

On the connections between Skyrme and Yang Mills theories

Joaquín Sánchez Guillén*

*Facultad de Física USC,
15706 Santiago de Compostela, Spain
E-mail joaquin@fpaxp1.usc.es*

L.A. Ferreira

*Instituto de Física Teórica - IFT/UNESP
Rua Pamplona 145
01405-900 São Paulo-SP, Brazil
E-mail laf@ift.unesp.br*

ABSTRACT: Skyrme theories on S^3 and S^2 , are analyzed using the generalized zero curvature. In the first case, new symmetries and integrable sectors, including the $|B|=1$ skyrmions, are unraveled. In S^2 the relation to QCD suggested by Faddeev is discussed.

1. Introduction

Skyrme theory, based on chiral fields with an stabilizing quartic derivative term [1] at the classical level, was an alternative to the standard field theory approach in ideas and methods. The theory was shown later to correspond to the non-abelian gauge theory with expansions in number of colours (soliton aspects) [2] and momentum (chiral aspects). Faddeev conjectured a more direct connection to pure QCD, restricting the Skyrme chiral fields to the coset $SU(2)/U(1)$ [3]. Non-perturbative progress generally used numerical methods both for ordinary Skyrme [4] as well as for Faddeev σ -model formulation [5], which has been also investigated on the lattice [6]. A generalization of the zero-curvature methods of two dimensional field theory to higher dimensions [7] offered a new possibility for analytical progress, in a scheme which uses gauge techniques and fields as auxiliary connections to study non-linear systems. A zero curvature representation for Skyrme-Faddeev theory was given in [8] among other examples of models defined on the sphere S^2 , and discussed in [9] in connection with QCD. The integrable sector of the S^3 Skyrme theory corresponding to $|B|=1$, was found in [10].

*Speaker.

Here we review and explore further both theories with that integrability method, which is very briefly summarized in next section 2, devoted to the ordinary Skyrme. The gauge ambiguities of the method are exhibited and the choice of the hedgehog Ansatz, found by direct computation in [10], is explained. In section 3 we work out in detail the Skyrme Faddeev case, and we discuss how the gauge formalism of the method clarifies the observations [11] and conjectures of a connection between Skyrme-Faddeev theory and the long distance limit of the non abelian gauge theory.

2. The Skyrme model

The Lagrangian density for the Skyrme model can be written as:

$$L = -\frac{f_\pi^2}{4} \text{tr} \left(U^\dagger \partial_\mu U U^\dagger \partial^\mu U \right) + \frac{1}{32e^2} \text{tr} \left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U \right]^2 \quad (2.1)$$

where f_π and e are phenomenological constants, and U is an unitary matrix representation of a compact Lie group G . The cases of physical interest correspond to G being $SU(2)$ or $SU(3)$. In terms of the Lie algebra valued field $A_\mu = U^\dagger \partial_\mu U = A_\mu^i T_i$ we can write the Lagrangian as:

$$L = -\frac{f_\pi^2}{4} \text{tr} (A_\mu A^\mu) + \frac{1}{32e^2} \text{tr} \{ [A_\mu, A_\nu] [A^\mu, A^\nu] \} \quad (2.2)$$

The equations of motion which can be derived from this Lagrangian are:

$$\partial_\mu (A^\mu - \epsilon [A_\nu, [A^\mu, A^\nu]]) = 0 \quad (2.3)$$

where $\epsilon = 1/4 f_\pi^2 e^2$.

Let us define the auxiliary field

$$\tilde{J}^\mu = A^\mu - \epsilon [A_\nu, [A^\mu, A^\nu]] \quad (2.4)$$

The equations of motion can then be written in the form $\partial_\mu \tilde{J}^\mu = 0$. The space components of the second term in (2.4) can be normalized to the degree of the map $S^3 \rightarrow S^3$, which gives a topological meaning to the baryon number of the solution. Squaring it one gets a lower bound for the energy functional in a given charge sector, which unfortunately can only be saturated in 3 spatial dimensions by $A = 0$. In addition, the bound does not lead to a lower degree equation of the BPS type [12]. In fact, the only known exact solution is the original $B = 1$ *hedgehog* Ansatz for the static Skyrme field:

$$U(\vec{x}) = \exp(i \hat{r} \cdot \vec{\tau} f(r)) \quad (2.5)$$

where $r = |\vec{x}|$ and $\hat{r} = \frac{\vec{x}}{r}$, $\vec{\tau}$ are the Pauli matrices, and $f(r)$ is the profile function. With this unique maximally symmetric Ansatz, it is well known [13] that the equations of motion (2.3) reduce to an ordinary differential equation in f , which has then to be handled numerically (but being an ODE it is an existence proof). One way to progress in the analytical understanding of the Skyrme model, is to study its equations of motion with the geometric approach of ref. [7] as we now explain.

The method is a generalization, for a $(d+1)$ -dimensional space-time, of the well known two dimensional Lax-Zakharov-Shabat zero curvature condition. The construction involves a flat connection on the space of $(d-1)$ -loops (closed $(d-1)$ -dimensional hypersurfaces) which is built from a 1-form \mathcal{A} and a d -form B on space-time. It is possible to find local sufficient conditions on the latter for the loop space connection to be flat. Those conditions involve a non-semisimple Poincaré type algebra which decomposes into a Lie algebra \mathcal{G} and an invariant abelian subalgebra P transforming under some representation R of \mathcal{G} . The local zero curvature conditions are given by

$$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu + [\mathcal{A}_\mu, \mathcal{A}_\nu] = 0; \quad D_\mu \tilde{B}^\mu = \partial_\mu \tilde{B}^\mu + [\mathcal{A}_\mu, \tilde{B}^\mu] = 0 \quad (2.6)$$

where \tilde{B}^μ is the dual of the d -form referred above.

In our $d = 3$ Skyrme case, since $A_\mu = U^\dagger \partial_\mu U$ is flat by construction, it is natural to start with $\mathcal{A}_\mu \equiv A_\mu$. Take $\tilde{B}_\mu \equiv \tilde{J}_\mu^a P_a$, where we have written $\tilde{J}_\mu = \tilde{J}_\mu^a T_a$, with T_a 's being the generators of \mathcal{G} , and P_a 's transforming under the adjoint representation of \mathcal{G} , i.e. $[T_a, T_b] = if_{ab}^c T_c$, $[T_a, P_b] = if_{ab}^c P_c$, and $[P_a, P_b] = 0$. Notice that the Jacobi identities require that $[A_\mu, \tilde{J}^\mu] = 0$. Then it is clear that (2.3) is equivalent to (2.6). We have then expressed Skyrme equations as local zero curvature conditions of [7].

2.1 Constraints. The most simple case

Notice that with $\mathcal{A}_\mu = U^\dagger \partial_\mu U$, the quantities $J_\mu \equiv U \tilde{B}_\mu U^\dagger$ are conserved currents as a consequence of (2.6). Together with \tilde{J}^μ themselves, those are the Noether currents associated to the $G \otimes G$ global symmetry of the Skyrme model. If the equivalence holds only for \tilde{B}_μ being in the adjoint representation, we have just reexpressed the equations of motion with the geometric gauge formalism, while *if it holds for any representation*, we can discover hidden symmetries, as in the $2d$ case of Sine Gordon and Toda theories. In the Skyrme model that can be implemented and the formulation (2.6) can be used to construct an infinite number of conserved currents for some sectors of the Skyrme theory. However, the sectors one gets depend crucially on the choice (gauge) of the zero curvature potentials. In [10] it was constructed an integrable sector containing the charge ± 1 skyrmions. Here we discuss the integrable sector of Skyrme theory obtained from the choice above of A and B . One can just follow the case of the chiral model [14] and introduce, for any integer spin j representation of $SU(2)$, the operator¹

$$\tilde{B}_\mu^{(j)} = -\tilde{J}_\mu^+ P_{+1}^{(j)} + \frac{1}{\sqrt{j(j+1)}} \tilde{J}_\mu^0 P_0^{(j)} + \tilde{J}_\mu^- P_{-1}^{(j)} \quad (2.7)$$

where we have denoted the quantity (2.4) as

$$\tilde{J}_\mu = \tilde{J}_\mu^+ T_+ + \tilde{J}_\mu^0 T_3 + \tilde{J}_\mu^- T_-$$

and where $T_{3,\pm}$ are the usual basis for the angular momentum algebra and $P_m^{(j)}$ transform under the spin j representation of $SU(2)$, i.e. $[T_3, T_\pm] = \pm T_\pm$, $[T_+, T_-] = 2T_3$, $[T_3, P_m^{(j)}] = m P_m^{(j)}$, $[T_\pm, P_m^{(j)}] = \sqrt{j(j+1) - m(m \pm 1)} P_{m \pm 1}^{(j)}$, and $[P_m^{(j)}, P_n^{(j)}] = 0$.

¹Notice that the normalization of the coefficients are due to the fact that $(-P_{+1}^{(1)})$, $(P_0^{(1)}/\sqrt{2})$ and $P_{-1}^{(1)}$ constitute the basis of the adjoint of $SU(2)$ that transforms exactly as T_+ , T_3 and T_- respectively.

Denoting $A_\mu = A_\mu^+ T_+ + A_\mu^0 T_3 + A_\mu^- T_-$, and using the fact that $[A_\mu, \tilde{J}^\mu] = 0$ we get that $A^{\mu,i} \tilde{J}_\mu^j - A^{\mu,j} \tilde{J}_\mu^i = 0$, for $i, j = 0, \pm$. Consequently, for the spin 1 representation we get $[A^\mu, \tilde{B}_\mu^{(1)}] = 0$. However, for $j > 1$ we get that $[A^\mu, \tilde{B}_\mu^{(j)}] = 0$ if and only if

$$A^{\mu,+} \tilde{J}_\mu^+ = A^{\mu,-} \tilde{J}_\mu^- = 0 \quad (2.8)$$

The conclusion we then reach is that if we substitute the operator (2.7) into (2.6) with $\mathcal{A}_\mu \equiv A_\mu = U^\dagger \partial_\mu U$, we get, for $j = 1$, just the equations of motion for the Skyrme model, namely $\partial^\mu \tilde{J}_\mu = 0$. However, if we impose the constraints (2.8) we can get the same equations but with the zero curvature potential \tilde{B} being in any integer spin j representation. That implies that the submodel of the Skyrme theory defined by the equations

$$\partial^\mu \tilde{J}_\mu = 0; \quad A^{\mu,+} \tilde{J}_\mu^+ = A^{\mu,-} \tilde{J}_\mu^- = 0 \quad (2.9)$$

possesses an infinite number of conserved currents given by

$$J_\mu^{(j)} = U \tilde{B}_\mu^{(j)} U^\dagger \equiv \sum_{m=-j}^j J_\mu^{(j),m} P_m^{(j)}; \quad \text{for any positive integer } j \quad (2.10)$$

2.2 The sector of the skyrmion solution

The restriction

$$A_\mu^\pm \tilde{J}_\pm^\mu = 0 \quad (2.11)$$

is highly non-trivial, and it is not clear whether the reduced model has any solutions at all.² For the only known Ansatz (2.5), it turns out that the constrained equations (2.11) in the static case, restricts the profile function $f(r)$ severely. One finds that the conditions (2.11) are solved by

$$f_R(r) = 2 \text{ArcCotan}(cr) \quad (2.12)$$

where c is a constant representing the (inverse) size parameter of the extended solution. The configuration (2.12) does not solve the static equations of motion $\partial_k \tilde{J}_k = 0$. However, it approximates the solution for an interval of the radial variable r which is of physical interest. Plugging (2.12) into the equations of motion, one gets a polynomial in r of order four. Solving it implicitly for c , for the physical values of the couplings, one finds that there exist admissible solutions for values of r up to half a Fermi *strongly peaked* around a very reasonable value of c , between 2 and 3 Fm^{-1} for f_π in the typical range of 60 to 120 GeV . The minimum of the energy for these lower and upper values is 1 and 2 GeV respectively, again as expected. So, for practical purposes, we conclude that the restricted solution (2.12) is in fact a good approximation for values of r of the order of the light particle sizes and for the physical values of the size parameter c .

It is also interesting that this simplified Ansatz was used in [15] to argue the absence of stable solutions in the Susy CP^1 , although the authors warn for the possibility that it might not be a solution, as we see here in the related Skyrme case.

²The constraint is analogous to the chiral model [14], where it is not difficult to obtain solutions, although subject to the scaling instabilities.

We have seen how the geometric method [7] works in the construction of integrable submodels of the Skyrme theory. The great problem is to find the gauge choice for the zero curvature potentials that produce constraints compatible with the equations of motion. The physical solution of the Skyrme turns out not to be in the simple gauge chosen above, namely by starting directly with the adjoint representation as in the case of the chiral model [14].

The correct choice of gauge to get the charge ± 1 skyrmions inside the integrable sector was presented in [10]. One has to write the group element as

$$U = W^\dagger e^{-i\zeta\tau_3} W \quad (2.13)$$

where τ_3 is the diagonal Pauli matrix and

$$W \equiv \frac{1}{\sqrt{1+|u|^2}} \begin{pmatrix} 1 & iu \\ iu^* & 1 \end{pmatrix} \quad (2.14)$$

with ζ being a real scalar field, and u a complex one. Then the zero curvature potentials are taken to be

$$\mathcal{A}_\mu \equiv -\partial_\mu W W^\dagger = \frac{1}{1+|u|^2} \left(-i\partial_\mu u \tau_+ - i\partial_\mu u^* \tau_- + \frac{1}{2} (u\partial_\mu u^* - u^*\partial_\mu u) \tau_3 \right) \quad (2.15)$$

$$\tilde{B}_\mu \equiv -iR_\mu\tau_3 + \frac{2\sin\zeta}{1+|u|^2} \left(e^{i\zeta} S_\mu \tau_+ - e^{-i\zeta} S_\mu^* \tau_- \right) \quad (2.16)$$

where

$$\begin{aligned} R_\mu &\equiv \partial_\mu \zeta - 8\lambda \frac{\sin^2 \zeta}{(1+|u|^2)^2} (N_\mu + N_\mu^*) \\ S_\mu &\equiv \partial_\mu u + 4\lambda \left(M_\mu - \frac{2\sin^2 \zeta}{(1+|u|^2)^2} K_\mu \right) \end{aligned} \quad (2.17)$$

and

$$\begin{aligned} K_\mu &\equiv (\partial^\nu u \partial_\nu u^*) \partial_\mu u - (\partial_\nu u)^2 \partial_\mu u^* \\ M_\mu &\equiv (\partial^\nu u \partial_\nu \zeta) \partial_\mu \zeta - (\partial_\nu \zeta)^2 \partial_\mu u \\ N_\mu &\equiv (\partial^\nu u \partial_\nu u^*) \partial_\mu \zeta - (\partial_\nu \zeta \partial^\nu u) \partial_\mu u^* \end{aligned} \quad (2.18)$$

One can check that the conditions (2.6) with the potentials (2.15) and (2.16) are equivalent to the equations of motion (2.3).

By extending the potential (2.16) to any integer spin j representation, in a manner similar to the one we did in (2.7), one gets highly non-trivial constraints. However, in the static case those constraints reduce to the conditions

$$\vec{\nabla} u \cdot \vec{\nabla} u = 0 ; \quad \vec{\nabla} u \cdot \vec{\nabla} \zeta = 0 \quad (2.19)$$

They are easily solved by the time independent configurations

$$\zeta = \zeta(r) \quad u = u(z) \quad u^* = u^*(z^*) \quad (2.20)$$

where the coordinates are such that the metric is

$$ds^2 = (dr)^2 + \frac{4r^2}{(1+|z|^2)^2} dz dz^* \quad (2.21)$$

If one takes $u = z$ and $u^* = z^*$ the decomposition (2.13) becomes the hedgehog ansatz (2.5), with $\zeta(r)$ playing the role of the profile function $f(r)$. So, the skyrmions of unity charge belong to the integrable sector. The rational map ansatz are particular cases of the configurations (2.20), and so solve the constraints (2.19). However, the rational maps associated to charge greater than 1 do not provide solutions for the Skyrme model, but just approximations to the true solutions.

Summarizing, we have a submodel of the Skyrme theory with an infinite number of local conserved currents, and that possesses the charges ± 1 skyrmions as solutions.

3. The Skyrme-Faddeev model

In view of the above results it is natural to attempt to go from the winding number charge of $S^3 \rightarrow S^3$ to the Hopf map $S^3 \rightarrow S^2$ reducing the target space to the sphere $S^2 \equiv SU(2)/U(1)$. The topological charge becomes the linking number of the preimages of points of S^2 . This is what Faddeev proposed, looking for the string of QCD. The solitons would have then knot configurations and the simplest allowed solution would be axially symmetric. The action for the Skyrme-Faddeev model is then given by

$$S = \int d^4x \left(m^2 (\partial \mathbf{n})^2 - \frac{1}{e^2} (\partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n})^2 \right) \quad (3.1)$$

where \mathbf{n} is a $SU(2)$ triplet of scalar field with unit norm, $\mathbf{n}^2 = 1$ and m is a parameter with dimensions of mass. A potential term can be added [16] to circumvent the global problems with colour in the glueball interpretation. Such explicit breaking of the global symmetry was first suggested in [6] to avoid spontaneous Goldstone modes, incompatible with the mass gap of pure QCD. These terms are also required for the pion mass and phenomenological application in the ordinary Skyrme case.

On the sphere the complex u field of the stereographic projection is very useful

$$\mathbf{n} = \frac{1}{1+|u|^2} (u + u^*, -i(u - u^*), |u|^2 - 1) ; \quad u \equiv u_1 + iu_2 = \frac{n_1 + in_2}{1 - n_3} \quad (3.2)$$

The energy for static configurations on the Skyrme-Faddeev model is easily found [3, 8],

$$E = E_1 + E_2 \quad (3.3)$$

with

$$E_1 \equiv 4m^2 \int d^3x \frac{|\nabla u|^2}{(1+|u|^2)^2}$$

$$E_2 \equiv \frac{8}{e^2} \int d^3x \frac{(|\nabla u|^4 - (\nabla u)^2 (\nabla u^*)^2)}{(1+|u|^2)^4} \quad (3.4)$$

All models on the sphere S^2 , independent of the dimension of space-time, have a convenient natural formulation of the zero curvature (2.6) in the approach of [7] given by

$$\begin{aligned} A_\mu &= -\partial_\mu W W^{-1} \\ &= \frac{-i}{(1+|u|^2)} \left((\partial_\mu u + \partial_\mu u^*) T_1 + i(\partial_\mu u - \partial_\mu u^*) T_2 + i(u\partial_\mu u^* - u^*\partial_\mu u) T_3 \right) \end{aligned} \quad (3.5)$$

where W is the group element given in (2.14), T_i being the usual basis of $SU(2)$, $[T_i, T_j] = i\varepsilon_{ijk} T_k$. To obtain the Skyrme-Faddeev's model equations of motion from (2.6) one takes

$$\tilde{B}_\mu = \frac{1}{1+|u|^2} \left(L_\mu P_1^{(1)} - L_\mu^* P_{-1}^{(1)} \right) \quad (3.6)$$

with $P_i^{(1)}$ being the same as in (2.7), and

$$L_\mu \equiv m^2 \partial_\mu u - \frac{4}{e^2} \frac{K_\mu}{(1+|u|^2)^2} \quad (3.7)$$

and K_μ is defined in (2.18).

3.1 The rotor spectrum

Models on the sphere have also in common an *integrable* sector given by the constraint ³

$$(\partial u)^2 = 0 \quad (3.8)$$

Indeed, if one replaces in (3.6) $P_{\pm 1}^{(1)}$ by $P_{\pm 1}^{(j)}$, with j integer, then the zero curvature (2.6) gives the Skyrme-Faddeev's model equations of motion plus the constraint (3.8). Consequently, such submodel has an infinite number of local conserved quantities.

We observe that the scaling stability of the static solutions under the Derrick's theorem requires that the two terms in the energy in (3.3) should be equal

$$E_1 = E_2 \quad (3.9)$$

For the submodel, the second term of E_2 in (3.4) does not exist and that relation implies

$$\int d^3x \mathcal{J} = \int d^3x \mathcal{J}^2 \quad (3.10)$$

where \mathcal{J} is

$$\mathcal{J} = \frac{2}{m^2 e^2} \frac{|\nabla u|^2}{(1+|u|^2)^2} \quad (3.11)$$

Therefore, the submodel presents a rotor like spectrum, with energy given by

$$E = 2m^4 e^2 \int d^3x \mathcal{J} (\mathcal{J} + 1) = 4m^4 e^2 \int d^3x \mathcal{J} = 4m^4 e^2 \int d^3x \mathcal{J}^2 \quad (3.12)$$

³For the simplest $O(3)$ model in $2+1$ this constraint generalizes the Cauchy Riemann conditions of the *baby Skyrminion* solution [7].

3.2 Gauge vacua and knots

The geometrical formulation of Skyrme-Faddeev model contains an intriguing property, which can be relevant for the connection with the gauge theory, as observed recently in the context of lattice approach [6] and [11].

Writing explicitly the components along the step operators of the auxiliary flat connection (3.5) (as in eq. (6.58) of [7]) as

$$A_j^1 \equiv \frac{\partial_j u + \partial_j u^*}{(1 + |u|^2)} \quad A_j^2 \equiv i \frac{\partial_j u - \partial_j u^*}{(1 + |u|^2)} \quad (3.13)$$

one has

$$A_j^1 A_j^1 + A_j^2 A_j^2 = 4 \frac{|\nabla u|^2}{(1 + |u|^2)^2} \quad (3.14)$$

and

$$(A_i^1 A_j^2 - A_j^1 A_i^2)^2 = 8 \frac{|\nabla u|^4 - (\nabla u)^2 (\nabla u^*)^2}{(1 + |u|^2)^4} \quad (3.15)$$

Consequently, the static energy (3.3) reads

$$\begin{aligned} E &= \int d^3x \left((A_j^1 A_j^1 + A_j^2 A_j^2) + (A_i^1 A_j^2 - A_j^1 A_i^2)^2 \right) \\ &= \int d^3x \left(4 \frac{|\nabla u|^2}{(1 + |u|^2)^2} + 8 \frac{|\nabla u|^4 - (\nabla u)^2 (\nabla u^*)^2}{(1 + |u|^2)^4} \right) \end{aligned} \quad (3.16)$$

Where $e = 1 = m$ has been taken (notice that eq. (12) of [11] corresponds to $e = \sqrt{2}$) As observed in [6] the first term is formally the functional used (upon minimization) to fix non-abelian theories to the so called maximal abelian gauge (MAG)⁴[18]. This suggests then that the minima of the Skyrme-Faddeev, knot configurations with topological charge given by linking numbers, may correspond to the vacua of the nonabelian theory, fixed to maximal abelian gauge.

Our analysis shows, firstly, that the static energy does not correspond strictly to the MAG, as it involves diagonal components from the commutator in the second term. Those diagonal colour components are absent in the *submodel*, since due to the constraint (3.8), the second term involves just $|\nabla u|^4$, the square of the first term, which only has transverse colour degrees of freedom. Moreover, it is more simple and it has a rotor spectrum. Therefore, definite results about exact (or approximate) solutions of the Skyrme-Faddeev model, will be relevant for the MAG procedure, and *vice versa*.

Another result from the analysis is that the commutator term for the energy of the full model, involves the diagonal component as a curl, i.e. as chromomagnetic potential, since the connection A is flat, which is relevant for the results of dual variables in the connection with with QCD [16]. It also shows that the Skyrme-Faddeev energy cannot be the functional given by space integral of A^2 , which has been investigated by numerical and

⁴This mimics the abelian Higgs phenomenon and it should correspond to the monopole condensation scenario of confinement [17]

analytical methods [19] and [6]. The idea of breaking splicitly the global $SO(3)$ symmetry, has been also discussed in our approach [9]. With such an additional potential term, while it is possible to have infinite conserved currents, the chances of finding stable solutions are reduced considerably.

4. Conclusions

We have reviewed applications of the generalized zero curvature approach, based on gauge techniques, to the Skyrme theories, which capture topological features of the gauge theory. The original Skyrme theory [10], is specially appropriate to understand how the method works and its difficulties. The results for the *integrable sector* of the Skyrminion Ansatz, found by direct computation in [10], are explained and some useful details are provided.

For the Skyrme Faddeev model we paid special attention to the observations that the auxiliar gauge formalism allows to look at the Skyrme Faddeev model as a gauge fixing of the nonabelian theory. Our analysis shows that the static energy corresponds strictly to the functional minimized in MAG fixing procedure only in the reduced submodel, which is more simple and it presents a rotor spectrum. In the full model it has still diagonal degrees of freedom, of the chromomagnetic type. We conclude, in agreement with results from perturbative [20] and lattice methods [6], that there is some evidence for the Skyrme Faddeev model representing global properties of the pure non-abelian theory in the infrared, but that some ingredients are missing and more work is requiered. And that the generalized zero curvature method can be useful for that, as it gives physical interpretation to the gauge dependent quantities from non-linear models, for which one learns in turn from the gauge theory.

Acknowledgments. J.S.G. gratefully acknowledges Fapesp for financial support, IFT for hospitality and the organizers for the stimulating and pleasant atmosphere. L.A.F. is partially supported by CNPq-Brasil.

References

- [1] T.H.R. Skyrme *Nucl.Phys.* **31**, 556 (1961) and *J. Math. Phys.* **12** (1971) 1735.
- [2] E.Witten *Nucl. Phys.* **B160** (57) 1979.
- [3] L. Faddeev *Einstein and several contemporary tendencies in the Field Theory of Elementary particles* in Relativity, Quanta and Cosmology voll. M. Pantaleo and F. De Finis (eds.) Johnson Reprint, 1979 and in *40 Years in Mathematical Physics* , (World Scientific, 1995) L. Faddeev and A. Niemi, *Nature* **387** (1997) 58.
- [4] C.J. Houghton, N.S. Manton and P. M. Sutcliffe, *Nucl. Phys.* **B510** (507) 1998; R.A. Battye, C.J. Houghton and P. M. Sutcliffe, hep-th/0210147.

- [5] R.A. Battye, P.M. Sutcliffe, *Phys. Rev. Lett.* **81** (1998) 4798;
J. Hietarinta and P. Salo, *Phys. Rev.* **D 62** (2000) 81701.
- [6] L. Dittmann, T. Heinzl and A. Wipf, *Nucl.Phys.Proc. Suppl.* **106** (2002) 649, hep-lat/0110026;
L.Dittmann, T. Heinzl and A. Wipf, hep-lat/0210021.
- [7] O. Alvarez, L. A. Ferreira and J. Sánchez Guillén, *Nucl. Phys.* **B529** (1998) 689,
hep-th/9710147.
- [8] H.Aratyn, L. A. Ferreira and A.H. Zimerman, *Phys. Lett.* **456B** (1999) 162, hep-th/9902141.
- [9] J. Sánchez-Guillén *Phys. Lett.* **548B** (2002) 252, *Err. Phys. Lett.* **550B** (2002) 220 ,
hep-th/0208215.
- [10] L. A. Ferreira, J. Sánchez-Guillén, *Phys. Lett.* **504B** (2001) 195, hep-th/0010168.
- [11] P.van Baal and A. Wipf, *Phys. Lett.* **515B** (2001) 181, hep-th/0105141.
- [12] B. A. Dubrovine, S.P. Novikov, A. T. Fomenko. Contemporary Geometry. Part II. " Nauka ",
Moscow 1979.
- [13] I. Zahed, G.E. Brown, *Physics Reports* **142**(1986) 1.
- [14] D.Gianzo, J.O. Madsen and J. Sánchez-Guillén,*Nucl. Phys.* **B537** (1999) 586;
E.E.Leite and L.A. Ferreira, *Nucl. Phys.* **B547** (1999) 471;
K. Fujii, Y.Homma and T. Suzuki *Phys. Lett.* **438B** (1998) 290.
- [15] E.A. Bergshoeff, R. Nepomechie and H. J. Schnitzer; *Nucl. Phys.* **B249** (1985) 93.
- [16] L. Faddeev and A.I. Niemi *Phys. Lett.* **525B** (2001) 195.
- [17] G.t'Hooft, *Nucl. Phys.* **B190** (1981) 555; S.Mandelstam *Phys. Lett.* **53B** (1975) 476.
- [18] A.S. Kronfeld, G. Schierholz and U.J. Wiese, *Nucl. Phys.* **B293** (1987) 461; see H. Ichie and
H. Suganuma, hep-lat/9906005 for a comprehensive review.
- [19] L. Gubarev, L. Stodolsky and V.I. Zakharov *Phys. Rev. Lett.* **86** (2001) 2220;
M.A. Semenov-Tyan-Shanskii and V.A. Franke, *Steklov AN SSSR*, **120** (1982), 159.
Translation Plenum Press, N.Y. 1986, p.999;
P. van Baal hep-th/9511119; D. Zwanziger *Progr.Theor. Phys. Suppl* **131** (1998) 233.
- [20] L. Faddeev and A. Niemi *Phys. Rev. Lett.* **82** (1999) 1624;
H.Gies, *Phys. Rev.* **D63** (2001) 125023, hep-th/0102026;
Y.M. Cho, H.W.Lee and D.G. Pak, hep-th/0105198.