_1) + D_N(x_1, x_4)\right] , \tag{3.30}$$

with $D_{N+\delta_{\vec{k},\vec{k}_1}}$ meaning that we substitute $N_{\vec{k}}$ by $N_{\vec{k}} + \delta_{\vec{k},\vec{k}_1}$ in the expression for the propagator, where $\vec{k}$ is the momentum to be integrated. The above expression is obtained by moving $\alpha(x_1)$ to the right hand side of the propagator and leaving the term in normal order. Note that, in consequence of commutating $\alpha(x_1)$ with $D_N(x_2, x_3)$, $N_{\vec{k}}$ inside the final propagator has been increased by $\delta_{\vec{k},\vec{k}_1}$.

With the notations defined above we obtain after standard manipulations

$$
\begin{aligned}
T(\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)) =&: \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) : + : \wick{\c1\phi(x_1)\c1\phi(x_2)\phi(x_3)\phi(x_4)} : \\
&+ : \wick{\c1\phi(x_1)\phi(x_2)\c1\phi(x_3)\phi(x_4)} : + : \wick{\c1\phi(x_1)\phi(x_2)\phi(x_3)\c1\phi(x_4)} : + : \wick{\phi(x_1)\c1\phi(x_2)\c1\phi(x_3)\phi(x_4)} : \\
&+ : \wick{\phi(x_1)\c1\phi(x_2)\phi(x_3)\c1\phi(x_4)} : + : \wick{\phi(x_1)\phi(x_2)\c1\phi(x_3)\c1\phi(x_4)} : + : \wick{\c1\phi(x_1)\c1\phi(x_2)\c2\phi(x_3)\c2\phi(x_4)} : \\
&+ : \wick{\c1\phi(x_1)\c2\phi(x_2)\c1\phi(x_3)\c2\phi(x_4)} : + : \wick{\c1\phi(x_1)\c2\phi(x_2)\c2\phi(x_3)\c1\phi(x_4)} : , \qquad (3.31)
\end{aligned}
$$

where

$$: \wick{\c1\phi(x_1)\c1\phi(x_2)\phi(x_3)\phi(x_4)} := D_N(x_1,x_2) : \phi(x_3)\phi(x_4) : , \qquad (3.32)$$

$$
\begin{aligned}
: \wick{\c1\phi(x_1)\phi(x_2)\c1\phi(x_3)\phi(x_4)} :=& D_N(x_1,x_3)\left[\alpha(x_2)\alpha(x_4) + \alpha^\dagger(x_4)\alpha(x_2)\right] \\
&+\alpha^\dagger(x_2)D_N(x_1,x_3)\phi(x_4) , \qquad (3.33)
\end{aligned}
$$

$$
\begin{aligned}
: \wick{\c1\phi(x_1)\phi(x_2)\phi(x_3)\c1\phi(x_4)} :=& D_N(x_1,x_4)\alpha(x_2)\alpha(x_3) + \alpha^\dagger(x_2)D_N(x_1,x_4)\alpha(x_3) + \\
&\alpha^\dagger(x_3)D_N(x_1,x_4)\alpha(x_2) + \alpha^\dagger(x_2)\alpha^\dagger D_N(x_1,x_4) , \qquad (3.34)
\end{aligned}
$$

$$
\begin{aligned}
: \wick{\phi(x_1)\c1\phi(x_2)\c1\phi(x_3)\phi(x_4)} :=& \alpha(x_1)D_N(x_2,x_3)\alpha(x_4) + \alpha^\dagger(x_1)D_N(x_2,x_3)\alpha(x_4) + \\
&\alpha^\dagger(x_4)\alpha(x_1)D_{N+\delta_{\vec{k},\vec{k}_4}}(x_2,x_3) + \alpha^\dagger(x_1)D_N(x_2,x_3)\alpha^\dagger(x_4) , \qquad (3.35)
\end{aligned}
$$

$$
\begin{aligned}
: \wick{\phi(x_1)\c1\phi(x_2)\phi(x_3)\c1\phi(x_4)} : =& \alpha(x_1)D_N(x_2,x_4)\alpha(x_3) + \alpha^\dagger(x_1)D_N(x_2,x_4)\alpha(x_3) + \\
&\alpha^\dagger(x_3)\alpha(x_1)D_N(x_2,x_4) + \alpha^\dagger(x_1)\alpha(x_3)D_N(x_2,x_4) , \qquad (3.36)
\end{aligned}
$$

$$: \wick{\phi(x_1)\phi(x_2)\c1\phi(x_3)\c1\phi(x_4)} : = : \phi(x_1)\phi(x_2) : D_N(x_3,x_4) , \qquad (3.37)$$

$$: \wick{\c1\phi(x_1)\c1\phi(x_2)\c2\phi(x_3)\c2\phi(x_4)} : = D_N(x_1,x_2)D_N(x_3,x_4) , \qquad (3.38)$$

$$: \wick{\c1\phi(x_1)\c2\phi(x_2)\c1\phi(x_3)\c2\phi(x_4)} : = D_N(x_1,x_3)D_N(x_2,x_4) , \qquad (3.39)$$

$$: \wick{\c1\phi(x_1)\c2\phi(x_2)\c2\phi(x_3)\c1\phi(x_4)} : = D_N(x_1,x_4)D_{N+\delta_{\vec{k},\vec{k}_4}}(x_2,x_3) . \qquad (3.40)$$

It is interesting to call attention to the fact that eqs. (3.35) and (3.40) present a propagator increased by $\delta_{\vec{k},\vec{k}_4}$ in accordance with the discussion presented to obtain eq. (3.30).

3.3 Second order computation

We are going now to present a computation done in [4]. The scattering process $1{+}2 \to 1'{+}2'$ to second order in λ is

$$\langle 1', 2'|S|1,2\rangle_2 = \frac{(-i)^2}{2}\lambda^2 \int\int d^4x\, d^4y \langle 1', 2'|T(:\phi^4(x)::\phi^4(y):)|1,2\rangle \ , \tag{3.41}$$

where T denotes the time-ordered product. In order to convert the time-ordered product into a normal product we use Wick's expansion taking into account that the propagator is not a c-number. Coming from the Wick's expansion of $T(:\phi^4(x)::\phi^4(y):)$ there are three representative terms that contribute to the scattering process of eq. (3.41) up to second order in λ, they are

$$\alpha^\dagger(x)\alpha^\dagger(x)\alpha(y)\alpha(y)D_F^N(x,y)D_F^N(x,y) \ , \tag{3.42}$$
$$\alpha^\dagger(y)\alpha^\dagger(y)\alpha(x)\alpha(x)D_F^N(x,y)D_F^N(x,y) \ , \tag{3.43}$$
$$\alpha^\dagger(x)\alpha^\dagger(y)\alpha(x)\alpha(y)D_F^N(x,y)D_F^N(x,y) \ . \tag{3.44}$$

All the other terms contributing to the second order scattering process are different from the above terms only by the position of the propagators in eqs. (3.42)-(3.44) or by a shift of the type $N_{\vec{k_i}} \to N_{\vec{k_i}} + \delta_{\vec{k_i},\vec{p_j}}$ in the propagators appearing in eqs. (3.42)-(3.44).

Let us first compute the second order contribution to the scattering under consideration coming from the term given in eq. (3.42). As seen in eq. (3.19) the propagator has two terms and we start considering only the first term of the propagator since, as it will be clear in what follows, the second term of the propagator gives a trivial contribution. Thus, putting the representative term given in eq. (3.42) into eq. (3.41), taking the exponentials and $S(N)$ outside the matrix element and using

$$\begin{aligned}\langle 0|A_{\vec{p'}_1}A_{\vec{p'}_1}A^\dagger_{\vec{k}_1}A^\dagger_{\vec{k}_2}A_{\vec{k}_3}A_{\vec{k}_4}A^\dagger_{\vec{p}_1}A^\dagger_{\vec{p}_2}|0\rangle = N_0^4\Big(N_0^4\delta_{\vec{k}_3\vec{p}_1}\delta_{\vec{k}_4\vec{p}_2}\delta_{\vec{k}_1\vec{p'}_1}\delta_{\vec{k}_2\vec{p'}_2}+\\ N_0^2\Delta E(\delta_{\vec{k}_2\vec{p'}_2})\delta_{\vec{k}_3\vec{p}_1}\delta_{\vec{k}_4\vec{p}_2}\delta_{\vec{k}_2\vec{p'}_1}\delta_{\vec{k}_1\vec{p'}_2} + N_0^2\Delta E(\delta_{\vec{k}_4\vec{p}_2})\delta_{\vec{k}_3\vec{p}_2}\delta_{\vec{k}_4\vec{p}_1}\delta_{\vec{k}_1\vec{p'}_1}\delta_{\vec{k}_2\vec{p'}_2}+\\ \Delta E(\delta_{\vec{k}_4\vec{p}_2})\Delta E(\delta_{\vec{k}_2\vec{p'}_2})\delta_{\vec{k}_3\vec{p}_2}\delta_{\vec{k}_4\vec{p}_1}\delta_{\vec{k}_2\vec{p'}_1}\delta_{\vec{k}_1\vec{p'}_2}\Big) \ ,\end{aligned} \tag{3.45}$$

we can sum over the $\vec{k}$'s coming from the Fourier expansion of $\alpha(x)$ given in eq. (3.18) obtaining, after a redefinition of the time as $t \to t/h(0)$, the following result

$$\begin{aligned}\langle 1', 2'|S|1,2\rangle_2^a = \frac{\lambda^2}{2\Omega^2 Q^2(2\pi)^8\sqrt{\omega_{\vec{p}_1}\omega_{\vec{p}_2}\omega_{\vec{p'}_1}\omega_{\vec{p'}_2}}}\int\frac{d^4xd^4yd^4k_1d^4k_2}{(k_1^2+m^2)(k_2^2+m^2)}\\ S(1+\delta_{\vec{k}_1,\vec{p}_1}+\delta_{\vec{k}_1,\vec{p}_2})^2 S(1+\delta_{\vec{k}_2,\vec{p}_1}+\delta_{\vec{k}_2,\vec{p}_2})^2 exp\left[i(\kappa_1+\kappa_2-P'_1-P'_2).x\right.\\ \left.+i(P_1+P_2-\kappa_1-\kappa_2).y\right] \ ,\end{aligned} \tag{3.46}$$

where

$$\kappa_i = \left(\vec{k}_i, h_{i,0}\, k_i^0\right) \ , i=1,2 \ , \tag{3.47}$$
$$k_i = \left(\vec{k}_i, k_i^0\right) \ , \tag{3.48}$$
$$h_{i,0} = h\left(\delta_{\vec{k}_i,\vec{p}_1}+\delta_{\vec{k}_i,\vec{p}_2}\right)/h(0) \ . \tag{3.49}$$

Using the property $\int_{-\infty}^{\infty} dx\, f(x+\delta_{x,x_0}) = \int_{-\infty}^{\infty} dx\, f(x)$ we can integrate eq. (3.46) over x, y and using standard properties of delta functions we can also integrate over k_1 or k_2 obtaining

$$\langle 1^{'}, 2^{'}|S|1,2\rangle_2^a = \frac{N_0^4 \lambda^2}{2\Omega^2 Q^2 \sqrt{\omega_{\vec{p}_1}\omega_{\vec{p}_2}\omega_{\vec{p'}_1}\omega_{\vec{p'}_2}}} \delta^4 \left(P_1 + P_2 - P'_1 - P'_2\right) \, I \ , \tag{3.50}$$

where I is the standard one loop divergent integral that appears in the usual λ-ϕ^4 model with value

$$I = \int d^4k \, \frac{1}{(k^2+m^2)\,[(-k+s)^2+m^2]} \ , \tag{3.51}$$

where $s = P_1 + P_2$. As usual the finite part of this integral can be computed using, for example, the method of dimensional regularization [10] giving the standard result.

We recall again that the propagator (see eq. (3.19)) has two terms and in the above computation we considered only the first term of the propagator. Now let us discuss the consequence of using the second term of the propagator in eq. (3.19) for the first propagator appearing in eq. (3.42) and the first term of the propagator for the second propagator appearing in eq. (3.42). After a similar computation as the one described above that resulted in eq. (3.50) we obtain

$$\frac{N_0^2 S(0)^2 \lambda^2}{2\Omega^2 Q^2 \sqrt{\omega_{\vec{p}_1}\omega_{\vec{p}_2}\omega_{\vec{p'}_1}\omega_{\vec{p'}_2}}} \delta^4 \left(P_1 + P_2 - P'_1 - P'_2\right) \, I \ , \tag{3.52}$$

which gives a trivial result since $S(0)^2 = 0$. We will have this trivial result every time the second term of the propagator enters the game. Thus, the second order contribution to the scattering under consideration coming from the term given in eq. (3.42) is given in eq. (3.50). We note that the difference from the standard spin-0 comes only from the constant N_0^8/Q^2 which goes to one when $q \to 1$.

Now, let us compute the contribution to the scattering in eq. (3.41) which arises from the term shown in eq. (3.43). This computation goes along the same lines as the previous computation and the result is

$$\langle 1^{'}, 2^{'}|S|1,2\rangle_2^b = \frac{N_0^4 \lambda^2}{2\Omega^2 Q^2 \sqrt{\omega_{\vec{p}_1}\omega_{\vec{p}_2}\omega_{\vec{p'}_1}\omega_{\vec{p'}_2}}} \delta^4 \left(P_1 + P_2 - P'_1 - P'_2\right) \, I' \ , \tag{3.53}$$

where $I' = I(s \to -s)$.

Finally, we discus the contribution to the scattering in eq. (3.41) coming from the term shown in eq. (3.44). The first thing to do is to put the representative term given in eq. (3.44) into eq. (3.41), in the sequence, perform the following steps:

1. Take the exponentials and $S(N)$ outside the matrix element,
2. use eq. (3.45),
3. sum over the $\vec{k}$'s coming from the Fourier expansion of $\alpha(x)$,
4. redefine $t \to t/h(0)$,

5. use the property given just below eq. (3.49),

6. integrate over d^4x and d^4y,

we obtain

$$\langle 1^{'}, 2^{'}|S|1,2\rangle_2^c = \frac{N_0^4\lambda^2}{8\Omega^2 Q^2\sqrt{\omega_{\vec{p}_1}\omega_{\vec{p}_2}\omega_{\vec{p'}_1}\omega_{\vec{p'}_2}}}\int \frac{d^4xd^4yd^4k_1d^4k_2}{(k_1^2+m^2)(k_2^2+m^2)}$$
$$\big[\delta^4(k_1+k_2+P_1-P'_1)\delta^4(-k_1-k_2+P_2-P'_2)+$$
$$\delta^4(k_1+k_2+P_2-P'_2)\delta^4(-k_1-k_2+P_1-P'_1)+$$
$$\delta^4(k_1+k_2+P_1-P'_2)\delta^4(-k_1-k_2+P_2-P'_1)+$$
$$\delta^4(k_1+k_2+P_2-P'_1)\delta^4(-k_1-k_2+P_1-P'_2)\big] \ . \tag{3.54}$$

Note that the first two terms in the main bracket correspond to a contribution in the t-channel while the last two in the u- channel. Considering separately the contributions in the two channels we have

$$\langle 1^{'}, 2^{'}|S|1,2\rangle_2^{c,t} = \frac{N_0^4\lambda^2}{8\Omega^2 Q^2\sqrt{\omega_{\vec{p}_1}\omega_{\vec{p}_2}\omega_{\vec{p'}_1}\omega_{\vec{p'}_2}}}\delta^4\left(P_1+P_2-P'_1-P'_2\right)\ I'' \ , \tag{3.55}$$

$$\langle 1^{'}, 2^{'}|S|1,2\rangle_2^{c,u} = \frac{N_0^4\lambda^2}{8\Omega^2 Q^2\sqrt{\omega_{\vec{p}_1}\omega_{\vec{p}_2}\omega_{\vec{p'}_1}\omega_{\vec{p'}_2}}}\delta^4\left(P_1+P_2-P'_1-P'_2\right)\ I''' \ , \tag{3.56}$$

where $I''=I(s\to t)$ and $I'''=I(s\to u)$ with $t=P_1-P'_1$ and $t=P_1-P'_2$.

We have computed so far the contribution to the scattering process $1+2\to 1'+2'$ to second order in λ coming from the representative terms given in eqs. (3.42)- (3.44). The other terms appearing in the generalized Wick's expansion of $T(:\phi^4(x)::\phi^4(y):)$ that contribute to the scattering are of the form given in eqs. (3.42)-(3.44) having the propagator in different positions of the product. Moreover, in these terms, N has possible shifts of the type $N_{\vec{q}_i}\to N_{\vec{q}_i}+n_1\delta_{\vec{q}_i,\vec{k}_1}+n_2\delta_{\vec{q}_i,\vec{k}_2}+n_3\delta_{\vec{q}_i,\vec{k}_3}+n_4\delta_{\vec{q}_i,\vec{k}_4}$, where $n_j=0,1,2,3$, $\vec{q}_i$ is the momentum associated with the propagator and $\vec{k}_j$ the momenta of the fields. But, since we have always a finite number of deltas in this shift and the functions $S(x)$ and $h(x)$ that will carry these shifts are finite at the shifted points, then it will be possible to exclude the finite number of the shifted points. The final result will be independent on the position where the propagator is inside the product shown in eqs. (3.42)-(3.44) and it is also independent of the shifts. Thus the result of using any other term of the Wick's expansion of $T(:\phi^4(x)::\phi^4(y):)$ in eq. (3.41), if it is different from zero, it will be necessarily one of the three results we presented in eqs. (3.50), (3.55) and (3.56).

In summary, the scattering process $1+2\to 1^{'}+2^{'}$ for $p_1\neq p_2\neq p'_1\neq p'_2$ with the initial state and the final state given in eqs. (3.20), (3.21) respectively, where A_{p_i}, $A^\dagger_{p_i}$ satisfy the algebraic relations in eqs. (2.3)-(2.5) and the particles are supposed to be described by the Hamiltonian given in eq. (3.12) with an interaction given by $\lambda\int:\phi(\vec{r},t)^4: d^3r$ is given up to second order in the coupling constant λ as

$$\langle 1^{'}, 2^{'}|S|1,2\rangle = \frac{\lambda N_0^2}{Q}A_1 + \frac{\lambda^2 N_0^4}{Q^2}\left(A_2^s+A_2^t+A_2^u\right) \ , \tag{3.57}$$

where A_1, A_2^s, A_2^t and A_2^u are the same contributions that we find in the *standard* λ-ϕ^4 (non-deformed) model corresponding to the tree level, the s, t and u channels for one-loop level respectively. Then, the contribution we find in the perturbation series due to the phenomenological way we consider the structure of a particle can be interpreted as changing the coupling constant that appears in the Hamiltonian as $\lambda' = N_0^2 \lambda / Q$ where λ' appears in the perturbation expansion of the scattering process $1 + 2 \rightarrow 1^{'} + 2^{'}$. Note that, depending on the values of the parameters of the algebra we could improve or even destroy the convergence of the perturbation series. This result shows that the structure of a particle can change in a non-trivial way the behavior of the perturbation series corresponding to the physical process involving these particles.

4. Final remarks

An approximate description of composite particles in the context of the formalism of second quantization was presented with the use of the q-oscillator algebra. Inspired by a result obtained in [2], we have constructed a QFT which creates at any space-time point, particles described by a q-deformed Heisenberg algebra.

The propagator for the deformed free theory, defined as the Dyson-Wick contraction between $\phi(x_1)$ and $\phi(x_2)$, depends on the number operator, thus being not a c-number anymore and this fact introduces differences in Wick's expansion. We have also shown what these differences are as compared with the standard expansion.

We have presented the scattering process $1+2 \rightarrow 1' + 2'$ up to second order in the coupling constant and the final result given in eq. (3.57) shows that the structure of a composite particle, viewed here by the algebraic structure, modify non-trivially the perturbation expansion of a specific process. As a consequence of our phenomenological way of treating a scattering of composite particles we find that the perturbation expansion corresponding to the scattering under consideration, computed with the standard λ-ϕ^4 interacting term, is given term by term by a factor coming from the algebraic structure multiplying the non-deformed result. This fact may provide interesting surprises when implementing the ideas developed here to specific phenomenological models. Its worth noticing that depending on the values that appear in the scattering coming from the algebra the convergence of the perturbation series can be improved.

Notice that the q-deformed algebra showed in eqs. (2.3)-(2.5) is a particular case of GHA [5],[6]

$$J_0 \, A^\dagger = A^\dagger \, f(J_0), \tag{4.1}$$

$$A \, J_0 = f(J_0) \, A, \tag{4.2}$$

$$\left[A, A^\dagger\right] = f(J_0) - J_0, \tag{4.3}$$

if we choose the function $f(x) = q^2 x + 1$. We believe that in all cases of this GHA where it is possible to realize the creation and annihilation operators as

$$A^\dagger = \sqrt{J_0 - \alpha_0} \, \bar{T}, \tag{4.4}$$

$$A = T \, \sqrt{J_0 - \alpha_0}, \tag{4.5}$$

it will be possible to construct a consistent QFT based on this new algebra following the steps we have done here for q-oscillators. The QFT based on the q-deformed algebra being thus the simplest example of this class of possible QFT.

One of the models describing the interaction of pions and nucleons at low energies [11],[12], the so-called linear sigma model, presents a problem concerning the convergence of the perturbation series. In the phenomenological approach of QFT we have presented to describe the scattering of composite particles, we have found that the convergence of perturbation series can be improved. We believe that this result could be used to investigate the above mentioned problem of convergence in the linear sigma model.

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