Precision tests of neutrino oscillations and lepton flavour violation

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ABSTRACT: I discuss the phenomenological framework for possible contributions of lepton flavour violating interactions to neutrino oscillations. It will be shown that in general the effects of those new interactions can be large and they can be of the same order of magnitude than CP effects in oscillations. Also some consequences for the ability of a planed neutrino factory to determine θ_{13} will be shown.

1. Introduction

In the last few years the evidence for neutrino oscillations has been steadily growing (for a review see [1]). The atmospheric neutrino anomaly [2, 3, 4, 5, 6, 7, 8, 9] as manifest in the data is best explained by oscillation of ν_μ to ν_τ with maximal mixing. The solar neutrino problem basically was solved by the SNO neutral current results [10, 11] since they show that the prediction of the solar ^8B neutrinos is correct and that the observed deficit of ν_e has to be attributed to the properties of the neutrino and not to the properties of the Sun. Also the solar data [12, 13, 14, 15, 16, 17] is most easily explained by neutrino oscillation. These results have been independently confirmed by KamLAND using the disappearance of $\bar{\nu}_e$ [18]. Also the atmospheric results have been tested in the long baseline experiment K2K [19]. All these results taken together lead to a picture where oscillation is the main mechanism for flavour transitions and any alternative mechanism can play at most a sub-leading role, see *e.g.* [20, 21]. However the current data is not precise enough to exclude a sizeable admixture of non-standard mechanisms like lepton flavour violating interactions, although some bounds can be derived already from the existing data [22].

Future plans for neutrino oscillation experiments include the search for leptonic CP violation [23, 24, 25]. The CP phase by itself can be large, however any effect in the

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oscillation probabilities is suppressed by θ_{13} and the hierarchy parameter $\alpha = \Delta m^2_{21}/\Delta m^2_{31}$. Thus the absolute size of CP effects is expected to be small and the experiments looking for them have to achieve a very high accuracy. At this level of accuracy any additional contribution to flavour transitions can become significant and therefore those corrections have to be well understood in order to extract an unbiased value for the CP phase.

In section 2 the phenomenological framework for the combined treatment of lepton flavour violating interactions (LFV) and oscillations is layed out and in section 3 the results for some special cases at a neutrino factory are presented.

2. Theoretical Framework

In order to quantify the impact of LFV on neutrino oscillations independent of a model for the origin of LFV it is useful to regard the low energy theory as an effective theory. In this effective theory LFV can be parameterized by the coupling strength of various four fermion vertices. In general those vertices can have any Lorentz structure which is invariant under the corresponding Lorentz transformation. In this general case there is a large number of vertices and couplings [1] which is extensively discussed in [26]. For the sake of simplicity I will restrict the discussion on the subclass of vertices which have the same Lorentz structure as the weak interaction in the Standard Model (SM), *i.e.* V-A, thus the new terms in the Lagrangian look like

$$\mathcal{L}_{\mathrm{LFV}} = \frac{G_F}{\sqrt{2}} \left[\bar{\nu}_l (1-\gamma_5)\gamma_\lambda l\right] \sum_\alpha \epsilon_{l'\alpha} \left[\bar{l}'(1-\gamma_5)\gamma^\lambda \nu_\alpha\right] + h.c.. \tag{2.1}$$

Thus the parameters $\epsilon_{l'\alpha}$ describe the coupling strength in units of the Fermi constant G_F. Furthermore only real couplings ϵ are considered. Similar expression can be written down for semi-leptonic processes which have independent couplings[2].

The existing bounds for the parameters ϵ are in the order of 10^{-6} for transitions involving muons[3] and 10^{-2} for the others (see *e.g.* [27]). Therefore the ϵ's involving muons will be set to zero in the following.

A neutrino experiment consists of three parts – the production (S), the propagation (P) and the detection (D) of the neutrino. The new interactions can influence each part of the experiment and therefore three sets of ϵ have to be considered since the vertices are usually not the same for the three parts. I will illustrate this with the example of a neutrino factory. The production is by muon decay, *i.e.* $\mu \to \nu_\mu + \bar{\nu}_e + e$, the propagation is influenced by charged current coherent forward scattering from electrons $\nu_e + e \to \nu_e + e$ and the detections happens via deep inelastic scattering from quarks $\nu_\mu + d \to \mu + u$. Now, one can imagine different contributions of LFV to each vertex, *e.g.* in the production the process $\mu \to \nu_\tau + \bar{\nu}_e + e$ might appear, whereas in the propagation the reaction $\nu_e + e \to \nu_\mu + e$ could occur and finally in the detection a non-standard coupling to the quarks is possible producing a τ instead of μ. Thus in case of looking for wrong sign muons 36 paths to go from μ^+ to μ^- exist as it is shown in figure 1. The black lines indicate strong links either

[1] in general complex

[2] in case no underlying model is used.

[3] based on assumptions about the size of $SU(2)_Y$ breaking effects

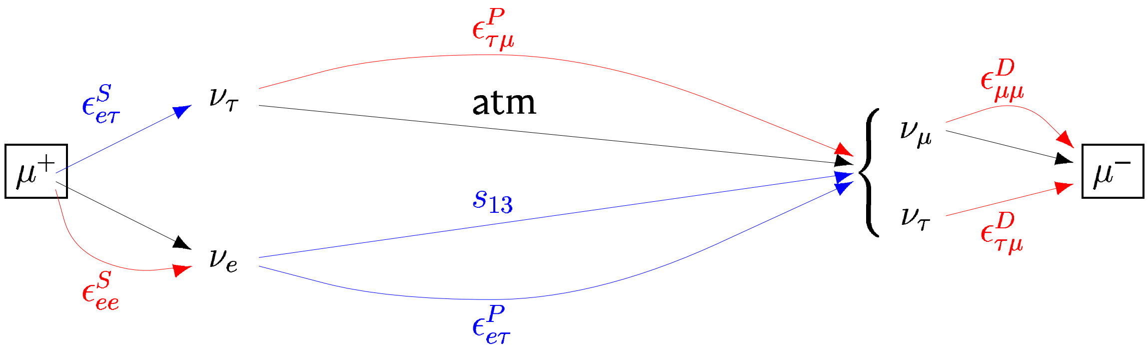

Figure 1: *Possible paths from μ^+ at the source to μ^- (wrong sign muon) in the detector in the presence of both oscillation and LFV. Figure taken from [27].*

by large mixing or a coupling of order G_F, whereas the colored lines indicate weak links, *i.e.* transitions which are either suppressed by θ_{13} or by couplings which are much smaller than G_F. The total transition rate is then given by [28]

$$\mathcal{R}_{e\mu} = \left| \sum_{\alpha\beta} \mathcal{A}^S_{e\alpha} \mathcal{A}^P_{\alpha\beta} \mathcal{A}^D_{\beta\mu} \right|^2 , \tag{2.2}$$

i.e. a coherent sum of all possible paths, where the amplitudes describing the neutrino source and detection process are defined as

$$\mathcal{A}^X_{\alpha\beta} \equiv \delta_{\alpha\beta} + \epsilon^X_{\alpha\beta} \quad \text{for} \quad X = S, D \,, \tag{2.3}$$

and the amplitude $\mathcal{A}^P_{\beta\gamma}$ describes the propagation of the neutrino state from the production point to the detector. This amplitude is obtained from the solution of a Schrödinger equation with the Hamiltonian

$$H_\nu = \frac{1}{2E_\nu} U \mathrm{diag}\,(0, \Delta m^2_{\mathrm{sol}}, \Delta m^2_{\mathrm{atm}})\, U^\dagger + \mathrm{diag}(V, 0, 0) + \sum_f V_f\, \epsilon^f \,, \tag{2.4}$$

which takes into account neutrino oscillations and SM interactions as well as non-standard interactions (NSI) with the matter crossed by the neutrino beam. Here E_ν is the neutrino energy and $V = \sqrt{2} G_F N_e$ is the matter potential due to the SM charged current interaction [29, 30, 31], where N_e is the electron number density. The last term in equation (2.4) describes the NSI with earth matter. The sum is over all fermions f present in matter, and $V_f\, \epsilon^f_{\alpha\beta}$ is the coherent forward scattering amplitude of the process $\nu_\alpha + f \to \nu_\beta + f$, where $V_f = \sqrt{2} G_F N_f$, with the number density of the fermion f along the neutrino path given by N_f. We define an effective NSI coefficient for the propagation by normalizing all contributions to the down-quark potential V_d:

$$\epsilon^P_{\alpha\beta} \equiv \sum_f \frac{V_f}{V_d}\, \epsilon^f_{\alpha\beta} \,. \tag{2.5}$$

The leading contributions to the $e \to \mu$ transition are indicated by the blue lines in figure 1, *i.e.* only s_{13}, $\epsilon^S_{e\tau}$ and $\epsilon^P_{e\tau}$ gives rise to leading order terms. Thus the problem can be considerably simplified by setting all other ϵ's to zero. To clean up the nomenclature I define $\epsilon_S := \epsilon^S_{e\tau}$ and $\epsilon_P := \epsilon^P_{e\tau}$.

3. Results

Before presenting the results for θ_{13}, following comparison can be made in order to estimate the relative magnitude of CP and LFV effects. To this end these two quantities are useful

$$\Delta\mathcal{R}_{\mathrm{CP}} = \int_{E_{\mathrm{min}}}^{E_{\mathrm{max}}} dE \left|\mathcal{R}(\epsilon^S = 0, \epsilon^D = 0, \delta = 0) - \mathcal{R}(\epsilon^S = 0, \epsilon^D = 0, \delta = \pi/2)\right| , \tag{3.1}$$

$$\Delta\mathcal{R}_{\mathrm{NSI}} = \int_{E_{\mathrm{min}}}^{E_{\mathrm{max}}} dE \left|\mathcal{R}(\epsilon^S = 0.03, \epsilon^D = 0.07, \delta = 0) - \mathcal{R}(\epsilon^S = 0, \epsilon^D = 0, \delta = 0)\right| .$$

$\Delta\mathcal{R}_{\mathrm{CP}}$ is a measure for the maximum size of CP effects in the pure oscillation case, whereas $\Delta\mathcal{R}_{\mathrm{NSI}}$ yields the maximal size of LFV effects in absence of any CP violation in oscillations. Clearly the relative size of the two indicates whether LFV is potentially problematic in looking for CP violation in oscillations. This estimate is conservative in the sense that the problem becomes worse once complex, *i.e.* CP violating, couplings ϵ are considered. The relative weight of both quantities is shown in figure 2.

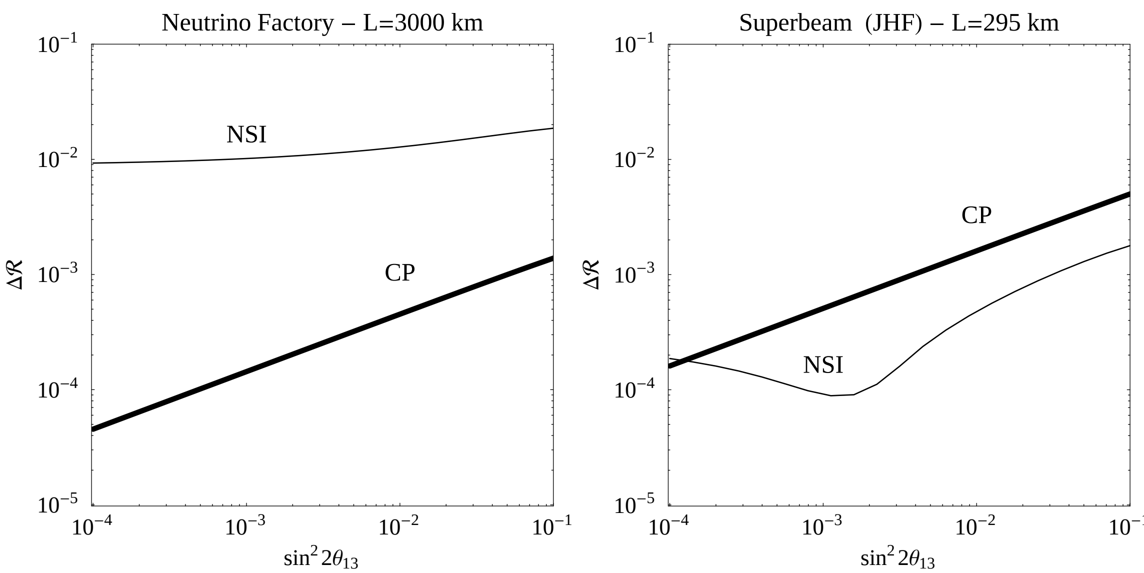

Figure 2: *The size of $\Delta\mathcal{R}$ in dependence of $\sin^2 2\theta_{13}$ for NSI and CP effects as explained in the text. The left hand panel is drawn for the JHF-SK experiment and the right hand panel for a neutrino factory. Both experiments are defined in [32].*

For the computation of figure 2 an energy range from 5 GeV to 50 GeV and a baseline of 3 000 km was used for the neutrino factory case and an energy range from 0.4 GeV to 1.2 GeV and a baseline of 295 km for JHF-SK. The transition rates $\mathcal{R}$ were obtained by numerically solving equations 2.3 and 2.4. For both panels the oscillation parameters were $\Delta m^2_{31} = +2.0\cdot 10^{-3}\,\mathrm{eV}^2$, $\Delta m^2_{21} = 7.0\cdot 10^{-5}\,\mathrm{eV}^2$, $\theta_{23} = \pi/4$ and $\sin^2\theta_{12} = 0.8$. From figure 2 it is obvious that LFV effects can have the same order of magnitude or be even larger than

CP effects for most values of $\sin^2 2\theta_{13}$ and therefore can present severe constraints on the achievable precision in the determination of the CP phase in long baseline experiments. One however observes a large difference in the behavior of a neutrino factory and JHF-SK. The LFV effects are much smaller relative to the CP effects at the superbeam experiment since the CP effects are enhanced by oscillation. This is possible because the superbeam is tuned to have most of its events in the first oscillation maximum, whereas a neutrino factory operates far beyond the first maximum. This problem field clearly deserves a much more detailed effort which however is beyond the scope of this work. A initial investigation of the possible new CP violating effects introduced by complex couplings ϵ can be found in [33].

The other issue is how the presence of LFV affects the θ_{13} sensitivity of a neutrino factory. In order to gain some insight in the problem I will present some analytical considerations in the limit where the solar mass splitting vanishes, *i.e.* $\Delta m^2_{21} = 0$. This approximation furthermore removes θ_{12} and the CP phase δ from the problem, leaving a much simpler set of equations. Expanding the exact result in the small quantities $s_{13} := \sin\theta_{13}$, ϵ_S and ϵ_P yields following form for $\mathcal{R}_{e\mu}$ [27]

$$\mathcal{R}_{e\mu} \simeq A s_{13}^2 + B s_{13}\epsilon^P_{e\tau} + C(\epsilon^D_{e\tau})^2 + D\epsilon^P_{e\tau}\epsilon^S_{e\tau} + E(\epsilon^S_{e\tau})^2 + F s_{13}\epsilon^S_{e\tau}\,, \tag{3.2}$$

where the coefficients A, B, C, D, E and F can be found in [27]. Now it turns out useful to regard the to limiting cases where there is only oscillation or only LFV. In case there is only oscillation one has $\epsilon_S = \epsilon_D = 0$ and equation 3.2 reduces to

$$\mathcal{R}_{e\mu} \simeq A s_{13}^2\,, \tag{3.3}$$

which is just the standard two neutrino result. In the other case, that there is only LFV, one has $s_{13} = 0$ and equation 3.2 becomes

$$\mathcal{R}_{e\mu} \simeq C(\epsilon^D_{e\tau})^2 + D\epsilon^P_{e\tau}\epsilon^S_{e\tau} + E(\epsilon^S_{e\tau})^2\,. \tag{3.4}$$

With the definition $\epsilon_S = r\epsilon_D$ one can find a value for r which makes the two equations 3.3 and 3.4 identical. This value for r is given by [27]

$$s_{13}^2 = r^2\epsilon_P^2\,\frac{1+\cos 2\theta_{23}}{2}\,. \tag{3.5}$$

Most interestingly this relation is independent of the energy which amounts to the so called "confusion theorem",*i.e.* for any value of θ_{13} it is possible to find a pair of ϵ_S and ϵ_D which can mimic the transition rate without the need for θ_{13}. This confusion theorem is however only valid on the level of probabilities as it was pointed out in [34].

In order to evaluate the θ_{13} sensitivity we simulate data for a neutrino factory as described in [35] for $\theta_{13} = \epsilon_S = \epsilon_D = 0$ and then perform a three dimensional fit to the data where the limit is given by the *largest* value of θ_{13} which is compatible with the simulated data at the given confidence level. The other oscillation parameters are kept fixed at $\Delta m^2_{31} = +3.0\,\mathrm{eV}^2$ and $\theta_{23} = \pi/4$. In figure 3 the result of this analysis is shown in several planes of parameter combinations. This way of presenting the results allows one to

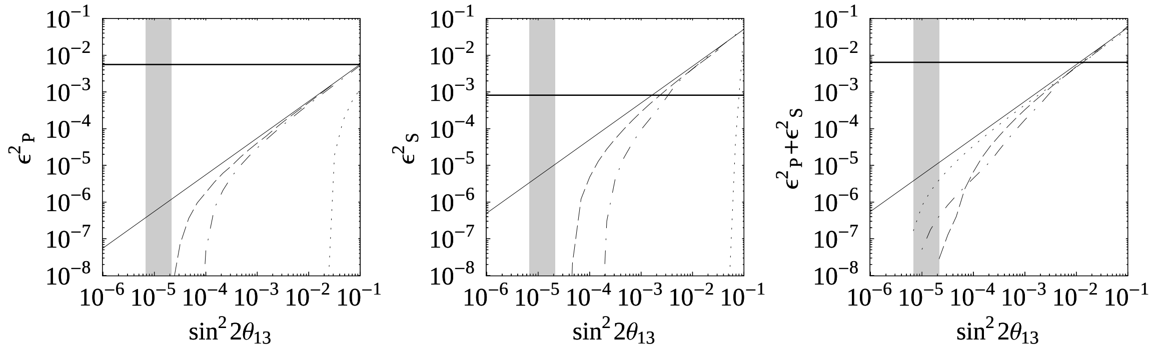

Figure 3: *Sensitivity limits at 90% CL on* $\sin^2 2\theta_{13}$ *attainable if a bound on* ϵ_P^2 *(left panel),* ϵ_S^2 *(middle panel) or* $\epsilon_P^2 + \epsilon_S^2$ *(right panel) is given. The dotted line is for a baseline of* 700 km, *the dash-dotted for* 3 000 km *and the dashed line for* 7 000 km. *The horizontal black line shows the current estimate of the limit on the NSI parameter. The vertical grey band shows the range of possible sensitivities without NSI [35]. The diagonal solid line is the theoretical bound derived from the confusion theorem. This figure is taken from [27].*

immediately read of the attainable sensitivity for $\sin^2 2\theta_{13}$ as function of the limit on the quantity shown on the vertical axes. The long dashed line shows the result for a baseline of 7,000 km, the dash-dotted line is for a baseline of 3 000 km and the dotted line is for 700 km. The grey shaded vertical bars indicated the limit in the absence of any LFV and the horizontal lines indicate the present bounds. Obviously an improvement on the bounds for LFV couplings is necessary for a neutrino factory to fully exploit its potential. This improvement can *e.g.* be achieved by the near physics program of a neutrino factory [36].

4. Conclusion

I have described the general framework needed for the combined treatment of LFV and oscillation and given a derivation of the so called confusion theorem. For some special case I also have presented some numerical results. In case of CP measurements it turned out that the relative magnitude of genuine CP effects and LFV is such that it may be very difficult to perform a precise determination of the CP phase without further improving the bounds on LFV couplings. Which is not straightforward since vertices involving two neutrinos are inherently difficult to measure. This may require a dedicated near physics program in future long baseline experiments. Furthermore it was shown that ability to limit θ_{13} is also strongly decreased by the presence of LFV. I just gave a set of simple examples to illustrate the importance of the topic but clearly further studies are needed.

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