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Testing the electroweak theory at e^+e^- Linear Colliders

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Abstract: Some selected tests of the electroweak theory at an e^+e^- Linear Collider are discussed. Improved measurements of fundamental parameters such as the effective weak mixing angle, the W-boson mass, or the top-quark mass, allow to strengthen the precision tests of the electroweak theory considerably. Moreover, an e^+e^- Linear Collider is an ideal instrument to investigate the Higgs sector of the Standard Model or, more generally, the mechanism of electroweak symmetry breaking. To this end, also the properties and interactions of the top quark can be studied accurately. An e^+e^- Linear Collider is also a well-suited tool to study physics beyond the Standard Model. Thus, for instance, the spectrum of supersymmetric theories can be explored with high precision, and the underlying fundamental theory can be reconstructed. In order to achieve all these goals, a lot of theoretical precision calculations are necessary.

1. Introduction

The Standard Model (SM) of particle physics was established during the past decades by an intensive interaction between theory and experiment. Gradually, experimental analyses have confirmed the basic physical concepts. Leptons and quarks were discovered as the fundamental constituents of matter. The photon, the W and Z bosons, and the gluons were identified as the carriers of the electromagnetic, weak, and strong interactions. Electromagnetic and weak interactions have been unified within the electroweak gauge-field theory, and the QCD gauge theory has been confirmed as the theory of the strong interactions.

In the last few years many aspects of the model, in particular in the electroweak sector, have been accurately tested, some to the per-mille level. The mass of the top quark was already constrained by quantum corrections before it was directly measured. Overall, the experimental analysis is in remarkable agreement with the electroweak SM, although some 2–3σ deviations have been identified, and neutrino oscillations have been established.

On the other hand, several aspects or the SM have not been confirmed or precisely tested so far. The most important issue in this respect is the Higgs boson. Apart from the

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direct lower bound, $M_{\mathrm{H}} > 114.4\,\mathrm{GeV}$ [1], and the indirect upper bound, $M_{\mathrm{H}} \lesssim 193\,\mathrm{GeV}$ [2], on the Higgs-boson mass, the Higgs sector of the SM is so far unexplored. The uncovering of the mechanism of electroweak symmetry breaking is one of the central problems to be solved by experiments at future colliders. But also the properties of the top quark and the self-couplings of the gauge bosons have not yet been tested with high precision.

Despite the great successes of the SM, it leaves many deep questions open. Here we list only some of them: The mass spectrum of the fermions is not explained, and CP violation is not understood at the level required for an explanation of the excess of matter over antimatter in the universe. The existence and number of families and the quantization of charge are not addressed. Moreover, a fundamental quantum-mechanical description of gravity has not been formulated.

A specific flaw of the SM is the so-called hierarchy problem. The SM is commonly understood as the low-energy limit of a more complete theory. This should eventually also include gravity. As a consequence, scales much higher than the electroweak scale appear, like the grand-unified scale $M_{\mathrm{GUT}} \approx 10^{16}\,\mathrm{GeV}$ or the Plank scale $M_{\mathrm{Pl}} = \sqrt{\hbar c/G_{\mathrm{N}}} \approx 10^{19}\,\mathrm{GeV}$. Since radiative corrections to M_{H}^2 are proportional to M_{GUT}^2 or M_{Pl}^2, the natural scale for the Higgs-boson mass is $M_{\mathrm{H}} \sim M_{\mathrm{GUT}}$ or $M_{\mathrm{H}} \sim M_{\mathrm{Pl}}$. On the other hand, the consistency of the SM requires $M_{\mathrm{H}} \lesssim 1\,\mathrm{TeV}$. Otherwise, e.g. the amplitude of W-boson scattering, $\mathrm{WW} \to \mathrm{WW}$, would violate unitarity at $\sqrt{s} \sim 1.2\,\mathrm{TeV}$. Thus, within the SM an enormous fine tuning is required to stabilize M_{H} at the TeV scale, order by order in perturbation theory.

Several solutions of the hierarchy problem have been proposed. Supersymmetry can be invoked to stabilize the low electroweak scale. By introducing extra space–time dimensions, the scale of D-dimensional gravity can be arranged to lie in the TeV region. On the other hand, if there is no light Higgs boson, the electroweak interaction becomes strong in the TeV region leading to a completely different phenomenology at these energies.

In order to improve the tests of the SM and to find evidence for physics beyond, one has to increase the energy and/or the accuracy of the experiments. To this end, high-energy colliders with high luminosity are required. The Large Hadron Collider (LHC) provides centre-of-mass energies up to 14 TeV, but this energy is distributed over the constituents of the colliding protons. Moreover, the interesting events have to be extracted from a huge background of standard hadronic strong-interaction events. An $\mathrm{e^+e^-}$ linear collider (LC), on the other hand will allow for cleaner and more precise experiments and can complement the research at the LHC even if the energy in only below 1 TeV.

Several versions of LCs have been proposed [3, 4, 5]. A typical example is TESLA [5]. It is planned to operate at energies between 90 and 500 GeV with a possible extension to 800 GeV. The projected luminosity ranges from $50\,\mathrm{fb}^{-1}$ at 90 GeV to $500\,\mathrm{fb}^{-1}$ at 800 GeV. An electron polarization of 80% is expected, and a positron polarization of 60% appears to be possible.

An $\mathrm{e^+e^-}$ linear collider allows to approach many open questions. Here we can only sketch some of them. The following discussion is essentially based on Refs. [6, 7, 8, 9, 10, 11].

2. Electroweak precision tests

The SM of the electroweak interaction is a renormalizable quantum field theory and thus allows accurate theoretical predictions. High precision is needed for the extraction of parameters, such as $M_{\mathrm{W}}, m_{\mathrm{t}}, M_{\mathrm{H}}$, Yukawa couplings, etc., for tests of the theory at its quantum level, and for the establishment of model-dependent parameter relations. In the SM these include the relation between the gauge-boson masses and the muon decay constant and relations between fermion masses and Yukawa couplings. In the past, high precision measurements allowed for a prediction of m_{t}, presently they constrain M_{H}. In a similar way, electroweak precision tests can also be performed for the Minimal Supersymmetric Standard Model (MSSM), which is also a renormalizable quantum field theory.

The electroweak SM has been tested in the past, in particular at LEP, with an accuracy at the per-mille level. A global fit to the electroweak precision data reveals a generally good agreement [2]. There are, however, some observables, such as the anomalous magnetic moment of the muon, the forward–backward asymmetry of bottom-quark production, and the on-shell weak mixing angle measured by NuTeV, which show deviations from the SM at the 2–3σ level. At present it is not clear whether these are due to statistical fluctuations, an underestimation of experimental or theoretical errors, or due to new physics. Ignoring these problems, one can extract an upper limit on the mass of the Higgs boson of $M_{\mathrm{H}} < 193\,\mathrm{GeV}$ (95% CL) from the fit to the SM [2].

An $\mathrm{e^+e^-}$ Linear Collider will allow us to considerably improve the precision tests [8]. In the GigaZ option of TESLA about 10^9 Z bosons can be produced per year which is about 100 times the yield of LEP1. Using polarized beams, the effective weak mixing angle $\sin^2\theta^{\ell}_{\mathrm{eff}}$, which is defined by the Z-boson–lepton couplings, can be measured from the left–right asymmetry A_{LR} with a precision of $\delta\sin^2\theta^{\ell}_{\mathrm{eff}} \sim 10^{-5}$. For the total and partial Z widths a factor 2–3 improvement in the experimental accuracy is possible. The experimental knowledge of the partial width $\Gamma(\mathrm{Z}\to\mathrm{b\bar{b}})$ can be improved by a factor five, the one of the b-quark forward–backward asymmetry even by a factor 15.

The W-boson mass can be determined from a scan over the W-pair production threshold. With TESLA one can collect an integrated luminosity of $100\,\mathrm{fb}^{-1}$ per year at $\sqrt{s}\sim 161\,\mathrm{GeV}$. A simulation shows that an error on the W-boson mass of $\delta M_{\mathrm{W}} \approx 6\,\mathrm{MeV}$ is reachable [12]. This requires that the knowledge of the absolute beam energy can be controlled to better than 2.5 MeV, which might be achievable with some additional effort.

The improved determinations of the effective electroweak mixing angle, $\sin^2\theta^{\ell}_{\mathrm{eff}}$, and the W-boson mass, M_{W}, enable improved electroweak precision tests. For example, they allow to predict the Higgs-boson mass with an uncertainty of 5%. The potential of future SM precision tests is illustrated in Figure 1 and Table 1 [11, 13].

In order to extract the SM parameters with the precision mentioned above adequate theoretical predictions are mandatory. The effective weak mixing angle is extracted from fermion-pair production. For this process presently the complete electroweak one-loop corrections, the leading two-loop corrections, and partial non-leading two-loop corrections are available. The present theoretical uncertainty can be estimated to $\delta\sin^2\theta^{\ell}_{\mathrm{eff}} \sim 7\times10^{-5}$ [11]. In order to reduce this uncertainty to the level required for a LC, the complete

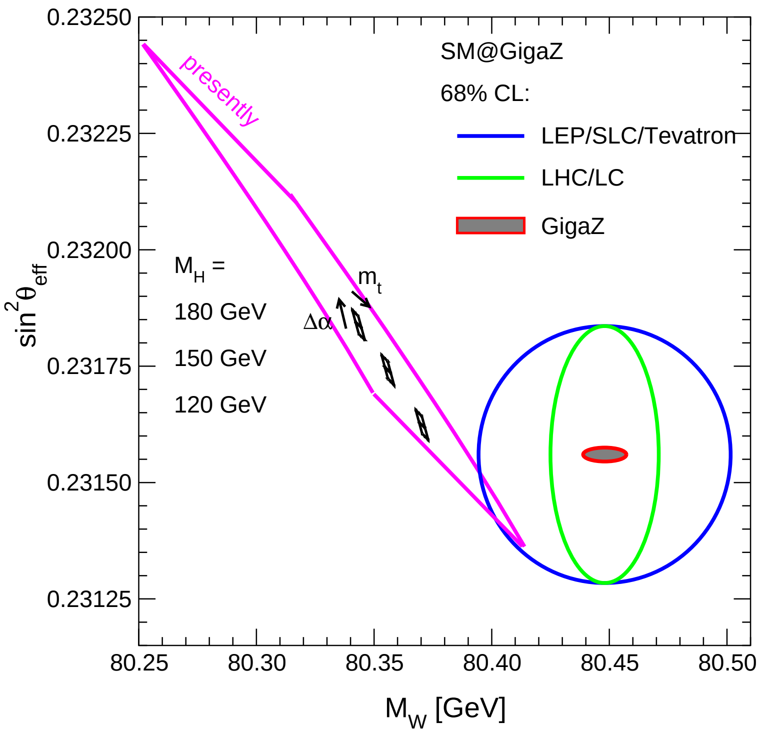

Figure 1: SM prediction for $M_{\rm W}$ and $\sin^2\theta^\ell_{\rm eff}$ versus prospective future accuracies assuming $\delta\Delta\alpha = \pm 7\times 10^{-5}$ and $\delta m_{\rm t} = \pm 200\,$MeV

electroweak two-loop corrections to $\mathrm{e^+e^-} \to \ell\bar\ell$ and the leading higher-order corrections have to be evaluated. A further important uncertainty arises from $\alpha(M_{\rm Z})$, i.e. from the hadronic contribution to the running of α, which is obtained from a dispersion integral over the hadronic cross section $\sigma(\mathrm{e^+e^-}\to \mathrm{hadrons})$ and thus from experimental data. The present experimental uncertainty $\delta\Delta\alpha = 3.9\times 10^{-4}$, induces an uncertainty $\delta\sin^2\theta^\ell_{\rm eff}\sim 1.4\times 10^{-4}$ [14]. By measuring the cross section $\sigma(\mathrm{e^+e^-}\to \mathrm{hadrons})$ with an accuracy below 1% for $\sqrt{s}\lesssim 10\,$GeV one could reduce the error on the hadronic contributions to $\alpha(M_{\rm Z})$ down to $\delta\Delta\alpha = 5\times 10^{-5}$ corresponding to $\delta\sin^2\theta^\ell_{\rm eff}\sim 1.8\times 10^{-5}$.

	now	TeV./LHC	GigaZ
$\delta\sin^2\theta_{\rm eff}\times 10^5$	17	17	1.3
$\delta M_{\rm W}$ [MeV]	33	15	6
$\delta m_{\rm t}$ [GeV]	5.1	2.0	0.13
$\delta M_{\rm H}$ [MeV]	—	100	50

Table 1: Expected improvement in precision at various colliders for $\sin^2\theta_{\rm eff}$, $M_{\rm W}$, $m_{\rm t}$ and the (lightest) Higgs boson mass, $M_{\rm H}$, assuming $M_{\rm H} = 115\,$GeV

The W-boson mass is measured in W-pair production either from the W-pair threshold or from the reconstruction of the W-decay products. Presently, the full electroweak $\mathcal{O}(\alpha)$ corrections in double-pole approximation (DPA) to $\mathrm{e^+e^-}\to\mathrm{W^+W^-}\to 4f$ are known and implemented into the Monte Carlo event generators YFSWW [15, 16] and RacoonWW [17, 18]. Higher-order initial-state-radiation corrections are included in the leading-logarithmic approximation. The theoretical uncertainty (TU) of the total W-pair-production cross section has been estimated to be $\lesssim 0.5\%$ in the energy range between 170 and 500 GeV [15, 17, 19].

Based on a comparison between YFSWW and RacoonWW, for the invariant-mass and angular distributions a TU of 0.5–1% has been derived. The TU on the reconstructed W-boson mass was estimated to be $\sim 5\,\mathrm{MeV}$ [20] and the TU for the anomalous triple gauge-boson coupling parameter λ, which is equal to 1 in the SM and contributes to the anomalous magnetic moment of the W boson, to be of the order of 0.005 [21]. Near the W-pair threshold ($\sqrt{s} \lesssim 170\,\mathrm{GeV}$) the TU of the DPA approach runs out of control because of the increasing importance of the non-doubly-resonant background. The TU of an improved Born approximation, which is valid in this region, is about 2%. All these TUs are sufficient for the present experiments at LEP2.

However, without reducing the TU to the level of 0.1%, the aimed precision of 6 MeV in the M_{W} determination from a threshold scan will be impossible. This level of precision is also required for measurements of the total cross section at LC energies, which will have an accuracy of a few per mille. To this end, the full $\mathcal{O}(\alpha)$ corrections to $\mathrm{e}^+\mathrm{e}^- \to 4f$ for all final states and the most important two-loop effects have to be included.

At high scattering energies, $\sqrt{s} \gg M_{\mathrm{W}}$, the radiative corrections are dominated by logarithms of the form $\alpha^n \ln^m(s/M_{\mathrm{W}}^2)$, $1 \le m \le 2n$; the leading ones ($m = 2n$) are known as Sudakov logarithms. While these terms are implicitly contained in the present DPA approaches at the one-loop level, the higher-order logarithms ($n \ge 2$) are not yet included in existing generators. These terms are potentially relevant for $\sqrt{s} \gtrsim 500\,\mathrm{GeV}$. The existing efforts in the calculation of these logarithms are reviewed in Ref. [22].

3. Higgs physics

Within the SM, the fundamental particles, electroweak gauge bosons, leptons, and quarks, acquire masses by interacting with the scalar Higgs field in the ground state. To accommodate the well-established electromagnetic and weak phenomena, the Higgs mechanism requires at least one weak iso-doublet scalar field. After absorbing three Goldstone modes to build up the longitudinal polarization states of the $\mathrm{W}^\pm$ and Z bosons, one degree is left which corresponds to a real scalar particle, the Higgs boson.

In the SM, the Higgs-boson mass M_{H} is the only unknown parameter. Once M_{H} is fixed, the profile of the Higgs boson is uniquely determined. If the Higgs boson exists and has no exotic properties, the LHC must find a particle compatible with it. However, the proof that this particle is responsible for mass generation requires a LC. Three steps are necessary to establish the Higgs mechanism experimentally as the mechanism for generating the masses of the fundamental particles:

- the Higgs boson must be discovered;
- the couplings of Higgs boson to gauge bosons and fermions must be proven to be proportional to their masses: $g_{ff\mathrm{H}} \propto m_f/v$, $g_{VV\mathrm{H}} \propto M_V/v$;
- the Higgs potential generating the non-zero vacuum expectation value must be reconstructed by determining the Higgs self-couplings.

The main SM Higgs-production processes at an $\mathrm{e^+e^-}$ LC are the Higgs-strahlung process $\mathrm{e^+e^-} \to \mathrm{ZH}$ [23] and the WW-fusion process $\mathrm{e^+e^-} \to \nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mathrm{H}$ [24]. The corresponding lowest-order diagrams are shown in Figure 2. The cross section for the Higgs-strahlung

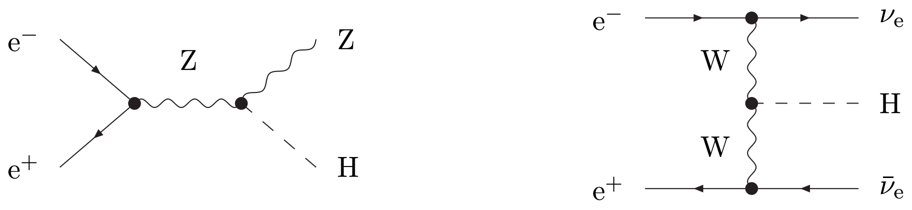

Figure 2: Lowest-order diagrams for main Higgs-production processes

process scales as $1/s$ and dominates at low energies,

$$\sigma_\mathrm{Born} = \frac{G_\mu^2 M_\mathrm{Z}^4}{8\pi}(v_\mathrm{e}^2 + a_\mathrm{e}^2)\frac{\beta_\mathrm{ZH} M_\mathrm{Z}^2}{(s - M_\mathrm{Z}^2)^2}\left(1 + \frac{s\beta_\mathrm{ZH}^2}{12 M_\mathrm{Z}^2}\right), \qquad (3.1)$$

where $\beta_\mathrm{ZH} = \sqrt{[s - (M_\mathrm{Z} + M_\mathrm{H})^2][s - (M_\mathrm{H} - M_\mathrm{Z})^2]}/s$, $a_\mathrm{e} = -1$, and $v_\mathrm{e} = -1 + 4s_\mathrm{w}^2$. The cross section for the WW-fusion process rises $\propto \ln(s/M_\mathrm{H}^2)$ and dominates at high energies

$$\sigma(\mathrm{e^+e^-} \to \nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mathrm{H}) = \frac{G_\mu^3 M_\mathrm{W}^4}{4\sqrt{2}\pi^3}\left[\left(1 + \frac{M_\mathrm{H}^2}{s}\right)\log\frac{s}{M_\mathrm{H}^2} - 2\left(1 - \frac{M_\mathrm{H}^2}{s}\right)\right]. \qquad (3.2)$$

The cross sections are shown in Figure 3 as a function of the Higgs-boson mass for different centre-of mass energies. They are of the order of 100 fb and 10 fb for $M_\mathrm{H} = 120\,\mathrm{GeV}$ and

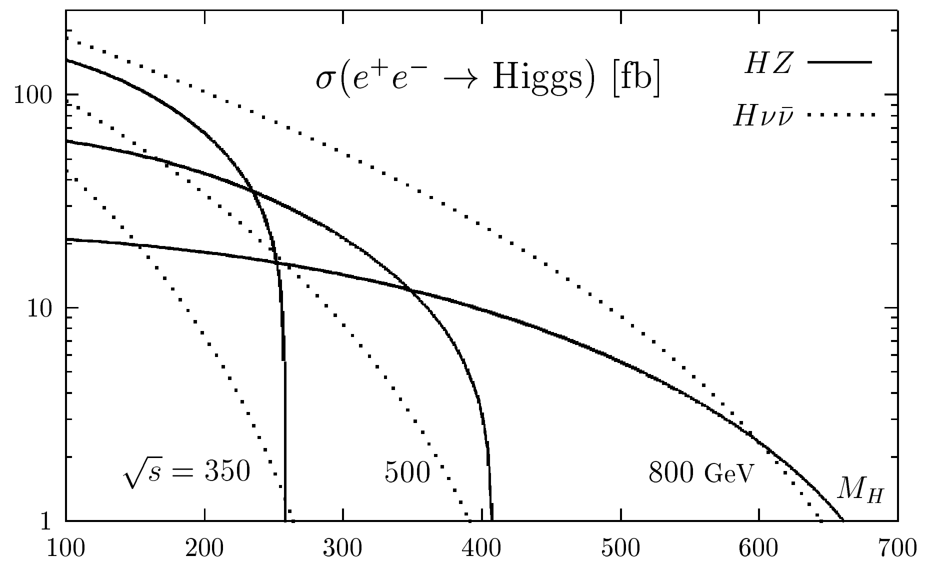

Figure 3: The Higgs-strahlung and WW-fusion production cross sections versus M_H for $\sqrt{s} = 350\,\mathrm{GeV}$, $500\,\mathrm{GeV}$ $800\,\mathrm{GeV}$ (taken from Ref. [8])

$M_\mathrm{H} = 500\,\mathrm{GeV}$, respectively (cf. Figure 3). Thus, of the order of 5×10^4 and 5×10^3 Higgs bosons can be produced for an integrated luminosity of $500\,\mathrm{fb}^{-1}$ per year. This allows to perform measurements with an accuracy at the per-cent level. Consequently, adequate theoretical predictions have to take into account radiative corrections and the effects of the finite decay widths of the Z boson and of the Higgs boson.

Since the couplings of the Higgs boson to other particles are proportional to the masses of those particles, the Higgs boson will predominantly decay into the heaviest particle that is kinematically allowed. If $M_\mathrm{H} \lesssim 140\,\mathrm{GeV}$, the Higgs-boson decays mainly into fermion–anti-fermion pairs, in particular, into bottom quarks. The corresponding partial decay width reads

$$\Gamma(\mathrm{H} \to f\bar{f}) = \frac{G_\mu}{4\sqrt{2}\pi} M_\mathrm{H} N_\mathrm{C}^f m_f^2 \left(1 - \frac{4m_f^2}{M_\mathrm{H}^2}\right)^{3/2} \tag{3.3}$$

with the colour factor $N_\mathrm{C}^f = 1$ for leptons and 3 for quarks. For larger Higgs-boson masses the decays into one real and one virtual gauge boson become important. If $M_\mathrm{H} > 2M_\mathrm{W}$ the Higgs boson can decay into a pair of real W bosons and for $M_\mathrm{H} > 2M_\mathrm{Z}$ also into real Z bosons. The corresponding partial decay widths

$$\begin{aligned}
\Gamma(\mathrm{H} \to \mathrm{WW}) &= \frac{G_\mu}{8\sqrt{2}\pi} M_\mathrm{H}^3 \left(1 - \frac{4M_\mathrm{W}^2}{M_\mathrm{H}^2}\right)^{1/2} \left(1 - \frac{4M_\mathrm{W}^2}{M_\mathrm{H}^2} + 12\frac{M_\mathrm{W}^4}{M_\mathrm{H}^4}\right), \\
\Gamma(\mathrm{H} \to \mathrm{ZZ}) &= \frac{G_\mu}{16\sqrt{2}\pi} M_\mathrm{H}^3 \left(1 - \frac{4M_\mathrm{Z}^2}{M_\mathrm{H}^2}\right)^{1/2} \left(1 - \frac{4M_\mathrm{Z}^2}{M_\mathrm{H}^2} + 12\frac{M_\mathrm{Z}^4}{M_\mathrm{H}^4}\right)
\end{aligned} \tag{3.4}$$

dominate for $M_\mathrm{H} \gtrsim 2M_\mathrm{W}$ even if the decay channel $\mathrm{H} \to \mathrm{t\bar{t}}$ opens. The Higgs-boson branching ratios and the total Higgs-boson decay width are shown in Figure 4 as a function of M_H [25].

The mass of the Higgs boson can be measured in a model-independent way from the Z recoil spectrum in ZH events ($M_\mathrm{H}^2 = s - 2\sqrt{s}E_\mathrm{Z} + M_\mathrm{Z}^2$). Experimental simulations show that the experimental error can be reduced to $\delta M_\mathrm{H} \approx 50\,\mathrm{MeV}$ in high-luminosity runs [8].

The width of the SM Higgs boson can be determined in a nearly model-independent way in the intermediate mass range by measuring branching ratios BR_i in the decays and the corresponding partial widths Γ_i in the production process. The total width can then be derived from $\Gamma_\mathrm{H} = \Gamma_i/BR_i$. From the branching ratio for the decay mode $\mathrm{H} \to \mathrm{WW}$ and the corresponding partial width extracted from the size of the WW-fusion cross section one obtains an accuracy of $\delta\Gamma_\mathrm{H}/\Gamma_\mathrm{H} \sim 4$–$13\%$ for $M_\mathrm{H} = 120$–$160\,\mathrm{GeV}$ [8]. In the high mass range ($M_\mathrm{H} \gtrsim 200\,\mathrm{GeV}$), the total width can be measured directly from the Higgs-boson line shape with an accuracy $\delta\Gamma_\mathrm{H}/\Gamma_\mathrm{H} \gtrsim 6\%$ [9].

The Higgs couplings to massive gauge bosons g_HZZ and g_HWW can be determined from the measurement of the cross sections for $\mathrm{e^+e^-} \to \mathrm{ZH}$ and $\mathrm{e^+e^-} \to \nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mathrm{H}$, respectively. The estimated experimental accuracy for these cross sections is of the order of $\delta\sigma/\sigma \approx 0.03$ for light Higgs bosons ($M_\mathrm{H} \lesssim 160\,\mathrm{GeV}$).

The couplings of the Higgs boson to τ leptons, charm and bottom quarks can be extracted from the measurement of the branching ratios for $\mathrm{H} \to f\bar{f}$ for light Higgs bosons with an accuracy of 2–8% [26]. By measuring the ratios of the $\tau\tau$ and cc to the bb branching

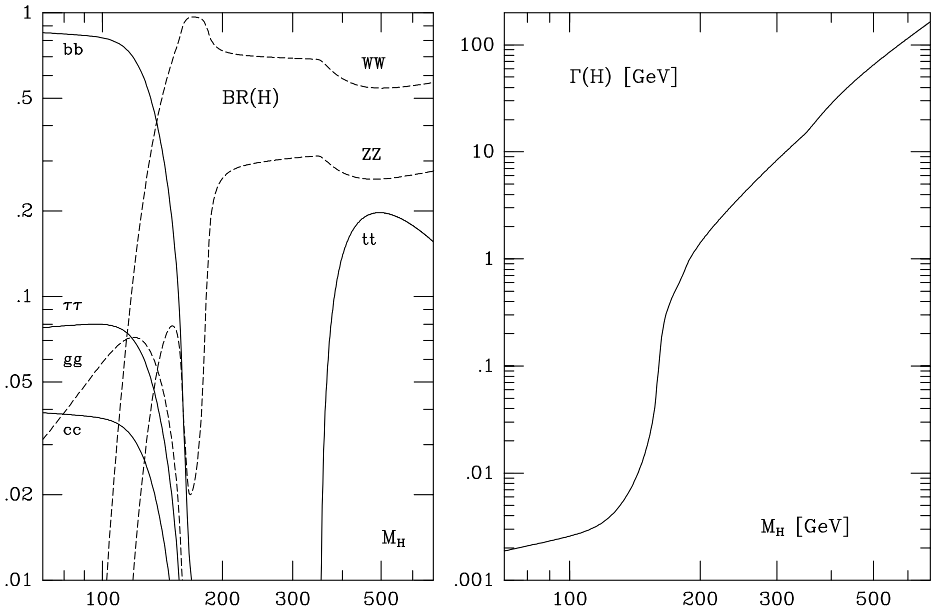

Figure 4: The branching ratios (left) and the total decay width (right) of the SM Higgs boson as a function of M_{H} (taken from Ref. [8])

ratio, the linear dependence of the Yukawa couplings on the fermion masses can be tested. The absolute values of the Yukawa couplings can be reconstructed by combining decay branching ratios with the production cross section. For a heavy-enough Higgs boson, the Yukawa coupling of the top quark can be measured directly to $\sim 6\%$ from the $\mathrm{e^+e^-} \to \mathrm{t\bar{t}H}$ cross-section normalization [27].

A LC allows for a model-independent determination of the quantum numbers J^{PC} of the Higgs boson [28]. The threshold rise of the process $\mathrm{e^+e^-} \to \mathrm{Z}X$ for a boson X with arbitrary spin J and normality $n = (-1)^J P$ has been studied in Ref. [29]. While for $J = 0$ the cross section at threshold rises $\propto \beta_{\mathrm{Z}X}$, for higher spins the cross section rises generally with higher powers of $\beta_{\mathrm{Z}X}$ except for some scenarios which can be distinguished through the angular dependence in the continuum. The angular distribution $\mathrm{d}\sigma/\mathrm{d}\cos\theta \propto \beta_{\mathrm{Z}X}\sin^2\theta + 8M_{\mathrm{Z}}^2/s$ approaches a spin-zero distribution $\propto \sin^2\theta$ asymptotically. This differs, in particular, from the angular distribution for a spin-less negative-parity state $\propto (1 + \cos^2\theta)$.

An unambiguous confirmation of the Higgs mechanism requires the reconstruction of the Higgs potential

$$V = \lambda\left(|\Phi|^2 - \frac{1}{2}v^2\right)^2 \to \frac{1}{2}M_{\mathrm{H}}^2 H^2 + \frac{1}{2v}M_{\mathrm{H}}^2 H^3 + \frac{1}{8v^2}M_{\mathrm{H}}^2 H^4, \tag{3.5}$$

where Φ is the SU(2) Higgs doublet and H the physical Higgs field, via a measurement of the triple and quartic Higgs self-couplings. At a high-luminosity LC the triple Higgs coupling can be measured from the process $\mathrm{e^+e^-} \to \mathrm{ZHH}$. This cross section amounts to 0.15 fb for $M_{\rm H} = 120\,{\rm GeV}$ at $\sqrt{s} = 500\,{\rm GeV}$. Nevertheless, with high luminosity ($1000\,{\rm fb}^{-1}$), excellent b-tagging, and excellent energy-flow reconstruction, a measurement of the triple Higgs coupling with an accuracy better than 20% seems to be possible [30]. In this way an essential element of the mechanism for spontaneous symmetry breaking can be established experimentally.

4. Top-quark physics

The top quark is the heaviest matter particle in the SM. Its mass close to the electroweak symmetry breaking scale renders the top quark an ideal object for studying the fundamental interactions. It is likely to play a key role in pinning down the origin of electroweak symmetry breaking and in the search for clues to the flavour problem. Moreover, its mass enters quadratically in precision tests of the SM and the MSSM. High-precision measurements of the properties and interactions of the top quark are therefore mandatory at any future collider.

The dominant decay mode of the SM top quark ($m_{\rm t} \sim 175\,{\rm GeV}$) is the channel ${\rm t} \to {\rm b} + {\rm W}$. Since the width of the top quark, $\Gamma_{\rm t} \sim 1.4\,{\rm GeV}$, is large compared to the scale Λ of the strong interaction, the top quark can be treated in good approximation as a free particle which is not dressed by non-perturbative strong-interaction effects. At an ${\rm e^+e^-}$ LC, the top quark is produced in pairs via the process ${\rm e^+e^-} \to {\rm t\bar{t}}$. At $\sqrt{s} = 350\,{\rm GeV}$ one expects of the order of 3×10^5 top pairs for $500\,{\rm fb}^{-1}$.

According to a recent study of the top-pair production threshold [31], the top-quark mass, top-quark width, and $\alpha_{\rm s}(M_{\rm Z})$ can be measured with an experimental precision of 20 MeV, 30 MeV, and 0.0012, respectively, from a multi-parameter fit to the shape of the top-pair production cross section near threshold, the forward–backward charge asymmetry, and the position of the peak of the top momentum distribution, assuming an integrated luminosity of $300\,{\rm fb}^{-1}$.

If the scale of physics beyond the SM is much larger than the collider energy, the electroweak top-pair-production currents can globally be described by form factors which reduce to anomalous vector and axial-vector couplings of the Z boson, anomalous magnetic dipole moments, and electric dipole moments. The vector and axial-vector couplings can be determined from the polarized production cross sections and the left–right asymmetry to $\sim 2\%$. The anomalous magnetic dipole moments can be bounded to the per-cent level by measuring the angular dependence of the top-pair cross section. Non-zero values for electric dipole moments can be detected by means of non-vanishing expectation values of CP-odd momentum tensors with a sensitivity $\lesssim 10^{-18} e{\rm cm}$. A possible $V + A$ admixture to the top-decay current can be measured from the energy distribution of the charged lepton ℓ resulting from the decay chain ${\rm t} \to {\rm W} \to \ell$ with an accuracy of about 1%.

5. Supersymmetry

Although supersymmetry has so far not been found in nature, it is a theoretically very appealing concept. It is the only possible non-trivial extension of the Poincaré group and relates fermions and bosons. Local supersymmetry includes naturally the algebra of general relativity. Supersymmetry implies a cancellation of quadratic divergences and, as a consequence, it stabilizes the masses of fundamental Higgs scalars with respect to radiative corrections in the presence of very high-energy scales associated, for instance, with grand unification.

The Minimal Supersymmetric Standard Model (MSSM) is the minimal extension of the Standard Model to incorporate supersymmetry. In addition to the particles of the SM, the MSSM contains their super-partners, sleptons $\tilde{\ell}^{\pm}, \tilde{\nu}_{\ell}$ ($\ell = \mathrm{e}, \mu, \tau$), squarks $\tilde{q}$, and gauginos $\tilde{\mathrm{g}}, \tilde{\mathrm{W}}^{\pm}, \tilde{\mathrm{Z}}, \tilde{\gamma}$. Two Higgs doublets are necessary to give masses to up- and down-type fermions and to cancel the anomalies of their super-partners, the Higgsinos. As a consequence, there are five physical Higgs particles, four CP-even ones, h, H, and $\mathrm{H}^{\pm}$, and one CP-odd one, A. The non-strongly interacting gauginos mix with the Higgsinos to form the corresponding mass eigenstates, two pairs of charginos, $\tilde{\chi}_i^{\pm}$ ($i = 1, 2$) and four neutralinos $\tilde{\chi}_i^0$ ($i = 1, \dots 4$). The MSSM conserves a multiplicative quantum number, R parity. The SM particles carry $R = 1$, their super-partners $R = -1$. As a consequence, supersymmetric particles are produced in pairs, and the lightest supersymmetric particle is stable.

Since the masses of the super-partners are obviously different from the masses of the SM particles, supersymmetry must be broken. In order not to spoil the cancellation of quadratic divergences, supersymmetry breaking is introduced in the MSSM via explicit soft terms with scale M_{SUSY}. This scale should be of the order of a few TeV or less if supersymmetry is to provide a generic solution of the hierarchy problem. Supersymmetry breaking introduces a large number of free parameters; in the most general case the MSSM contains 105 parameters in addition to the SM parameters. This number can be considerably reduced by invoking additional assumptions. In the minimal super-gravity model, owing to universal unification conditions at $M_{\mathrm{GUT}} \approx 10^{16}\,\mathrm{GeV}$, there are only five free parameters, the scalar mass parameter m_0, the gaugino mass parameter $m_{1/2}$, the universal trilinear coupling parameter A_0, the ratio of vacuum expectation values $\tan\beta = v_1/v_2$, and the sign of the Higgsino mass parameter μ.

In addition to the solution of the hierarchy problem, the MSSM has further attractive features. The gauge couplings unify, the electroweak symmetry breaking originates as a natural result of renormalization group running from the GUT scale to the electroweak scale, and the lightest supersymmetric particle is a good candidate for cold dark matter. Most motivations for supersymmetry imply that supersymmetric particles should be found in the TeV region or below.

Besides their masses, two mixing angles define the properties of the Higgs bosons and their interactions, the mixing angle β of the neutral CP-odd and charged Higgs sector and and the mixing angle α of the neutral CP-even Higgs sector. Because of supersymmetry, all these parameters can be expressed by only two independent ones. A convenient choice consists in the mass of the pseudoscalar, M_{A}, and $\tan\beta$. The MSSM predicts an upper bound on the mass of the lightest Higgs boson, $M_{\mathrm{h}} \lesssim 135\,\mathrm{GeV}$ [32]. The search for the neutral MSSM Higgs bosons at $\mathrm{e}^+\mathrm{e}^-$ colliders will be a straightforward extension of the search performed at LEP2. The main production mechanisms are Higgs strahlung $\mathrm{e}^+\mathrm{e}^- \to$

Zh/ZH and associated pair production $\mathrm{e^+e^-} \to \mathrm{Ah/AH}$, as well as the related fusion processes. The cross sections for the four Higgs-strahlung and pair-production processes can be expressed as

$$\begin{aligned} \sigma(\mathrm{e^+e^-} \to \mathrm{Zh/ZH}) &= \sin^2(\alpha-\beta)/\cos^2(\alpha-\beta)\,\sigma_{\mathrm{SM}} \\ \sigma(\mathrm{e^+e^-} \to \mathrm{Ah/AH}) &= \cos^2(\alpha-\beta)/\sin^2(\alpha-\beta)\,\bar{\lambda}\,\sigma_{\mathrm{SM}}, \end{aligned} \tag{5.1}$$

where σ_{SM} is the SM cross section for Higgs strahlung, and the coefficient $\bar{\lambda} = \beta^3_{\mathrm{A}j}/[\beta_{\mathrm{Z}j} \times (12M_{\mathrm{Z}}^2 + \beta^2_{\mathrm{Z}j})]$ accounts for the P wave suppression of the Ah/AH cross sections near threshold. As a consequence, at least the lightest CP-even Higgs boson h will be accessible at a LC. A supersymmetric Higgs boson h can be disentangled from a SM Higgs boson by measuring branching ratios. For $M_A \lesssim \sqrt{s}/2$ all supersymmetric Higgs bosons can be discovered independently of the value of $\tan\beta$.

In extended supersymmetric models an absolute upper limit on the mass of the lightest Higgs boson of $M_{\mathrm{h}} \lesssim 200\,\mathrm{GeV}$ has been derived assuming gauge-coupling unification near the GUT scale [33]. Thus, at least one Higgs boson must be discovered at the LHC or LC if low-energy supersymmetry is realized in nature.

If their masses are small enough, supersymmetric particles can be copiously produced at a LC. Charginos and neutralinos are produced in pairs through s-channel γ and Z exchange and t-channel selectron or sneutrino exchange (cf. Figure 5). Since the cross

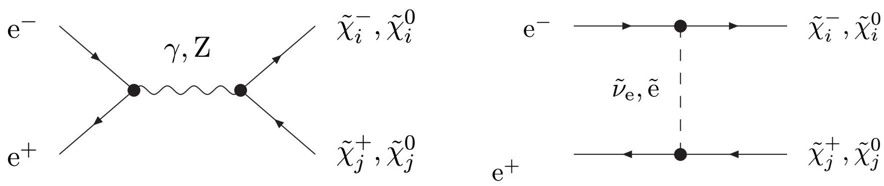

Figure 5: Lowest-order diagrams for chargino and neutralino pair production

sections are $\mathcal{O}(100\,\mathrm{fb})$, and these particles are easy to detect via their decays, they can be discovered nearly up to the kinematical limit. The properties of charginos and neutralinos can be studied in great detail and the masses can be determined from threshold scans to better than 100 MeV. Note, however, that this requires adequate theoretical predictions. The mass of the lightest supersymmetric particle, which is typically a neutralino, can be measured from decay spectra of heavier supersymmetric particles.

Sleptons and squarks are pair-produced through s-channel γ and Z exchange and t-channel neutralino or chargino exchange. The corresponding lowest-order diagrams are shown in Figure 6. The cross sections are in the range 10–100 fb and the neutralinos and charginos are easy to detect via their decays. Consequently, their discovery is very easy up to the kinematical limit. The masses can be determined from the P-wave onset of the annihilation cross sections at a level of 200 to 300 MeV. They production rates are high enough to allow for a detaild study of their properties.

From the precisely measured properties of the supersymmetric particles and their production and decay characteristics, the basic low-energy parameters of the MSSM can be

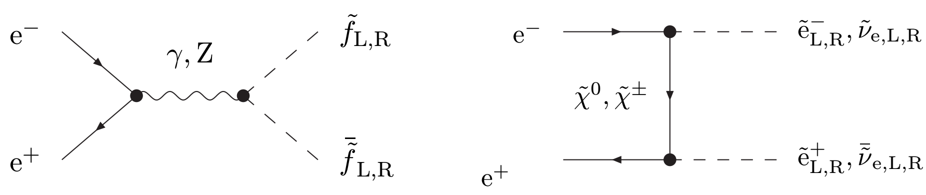

Figure 6: Lowest-order diagrams for sfermion–antisfermion production

extracted, typically with accuracies at the per-cent level. These parameters can then be extrapolated to higher energies using the renormalization group. The emerging unification pattern yields information on the supersymmetry breaking scenario. For example, in the minimal supergravity model, universal gaugino and scalar masses should arise at the GUT scale. In the gauge-mediated symmetry breaking (GMSB) model, the scalar masses do not unify, but the masses of particles carrying the same quantum numbers are equal at the intermediate GMSB scale. Using LC results together with LHC results improves errors on parameters at the unification scale by one order of magnitude. A model-independent analysis requires input from a LC. More details can be found in Ref. [34].

6. Summary

One of the most pressing problems in particle physics is the mechanism of electroweak symmetry breaking. The LHC will find the Higgs boson, if it exists and has no particular exotic properties. The proof that this particle actually generates the masses requires an e^+e^- linear collider. Such a machine would allow to challenge the SM by studying in detail the profile of the Higgs boson and of the top quark and by performing improved electroweak precision tests. Beyond the Standard Model, it would be an ideal tool to explore supersymmetry, extra dimensions, etc. An e^+e^- linear collider is in many aspects complementary to the LHC, and both are needed to understand electroweak symmetry breaking.

In order to successfully carry out the proposed experimental program at a LC, many new theoretical investigations and calculations are necessary. The required precision calculations enter a new level of complexity, requiring the development of new techniques, new concepts, and a lot of (wo)manpower.

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References

[1] ALEPH, DELPHI, L3, and OPAL collaborations, the LEP wqorking group for Higgs-boson searches, LHWG Note/2002-01,
http://lephiggs.web.cern.ch/LEPHIGGS/www/Welcome.html.

[2] The LEP Collaborations ALEPH, DELPHI, L3, OPAL, the LEP EWWG, and the SLD Heavy Flavor Group, LEPEWWG/2002-01, http://lepewwg.web.cern.ch/LEPEWWG/.

[3] C. Adolphsen *et al.*, [The NLC design group], Zeroth Order Design Report for the Next Linear Collider, SLAC Report 474, 1996.

[4] N. Akasaka *et al.*, JLC design study, KEK-REPORT-97-1.

[5] R. Brinkmann *et al.*, TESLA Technical Design Report Part III: The accelerator, DESY-01-011.

[6] E. Accomando *et al.* [ECFA/DESY LC Physics Working Group Collaboration], *Phys. Rept.* **299** (1998) 1 [hep-ph/9705442].

[7] P. M. Zerwas, *Prepared for 5th International Linear Collider Workshop (LCWS 2000), Fermilab, Batavia, Illinois, 24-28 Oct 2000.*

[8] J. A. Aguilar-Saavedra *et al.*, TESLA Technical Design Report Part III: Physics at an e^+e^- Linear Collider, hep-ph/0106315.

[9] T. Abe *et al.* [American Linear Collider Working Group Collaboration], in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)*, ed. R. Davidson and C. Quigg, SLAC-R-570, *Resource book for Snowmass 2001*, [hep-ex/0106055, hep-ex/0106056, hep-ex/0106057, hep-ex/0106058].

[10] K. Abe *et al.* [ACFA Linear Collider Working Group Collaboration], ACFA Linear Collider Working Group report, [hep-ph/0109166].

[11] U. Baur *et al.* [The Snowmass Working Group on Precision Electroweak Measurements Collaboration], in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)*, ed. N. Graf, hep-ph/0202001.

[12] G. Wilson, LC-PHSM-2001-009, http://www.desy.de/Ĩcnotes/notes.html.

[13] J. Erler, S. Heinemeyer, W. Hollik, G. Weiglein and P. M. Zerwas, *Phys. Lett.* **B 486** (2000) 125 [hep-ph/0005024].

[14] F. Jegerlehner, LC-TH-2001-035, http://www.desy.de/Ĩcnotes/notes.html, hep-ph/0105283.

[15] S. Jadach, W. Płaczek, M. Skrzypek, B. F. Ward and Z. Wąs, *Phys. Rev.* **D65** (2002) 093010 [hep-ph/0007012]; *Comput. Phys. Commun.* **140** (2001) 432 [hep-ph/0103163]; *Comput. Phys. Commun.* **140** (2001) 475 [hep-ph/0104049].

[16] S. Jadach, W. Płaczek, M. Skrzypek and B. F. Ward, *Phys. Rev.* **D54** (1996) 5434 [hep-ph/9606429]; *Phys. Lett.* **B 417** (1998) 326 [hep-ph/9705429]; *Phys. Rev.* **D 61** (2000) 113010 [hep-ph/9907436].

[17] A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, *Nucl. Phys.* **B 587** (2000) 67, [hep-ph/0006307].

[18] A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, *Nucl. Phys.* **B 560** (1999) 33 [hep-ph/9904472]; *Eur. Phys. J.* **C 20** (2001) 201 [hep-ph/0104057]; *Phys. Lett.* **B 475** (2000) 127 [hep-ph/9912261]; hep-ph/9912447; in *Proc. of the 5th International Symposium on Radiative Corrections (RADCOR 2000)*, ed. H. E. Haber, [hep-ph/0101257]; hep-ph/0209330.

[19] M. W. Grünewald *et al.*, in *Reports of the Working Groups on Precision Calculations for LEP2 Physics*, eds. S. Jadach, G. Passarino and R. Pittau (CERN 2000-009, Geneva, 2000), p. 1 [hep-ph/0005309].

[20] S. Jadach, W. Płaczek, M. Skrzypek, B. F. Ward and Z. Wąs, *Phys. Lett.* **B 523** (2001) 117 [hep-ph/0109072].

[21] R. Brunelière *et al.*, *Phys. Lett.* **B 533** (2002) 75 [hep-ph/0201304].

[22] A. Denner, in *Proc. of the International Europhysics Conference on High-Energy Physics (HEP 2001)*, hep-ph/0110155;
M. Melles, hep-ph/0104232.

[23] J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, *Nucl. Phys.* **B 106** (1976) 292;
B. L. Ioffe and V. A. Khoze, *Sov. J. Part. Nucl.* **9** (1978) 50 [*Fiz. Elem. Chast. Atom. Yadra* **9** (1978) 118];
J. D. Bjorken, in *Weak Interactions At High-Energy And The Production Of New Particles: Proceedings The 4th Slac Summer Institute On Particle Physics*, ed. M. C. Zipf, SLAC-198, Stanford, Calif., SLAC (1976) p. 1.

[24] D. R. Jones and S. T. Petcov, *Phys. Lett.* **B 84** (1979) 440;
G. Altarelli, B. Mele and F. Pitolli, *Nucl. Phys.* **B 287** (1987) 205;
W. Kilian, M. Krämer and P. M. Zerwas, *Phys. Lett.* **B 373** (1996) 135 [hep-ph/9512355].

[25] A. Djouadi, M. Spira and P. M. Zerwas, *Z. Physik* **C 70** (1996) 427 [hep-ph/9511344];
A. Djouadi, J. Kalinowski and M. Spira, *Comput. Phys. Commun.* **108** (1998) 56 [hep-ph/9704448].

[26] M. Battaglia, in Proceedings of the LC conference, Sitges 1999, hep-ph/9910271.

[27] A. Juste and G. Merino, hep-ph/9910301.

[28] V. D. Barger, K. M. Cheung, A. Djouadi, B. A. Kniehl and P. M. Zerwas, *Phys. Rev.* **D 49** (1994) 79 [hep-ph/9306270];
K. Hagiwara and M. L. Stong, *Z. Physik* **C 62** (1994) 99 [hep-ph/9309248].

[29] D. J. Miller, S. Y. Choi, B. Eberle, M. M. Mühlleitner and P. M. Zerwas, *Phys. Lett.* **B 505** (2001) 149 [hep-ph/0102023].

[30] C. Castanier, P. Gay, P. Lutz and J. Orloff, LC-PHSM-2000-061, hep-ex/0101028.

[31] M. Martinez and R. Miquel, hep-ph/0207315.

[32] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, hep-ph/0212020.

[33] J. R. Espinosa and M. Quiros, *Phys. Rev. Lett.* **81** (1998) 516 [hep-ph/9804235].

[34] G. A. Blair, W. Porod and P. M. Zerwas, Phys. Rev. D **63** (2001) 017703 [hep-ph/0007107].