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R-Parity violation in a SUSY GUT model and radiative neutrino masses

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Abstract: Within the framework of an SU(5) SUSY GUT model, a mechanism which effectively induces R-parity-violating terms below the unification energy scale M_X is proposed. The model has matter fields $\overline{5}_{L(+)} + 10_{L(-)}$ and Higgs fields $H_{(-)}$ and $\overline{H}_{(+)}$ in addition to the ordinary Higgs fields $H_{(+)}$ and $\overline{H}_{(-)}$ which contribute to the Yukawa interactions, where $(\pm)$ denote the transformation properties under a discrete symmetry Z_2. The Z_2 symmetry is only broken by the μ-term $\overline{H}_{(+)}H_{(-)}$ softly, so that the $\overline{5}_{(+)} \leftrightarrow \overline{H}_{(+)} \leftrightarrow \overline{H}_{(-)}$ mixing appears at $\mu < m_{SB}$, and R-parity violating terms $\overline{5}_L\overline{5}_L10_L$ are effectively induced from the Yukawa interactions $\overline{H}_{(-)}\overline{5}_{L(+)}10_{L(-)}$, i.e. the effective coupling constants λ_{ijk} of $\nu_{Li}e_{Lj}e^c_{Rk}$ and $\nu_{Li}d^c_{Rj}d_{Lk}$ are proportional to the mass matrices $(M_e^*)_{jk}$ and $(M_d^\dagger)_{jk}$, respectively. The parameter regions which are harmless for the proton decay are investigated. Possible forms of the radiatively induced neutrino mass matrix are also investigated.

1. introduction

The origin of the neutrino mass generation is still a mysterious problem in the unified understanding of the quarks and leptons. As the origin, from the standpoint of a grand unification theory (GUT), currently, the idea of the so-call seesaw mechanism[2] is influential. On the other hand, an alternative idea that the neutrino masses are radiatively induced is still attractive. As an example of such a model, the Zee model [3] is well known. Regrettably, the original Zee model is not on the framework of GUT. A possible idea to embed the Zee model into GUTs is to identify the Zee scalar h^+ as the slepton $\widetilde{e}_R$ in an R-parity-violating supersymmetric (SUSY) model [4]. However, usually, it is accepted that SUSY models with R-parity violation are incompatible with a GUT scenario, because the

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R-parity-violating interactions induce proton decay [5, 6]. By the way, there is another problem in a GUT scenario, i.e. how to give doublet-triplet splitting in SU(5) 5-plet Higgs fields. There are many ideas to solve this problem [7]. Although these mechanisms are very attractive, in the present paper, we will take another choice, that is, fine tuning of parameters: we consider a possibility that a mechanism which provides the doublet-triplet splitting gives a suppression of the R-parity violating terms with baryon number violation while it gives visible contributions of the doublet component to the low energy phenomena (neutrino masses, lepton flavor processes, and so on)[5]. In the present paper, we will try to give an example of such a scenario.

In the present paper, in order to suppress the dim-5 proton decay, a discrete symmetry Z_2 is introduced. The essential idea is as follows: we consider matter fields $\overline{5}_{L(+)} + 10_{L((-)}$ of SU(5) and two types of SU(5) 5-plet and $\overline{5}$-plet Higgs fields $H_{(\pm)}$ and $\overline{H}_{(\pm)}$, where $(\pm)$ denote the transformation properties under a discrete symmetry Z_2 (we will call it "Z_2-parity" hereafter). The superpotential in the present model is given by

$$W = W_Y + W_H + W_{mix} \ , \tag{1.1}$$

where W_Y denotes Yukawa interactions

$$W_Y = \sum_{i,j}(Y_u)_{ij}H_{(+)}10_{L(-)i}10_{L(-)j} + \sum_{i,j}(Y_d)_{ij}\overline{H}_{(-)}\overline{5}_{L(+)i}10_{L(-)j} \ . \tag{1.2}$$

Under the discrete symmetry Z_2, R-parity violating terms $\overline{5}_{L(+)}\overline{5}_{L(+)}10_{(-)}$ are exactly forbidden. The discrete symmetry Z_2 is softly violated only by the following μ-terms

$$W_H = \overline{H}_{(+)}(m_+ + g_+\Phi)H_{(+)} + \overline{H}_{(-)}(m_- + g_-\Phi)H_{(-)} + m_{SB}\overline{H}_{(+)}H_{(-)} \ , \tag{1.3}$$

where Φ is an SU(5) 24-plet Higgs field with the vacuum expectation value (VEV) $\langle\Phi\rangle = v_{24}\mathrm{diag}(2,2,2,-3,-3)$, and it has been introduced in order to give doublet–triplet splittings in the SU(5) 5- and $\overline{5}$-plets Higgs fields at an energy scale $\mu < M_X$ (M_X is an SU(5) unification scale). The Z_2-parity is violated only by[1] the term $\overline{H}_{(+)}H_{(-)}$. Note that $H_{(-)}$ and $\overline{H}_{(+)}$ in the m_{SB}-term do not contribute to the Yukawa interaction (1.2) directly, so that proton decay via the dimension-5 operator is suppressed in the limit of $m_{SB} \to 0$. (A similar idea, but without Z_2 symmetry, has been proposed by Babu and Barr [8].) The terms W_{mix} have been introduced in order to bring the $\overline{H}_{(+)} \leftrightarrow \overline{5}_{(+)}$ mixing:

$$W_{mix} = \sum_i \overline{5}_{L(+)i}\left(b_i m_5 + c_i g_5\Phi\right)H_{(+)} \ , \tag{1.4}$$

where $\sum_i |b_i|^2 = \sum_i |c_i|^2 = 1$. At the energy scale $\mu < M_X$, the terms $W_H + W_{mix}$ are effectively given by

$$\begin{aligned} W_H + W_{mix} = & \sum_{a=2,3} m_+^{(a)}\left[\overline{H}^{(a)}_{(+)}\cos\alpha^{(a)} + \sum_i d_i\overline{5}^{(a)}_{L(+)i}\sin\alpha^{(a)}\right]H^{(a)}_{(+)} \\ & + \sum_{a=2,3} m_-^{(a)}\overline{H}^{(a)}_{(-)}H^{(a)}_{(-)} + m_{SB}\sum_{a=2,3}\overline{H}^{(a)}_{(+)}H^{(a)}_{(-)} \ , \end{aligned} \tag{1.5}$$

[1] The Z_2 symmetry can softly violated not only by the term $\overline{H}_{(+)}H_{(-)}$, but also by terms $\overline{H}_{(-)}H_{(+)}$ and $\overline{5}_{L(+)1}H_{(-)}$. However, in the present scenario, the existence of $\overline{H}_{(+)}H_{(-)}$ is essential. The details are discussed in Appendix A of [1].

where $\sum_i |d_i|^2 = 1$, the index (a) denotes that the fields with (2) and (3) are doublet and triplet components of SU(5)→SU(2)×SU(3), respectively, and

$$m_+^{(2)} \cos\alpha^{(2)} = m_+ - 3g_+ v_{24} \ , \quad m_+^{(3)} \cos\alpha^{(3)} = m_+ + 2g_+ v_{24} \ , \tag{1.6}$$

$$m_+^{(2)} \sin\alpha^{(2)} d_i = m_5 b_i - 3g_5 v_{24} c_i \ , \quad m_+^{(3)} \sin\alpha^{(3)} d_i = m_5 b_i + 2g_5 v_{24} c_i \ , \tag{1.7}$$

$$m_-^{(2)} = m_- - 3g_- v_{24} \ , \quad m_-^{(3)} = m_- + 2g_- v_{24} \ . \tag{1.8}$$

Therefore, the m_{SB}-term together with $m_+ \sin\alpha$-term induces the $\overline{H}_{(-)} \leftrightarrow \overline{5}_{L(+)}$ mixing, so that the R-parity violating terms $\overline{5}_L\overline{5}_L 10_L$ are generated from the Yukawa interactions $\overline{H}_{(-)}\overline{5}_{L(+)}10_{L(-)}$. The coupling constants λ_{ijk} of $\overline{5}_i\overline{5}_j 10_k$ will be proportional to the charged lepton mass matrix $(M_e^*)_{jk}$ or down-quark mass matrix $(M_d^\dagger)_{jk}$. (The details are discussed in the next section 2.) As we demonstrate in Sec. 2, we can show that the mixing $\overline{5}_{L(+)} \leftrightarrow \overline{H}_{(-)}$ is negligibly small for the colored sector, while it is sizable for SU(2)-doublet sector.

The parameters in the present model need fine-tuning. For example, we will find that a large value of m_{SB} is not acceptable, because for such a large value of m_{SB} the proton decay due to the dimension five operator becomes visible. On the other hand, we will find that a smaller value of m_{SB} leads to a small bottom quark mass, so that a small value of m_{SB} is not acceptable. We will take $m_{SB} \sim 10^{14}$ GeV. In Sec. 3, we will investigate a possible form of the radiatively induced neutrino mass matrix due to the R-parity violation term $\overline{5}_L\overline{5}_L 10_L$. The radiatively induced neutrino mass matrix M_ν^{rad} will be expressed by the sum of two rank-1 matrices. On the other hand, we also have contributions $M_{\tilde{\nu}}$ from VEVs $\langle\tilde{\nu}_i\rangle$ of the sneutrinos to the neutrino mass matrix M_ν. Finally, Sec. 4 will be devoted to the summary.

2. $\overline{H}_{(-)}$-$\overline{5}_{(+)}$ mixing

In order to suppress the proton decay, we want to take $m_+^{(2)} \sim M_W$ with a sizable $\alpha^{(2)}$, but $m_+^{(3)} \sim M_X$ with a negligibly small $\alpha^{(3)}$. However, from the relations (1.6) and (1.7), we obtain the relation

$$d_i \tan\alpha^{(3)} = \frac{m_5 b_i + 2g_5 v_{24} c_i}{m_+ + 2g_+ v_{24}} = \frac{m_+^{(2)} \sin\alpha^{(2)} d_i + 5g_5 v_{24} c_i}{m_+^{(2)} \cos\alpha^{(2)} + 5g_+ v_{24}} \ . \tag{2.1}$$

The requirement $|\alpha^{(3)}| \leq M_W/M_X$ leads to the constraint $|g_5| \leq M_W/M_X$ for $|g_+| \sim 1$. We do not like to introduce such a small dimensionless parameter g_5. Therefore, for simplicity, we will put $g_5 = 0$ hereafter. Then, without loss of generality, we can put

$$\overline{5}'_{L(+)1} = \sum_i b_i \overline{5}_{L(+)i} \tag{2.2}$$

where $\overline{5}'_{L(+)1}$ does not mean the observed first generation particle. (Hereafter, for convenience, we denote $\overline{5}'_{L(+)1}$ as $\overline{5}_{L(+)1}$ simply. The effective parameters $m_+^{(a)}$, $m_-^{(a)}$ and $\alpha^{(a)}$

are given as follows:

$$
\begin{aligned}
m_+^{(2)} &= \sqrt{(m_+ - 3g_+ v_{24})^2 + m_5^2}\ , & m_+^{(3)} &= \sqrt{(m_+ + 2g_+ v_{24})^2 + m_5^2}\ , \\
m_-^{(2)} &= m_- - 3g_- v_{24}\ , & m_-^{(3)} &= m_- + 2g_- v_{24}\ , \\
\tan\alpha^{(2)} &= \tfrac{m_5}{m_+ - 3g_+ v_{24}} \simeq \tfrac{m_5}{m_+^{(2)}}\ , & \tan\alpha^{(3)} &= \frac{m_5}{m_+ + 2g_+ v_{24}} \simeq \frac{m_5}{m_+^{(3)}}\ .
\end{aligned}
\tag{2.3}
$$

We will take

$$
\begin{aligned}
m_+^{(2)} &\sim M_W\ , & m_+^{(3)} &\sim M_X\ , \\
m_-^{(2)} &\sim M_I\ , & m_-^{(3)} &\sim M_X\ , \\
\tan\alpha^{(2)} &\sim \tfrac{m_5}{M_W}\ , & \tan\alpha^{(3)} &\sim \tfrac{m_5}{M_X}\ ,
\end{aligned}
\tag{2.4}
$$

where $M_I \sim 10^{14}$ GeV and $m_5 \sim 10^1$ GeV as we state later. The mass matrix in the basis of $(\overline{H}_{(-)}, \overline{H}_{(+)}, \overline{5}_{L(+)1})$ and $(H_{(+)}, H_{(-)})$ is given by

$$
M = \begin{pmatrix} 0 & m_- \\ m_+ \cos\alpha & m_{SB} \\ m_+ \sin\alpha & 0 \end{pmatrix}\ . \tag{2.5}
$$

Here and hereafter, for simplicity, we drop the index (a). The mass matrix (2.5) is diagonalized as

$$
\overline{U}^\dagger M U = D \equiv \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \\ 0 & 0 \end{pmatrix}\ . \tag{2.6}
$$

Note that the matter field $\overline{5}'_{L1}$ is still massless, and also note that it is not in the eigenstate of the Z_2 parity.

The mixing matrix U is easily obtained from the diagonalization of

$$
M^\dagger M = \begin{pmatrix} |m_+|^2 & m_{SB} m_+^* \cos\alpha \\ m_{SB}^* m_+ \cos\alpha & |m_{SB}|^2 + |m_-|^2 \end{pmatrix}\ . \tag{2.7}
$$

For real m_1, m_{SB} and $m_\pm$, we obtain

$$
U = \begin{pmatrix} \cos\theta_u & \sin\theta_u \\ -\sin\theta_u & \cos\theta_u \end{pmatrix}\ , \tag{2.8}
$$

$$
\tan 2\theta_u = \frac{2 m_{SB} m_+ \cos\alpha}{m_{SB}^2 + m_-^2 - m_+^2}\ , \tag{2.9}
$$

$$
m_1^2 = \frac{1}{2}\left(m_{SB}^2 + m_+^2 + m_-^2\right) - \frac{1}{2} Q\ , \tag{2.10}
$$

$$
m_2^2 = \frac{1}{2}\left(m_{SB}^2 + m_+^2 + m_-^2\right) + \frac{1}{2} Q\ , \tag{2.11}
$$

where

$$
Q = (m_{SB}^2 - m_+^2 + m_-^2)\cos 2\theta_u + 2 m_{SB} m_+ \cos\alpha \sin 2\theta_u\ . \tag{2.12}
$$

When we define

$$A \equiv m_{SB}^2 - m_+^2 + m_-^2 \ , \quad B \equiv 2m_{SB}m_+ \cos\alpha \ , \tag{2.13}$$

$$\cos 2\theta_u = \frac{A}{\sqrt{A^2+B^2}} \ , \quad \sin 2\theta_u = \frac{B}{\sqrt{A^2+B^2}} \ , \tag{2.14}$$

the quantity Q is given by

$$Q = \sqrt{A^2+B^2} = \sqrt{\left[m_{SB}^2 + (m_+ - m_-)^2\right]\left[m_{SB}^2 + (m_+ + m_-)^2\right] - 4m_{SB}^2 m_+^2 \sin^2\alpha} \ . \tag{2.15}$$

The rotation $\overline{U}$ is also obtained from the diagonalization of

$$MM^\dagger = \begin{pmatrix} m_-^2 & m_{SB}m_- & 0 \\ m_{SB}m_- & m_{SB}^2 + m_+^2\cos^2\alpha & m_+^2\cos\alpha\sin\alpha \\ 0 & m_+^2\cos\alpha\sin\alpha & m_+^2\sin^2\alpha \end{pmatrix} \ . \tag{2.16}$$

The mixing matrix elements $\overline{U}_{i3}$ are easily obtain as follows:

$$\overline{U}_{13} = \frac{1}{N_3} m_{SB}\sin\alpha \ , \overline{U}_{23} = -\frac{1}{N_3} m_- \sin\alpha \ , \overline{U}_{33} = \frac{1}{N_3} m_- \cos\alpha \ , \tag{2.17}$$

where

$$N_3^2 = -m_-^2 + m_{SB}^2 \sin^2\alpha \ . \tag{2.18}$$

Other matrix elements are obtained as follows: We express the mixing matrix $\overline{U}$ as

$$\overline{U} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13} & c_{23}c_{12} - s_{23}s_{12}s_{13} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13} & -s_{23}c_{12} - c_{23}s_{12}s_{13} & c_{23}c_{13} \end{pmatrix} \ , \tag{2.19}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. Then, by comparing (2.19) with (2.17), we obtain

$$s_{13} = \overline{U}_{13} = \frac{m_{SB}\sin\alpha}{\sqrt{m_-^2 + m_{SB}^2\sin^2\alpha}} \ , \quad c_{13} = \frac{1}{\sqrt{1 + (m_{SB}/m_-)^2\sin^2\alpha}} \ , \tag{2.20}$$

$$s_{23} = \frac{\overline{U}_{23}}{c_{13}} = -\sin\alpha \ , \quad c_{23} = \cos\alpha \ . \tag{2.21}$$

By using the relation $(M^\dagger M)_{11} = \overline{U}_{11}\overline{U}_{11}(m_1')^2 + \overline{U}_{12}\overline{U}_{12}(m_2')^2$, the mixing angle θ_{12} is obtained as follows:

$$\begin{aligned} \cos 2\theta_{12} &= \frac{1}{m_2^2 - m_1^2}\left[m_1^2 + m_2^2 - 2\frac{m_-^2}{c_{13}^2}\right] = \frac{1}{Q}(m_{SB}^2 + m_+^2 - m_-^2 - 2m_{SB}^2\sin^2\alpha) \\ &= -\frac{m_-^2 - m_2^2\cos 2\alpha - m_+^2}{\sqrt{m_-^4 + 2(m_2^2 - m_+^2)m_-^2 + m_2^4 + 2m_2^2m_+^2\cos 2\alpha + m_+^4}} \ . \end{aligned} \tag{2.22}$$

Note that $\cos 2\theta_{12} \simeq -1$ for $m_-^2 \gg m_{SB}^2, m_+^2$, so that $\theta_{12} \simeq \pi/2$.

Since the physical fields $(\overline{H}_1, \overline{H}_2, \overline{5}'_{L1}, \overline{5}'_{L2}, \overline{5}'_{L3})$ are given by

$$\begin{pmatrix} \overline{H}_{(-)} \\ \overline{H}_{(+)} \\ \overline{5}_{L(+)1} \\ \overline{5}_{L(+)2} \\ \overline{5}_{L(+)3} \end{pmatrix} = \begin{pmatrix} \overline{U}_{11} & \overline{U}_{12} & \overline{U}_{13} & 0 & 0 \\ \overline{U}_{21} & \overline{U}_{22} & \overline{U}_{23} & 0 & 0 \\ \overline{U}_{31} & \overline{U}_{32} & \overline{U}_{33} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \overline{H}_1 \\ \overline{H}_2 \\ \overline{5}'_{L1} \\ \overline{5}'_{L2} \\ \overline{5}'_{L3} \end{pmatrix} , \tag{2.23}$$

the Yukawa interactions $\overline{H}_{(-)}\overline{5}_{L(+)}10_{L(-)}$ lead to the effective Yukawa interactions at a low energy scale

$$(Y_d)_{ij}\overline{H}_1 \left[\delta_{i1}(\overline{U}_{11}\overline{U}_{33} - \overline{U}_{13}\overline{U}_{31})\overline{5}'_{L1} + \overline{U}_{11}(\delta_{i2}\overline{5}'_{L2} + \delta_{i3}\overline{5}'_{L3})\right] 10_{L(-)j} , \tag{2.24}$$

and the induced R-parity violating terms

$$(Y_d)_{ij}\overline{U}_{13}\overline{5}'_{L1}\left(\delta_{i1}\overline{U}_{33}\overline{5}'_{L1} + \delta_{i2}\overline{5}'_{L2} + \delta_{i3}\overline{5}'_{L3}\right) 10_{L(-)j} , \tag{2.25}$$

where we have assumed that $|m_1| << |m_2|$, i.e. the Higgs field surviving at a low energy scale is not $\overline{H}_2$, but $\overline{H}_1$.

The effective Yukawa interactions (2.24) give the fermion mass matrices

$$(M_e^*)_{ij} = \begin{cases} (\overline{U}^{(2)}_{11}\overline{U}^{(2)}_{33} - \overline{U}^{(2)}_{13}\overline{U}^{(2)}_{31})(Y_d)_{ij}v_d = \overline{U}^{(2)*}_{22}(Y_d)_{ij}v_d & \text{for } i = 1 , \\ \overline{U}^{(2)}_{11}(Y_d)_{ij}v_d & \text{for } i = 2, 3 , \end{cases} \tag{2.26}$$

$$(M_d^\dagger)_{ij} = \begin{cases} (\overline{U}^{(2)}_{11}\overline{U}^{(3)}_{33} - \overline{U}^{(3)}_{13}\overline{U}^{(2)}_{31})(Y_d)_{ij}v_d & \text{for } i = 1 , \\ \overline{U}^{(2)}_{11}(Y_d)_{ij}v_d & \text{for } i = 2, 3 , \end{cases} \tag{2.27}$$

where $v_d = \langle \overline{H}_1 \rangle$, and, in (2.26), we have used the general formula $U_{ik}U_{jl} - U_{il}U_{jk} = \varepsilon_{ijm}\varepsilon_{kln}U^*_{mn}$ for an arbitrary 3×3 unitary matrix U. Note that in the present model, the relation $M_d = M_e^T$ does not hold.

From the R-parity violating terms (2.25), we obtain coefficients $\lambda^{(2,2)}_{ijk}$, $\lambda^{(2,3)}_{ijk}$, $\lambda^{(3,2)}_{ijk}$ and $\lambda^{(3,3)}_{ijk}$, which are the coefficients of the interactions $(\nu_{L1}e_{Li} - e_{L1}\nu_{Li})e^c_{Rj}$, $(\nu_{L1}d^c_{Ri}d_{Lj} - e_{L1}d^c_{Ri}u_{Lj})$, $(d^c_{R1}e_{Li}u_{Lj} - d^c_{R1}\nu_{Li}d_{Lj})$, and $\varepsilon_{\alpha\beta\gamma}d^{c\alpha}_{R1}d^{c\beta}_{Ri}u^{c\gamma}_{Rj}$, respectively, as follows:

$$\lambda^{(2,2)}_{11j} = 0 , \qquad \lambda^{(2,2)}_{1ij} = \kappa(M_e^*)_{ij}/v_d \quad (i = 2, 3), \tag{2.28}$$

$$\lambda^{(2,3)}_{11j} = \frac{\kappa(M_d^\dagger)_{1j}/v_d}{1 - \xi\kappa\overline{U}^{(2)}_{31}/\overline{U}^{(3)}_{33}} , \quad \lambda^{(2,3)}_{1ij} = \kappa(M_d^\dagger)_{ij}/v_d \quad (i = 2, 3), \tag{2.29}$$

$$\lambda^{(3,2)}_{11j} = \frac{\xi\kappa(M_e^*)_{1j}/v_d}{1 - \kappa\overline{U}^{(2)}_{31}/\overline{U}^{(2)}_{33}} , \quad \lambda^{(3,2)}_{1ij} = \xi\kappa(M_e^*)_{ij}/v_d \quad (i = 2, 3), \tag{2.30}$$

$$\lambda^{(3,3)}_{11j} = 0 , \qquad \lambda^{(3,3)}_{1ij} = \xi\kappa(M_d^\dagger)_{ij}/v_d \quad (i = 2, 3), \tag{2.31}$$

where

$$\kappa = \frac{\overline{U}^{(2)}_{13}}{\overline{U}^{(2)}_{11}} , \qquad \xi = \frac{\overline{U}^{(3)}_{13}}{\overline{U}^{(2)}_{13}}. \tag{2.32}$$

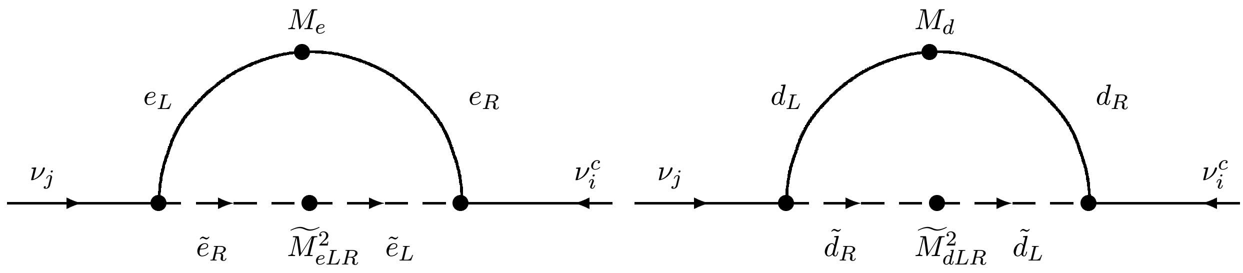

Figure 1: Radiative generation of neutrino Majorana mass

Note that the proton decay due to the exchange of squarks $\widetilde{d}_i$ is forbidden in the limit of $\xi \to 0$, while the radiatively-induced neutrino masses do not become zero even if $\xi \to 0$. To suppress the proton decay due to these couplings, κ should be small while to have neutrino masses κ should have a sufficient strength. In this respect we can show that $\kappa \sim O(0.1)$ is welcome. Also to suppress the proton decay through dimension 5 operators it is necessary that m_{SB} must be small, though it is also bounded from below to make yukawa couplings be sufficiently large. [1].

3. Radiatively induced neutrino mass matrix

In a SUSY GUT scenario, there are many origins of the neutrino mass generations [10]. For example, the sneutrinos $\widetilde{\nu}_{iL}$ can have VEVs $\langle\widetilde{\nu}\rangle \neq 0$, and the neutrinos ν_{Li} acquire their masses thereby [11]. In the present model, there is a R-parity violating bilinear term $\overline{5}_{L(+)}H_{(+)}$, while there is no $\overline{H}_{(-)}H_{(+)}$ term (the so-called μ-term). In the physical field basis (the basis on which the mass matrix (2.5) is diagonal), the so-called μ-term, $m_1\overline{H}_1 H_1$, appears, while the $\overline{5}_L H_1$ term is absent. Therefore, in the present model, the sneutrinos cannot have VEVs $\langle\tilde{\nu}_i\rangle$ at tree level. The VEVs $\langle\tilde{\nu}_i\rangle \neq 0$ will appear only through the renormalization group equation (RGE) effect. The contribution highly depends on an explicit model of the SUSY symmetry breaking. Since the purpose of the present paper is to investigate a general structure of the radiative neutrino masses, for the moment, we confine ourselves to discussing possible forms of the radiative neutrino mass matrix .

The radiative neutrino mass matrix M_ν^{rad} is given by

$$M_\nu^{rad} = M_\nu^e + M_\nu^d \ , \tag{3.1}$$

where M_ν^e is generated by the interactions $\nu_L e_L \widetilde{e}^c_R$ and $\nu_L \widetilde{e}_L e^c_R$ (i.e. by the charged lepton loop) and M_ν^d is generated by $\nu_L d^c_R \widetilde{d}_L$ and $\nu_L \widetilde{d}^c_R d_L$ (i.e. by the down-quark loop). We assume that the contributions from Zee-type diagrams due to $\overline{H}^+ \leftrightarrow \widetilde{e}^+_R$ mixing is negligibly small because the term $\overline{H}\,\overline{H}\,10_L$ must be not $\overline{H}_1\overline{H}_1 10_L$, but $\overline{H}_1\overline{H}_2 10_L$ (recall that only the field $\overline{H}_1$ has the VEV in the present model).

We consider the radiative diagram with $(\nu_L)_j \to (e_R)_l + (\widetilde{e}^c_L)_n$ and $(e_L)_k + (\widetilde{e}^c_L)_m \to (\nu^c_L)_i$. The contributions $(M_\nu^e)_{ij}$ from the charged lepton loop are, except for the common factors, given as follows:

$$(M_\nu^e)_{ij} = (\lambda_{1km}\delta_{i1} - \lambda_{1im}\delta_{k1})(\lambda_{1jl}\delta_{n1} - \lambda_{1nl}\delta_{j1})(M_e)_{kl}(\widetilde{M}^2_{eLR})_{nm} + (i \leftrightarrow j) \ , \tag{3.2}$$

where M_e and $\widetilde{M}^2_{eLR}$ are charged-lepton and charged-slepton-LR mass matrices, respectively. Here and hereafter, we will drop the common factor in $(M_\nu^{rad})_{ij}$, because we have an interest only in the relative structure of the matrix elements $(M_\nu^{rad})_{ij}$. Since, as usual, we assume that the structure of $\widetilde{M}^2_{eLR}$ is proportional to that of M_e, we obtain

$$\begin{aligned}(M_\nu^e)_{ij} =& \lambda_{1im}\lambda_{1jl}(M_e)_{1l}(M_e)_{1m} + \delta_{i1}\delta_{j1}\lambda_{1km}\lambda_{1nl}(M_e)_{kl}(M_e)_{nm} \\ &-\delta_{i1}\lambda_{1jl}\lambda_{1km}(M_e)_{1m}(M_e)_{kl} - \delta_{j1}\lambda_{1il}\lambda_{1km}(M_e)_{1m}(M_e)_{kl} \ .\end{aligned} \tag{3.3}$$

Since $\lambda^e_{1ij} \equiv \lambda^{(2,2)}_{1ij} = \kappa(1-\delta_{i1})(M_e^\dagger)_{ji}$ from the expression (2.29), we obtain the contribution from the charged lepton loop:

$$M_\nu^e = H_e^T S_1 H_e - S_1 H_e H_e - H_e^T H_e^T S_1 + S_1 \mathrm{Tr}(H_e H_e) \ . \tag{3.4}$$

where we have dropped the common factor κ, and the Hermitian matrix H_e and the rank-1 matrix S_1 are defined by

$$H_e = M_e M_e^\dagger \ , \tag{3.5}$$

$$S_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ . \tag{3.6}$$

Similarly, we can obtain the contributions from the down-quark loop. From the expression (2.30), we denote $\lambda^d_{1ij} \equiv \lambda^{(2,3)}_{1ij}$ as

$$\lambda^d_{1ij} = \kappa[\rho\delta_{1i} + (1-\delta_{i1})](M_d^\dagger)_{ij} \ , \tag{3.7}$$

where

$$\rho = \frac{1}{1 - \xi\kappa U^{(2)}_{31}/U^{(3)}_{33}} \simeq 1 \ . \tag{3.8}$$

Then, we obtain

$$M_\nu^d = S_1 \mathrm{Tr}(H_d H_d), \tag{3.9}$$

where

$$H_d = M_d^\dagger M_d \ . \tag{3.10}$$

Note that the result (3.9) is independent of the value of ρ.

The field $\overline{5}_{L(+)1}$ defined in Eq. (2.2) does not mean the observed first-generation field $(d^c, \nu, e)_L$ (and its SUSY partner). The forms of M_ν^e and M_ν^d on the general basis are given by

$$M_\nu^e = H_e^T S H_e - S H_e H_e - H_e^T H_e^T S + S \mathrm{Tr}(H_e H_e) \ , \tag{3.11}$$

$$M_\nu^d = S \mathrm{Tr}(H_d H_d), \tag{3.12}$$

where S is an arbitrary rank-1 matrix $S = U_5^T S_1 U_5$, which is given by the rebasing $\overline{5}_i \to \overline{5}'_i = (U_5^\dagger \overline{5})_i$.

It is convenient to investigate the form M_ν^{rad} on the basis on which the charged lepton mass matrix M_e is diagonal: $H_e = D_e^2 = \mathrm{diag}(m_{e1}^2, m_{e2}^2, m_{e3}^2) \equiv \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2)$. Then, the matrix M_ν^e is given by

$$M_\nu^e = \begin{pmatrix} S_{11}(m_{e2}^4 + m_{e3}^4) & S_{12}(m_{e3}^4 + m_{e1}^2 m_{e2}^2) & S_{13}(m_{e2}^4 + m_{e1}^2 m_{e3}^2) \\ S_{21}(m_{e3}^4 + m_{e1}^2 m_{e2}^2) & S_{22}(m_{e3}^4 + m_{e1}^4) & S_{23}(m_{e1}^4 + m_{e2}^2 m_{e3}^2) \\ S_{31}(m_{e2}^4 + m_{e1}^2 m_{e3}^2) & S_{32}(m_{e1}^4 + m_{e2}^2 m_{e3}^2) & S_{33}(m_{e1}^4 + m_{e2}^4) \end{pmatrix} \quad (3.13)$$

$$\simeq m_\tau^4 \begin{pmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} = m_\tau^4 PSP \ , \quad (3.14)$$

$$P = \mathrm{diag}(1,1,0) \ . \quad (3.15)$$

Therefore, we can express the neutrino mass matrix M_ν^{rad} as the following form:

$$M_\nu^{rad} = m_0 \left(PSP + k^2 S \right) , \quad (3.16)$$

where k is given by $k \simeq (m_b/m_\tau)^2$ and m_0 will be given later [in Eq. (3.18)]. The matrix S is a rank-1 matrix, so that PSP is also rank-1 matrix. In other words, the radiative neutrino mass matrix M_ν^{rad} has the following form by assuming $(U_5)_{1e} \sim O(\epsilon), (U_5)_{1\mu}, (U_5)_{1\tau} \sim O(1)$:

$$M_\nu^{rad} = m_0 \left\{ \begin{pmatrix} \epsilon^2 & \epsilon & \\ \epsilon & 1 & \\ & & \end{pmatrix} + k \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \right\} \quad (3.17)$$

It shows a good hierarchical pattern [15].

So far, we have not discussed the absolute magnitude of the neutrino mass matrix M_ν^{rad}. When we assume $m^2(\widetilde{e}_R) \equiv m^2(\widetilde{e}_{R3}) \simeq m^2(\widetilde{e}_{R2}) \simeq m^2(\widetilde{e}_{R1})$ and $m^2(\widetilde{e}_L) \equiv m^2(\widetilde{e}_{L3}) \simeq m^2(\widetilde{e}_{L2}) \simeq m^2(\widetilde{e}_{L1})$ and the rank-1 matrix S is normalized as $\mathrm{Tr}(SS^\dagger) = 1$, the coefficient m_0 in the expression (3.16) is given by

$$m_0 = \frac{1}{16\pi^2} \kappa^2 \frac{m_1^{(2)} m_\tau^4}{v^2} F(m^2(\widetilde{e}_R), m^2(\widetilde{e}_L)) \ , \quad (3.18)$$

where

$$F(m_R^2, m_L^2) = \frac{1}{m_R^2 - m_L^2} \ln \frac{m_R^2}{m_L^2} \ . \quad (3.19)$$

If $F(m^2(\widetilde{e}_R), m^2(\widetilde{e}_L)) \simeq F(m^2(\widetilde{d}_R), m^2(\widetilde{d}_L))$, the factor k is given by $k \simeq (m_b/m_\tau)^4 = 8.6$. However, in the present paper, we regard k as a free parameter. By using $1/16\pi^2 = 6.33 \times 10^{-3}$, $m_1^{(2)} \equiv m(H_1^{(2)}) = 2 \times 10^2$ GeV, $m_\tau(m_Z) = 1.75$ GeV, $v = 174$ GeV and $\tan\beta = 1.5$, we obtain

$$m_0 \simeq 1.9 \kappa^2 F \text{ eV} \ , \quad (3.20)$$

where F is the value of $F(m^2(\widetilde{e}_R), m^2(\widetilde{e}_L))$ in the unit of TeV. If the neutrino mass matrix M_ν is dominated by the radiative mass terms M_ν^{rad} and we wish that the largest one of $m_{\nu i}$ is of the order of $\sqrt{\Delta m_{atm}^2} \simeq 0.05$ eV, the value $\kappa \sim 10^{-1}$ is favorable.

4. Summary

In conclusion, within the framework of an SU(5) SUSY GUT model, we have proposed a mechanism which effectively induces R-parity-violating terms at $\mu < m_{SB}$. In our model, those terms with lepton number violation are large enough to generate neutrino Majorana masses while those with baryon number violation are strongly suppressed so that the experimental bound of proton decay is evaded. This is related with doublet-triplet splitting. We have matter fields $\overline{5}_{L(+)} + 10_{L(-)}$ and two types of Higgs fields $H_{(\pm)}$ and $\overline{H}_{(\pm)}$, where $(\pm)$ denote the transformation properties under a discrete symmetry Z_2. The Higgs fields $H_{(+)}$ and $\overline{H}_{(-)}$ couple to $10_{L(-)}10_{L(-)}$ and $\overline{5}_{L(+)}10_{L(-)}$, respectively, to make the Yukawa interactions. The Z_2 symmetry is only broken by the μ-term, $m_{SB}\overline{H}_{(+)}H_{(-)}$, so that the $\overline{H}_{(-)} \leftrightarrow \overline{5}_{(+)}$ mixing is effectively induced at $\mu < m_{SB}$. Because of the heaviness of the color triplet components of the Higgs fields, the mixing is sizable in the $SU(2)_L$ doublet sector, while it is negligibly small in the $SU(3)_c$ triplet sector.

Whether the model is harmless or not for proton decay is highly sensitive to the choice of the parameter values, especially, m_{SB} and m_5. A smaller value of m_{SB} gives a lighter mass for the massive Higgs fields H_2 (another one, H_1, corresponds to the Higgs field in the conventional model), so that the case spoils the unification of the gauge coupling constants at $\mu = M_X$. On the other hand, a large value of m_{SB} induces the proton decay due to the dimension-5 operator. We have taken $m_{SB} \sim 10^{14}$ GeV. Also, a large value of m_5 induce the proton decay due to the exchange of squark $\widetilde{d}$. We have taken $m_5 \sim 10^1$ GeV. Those parameter values can give a reasonable magnitude of the neutrino mass. However, the choice of such a small m_5 gives a small mixing between $\overline{H}_{(+)}$ and $\overline{5}_{(+)}$, so that the case gives $\overline{U}^{(2)}_{22} \simeq \overline{U}^{(2)}_{11}$ and $\overline{U}^{(2)}_{11}\overline{U}^{(2)}_{33} - \overline{U}^{(2)}_{13}\overline{U}^{(2)}_{31} \simeq \overline{U}^{(2)}_{11}\overline{U}^{(3)}_{33} - \overline{U}^{(3)}_{13}\overline{U}^{(2)}_{31}$. Therefore, the case with $|\alpha^{(2)}| \ll 1$ cannot give a sizable deviation from $M_d^T = M_e$. However, this is critical for each parameter value. The details are dependent on the explicit model, i.e. on the choice of the forms S and $U \equiv U_R^d$. A further careful study based on an explicit model will be required.

Anyhow, if the present scenario is working, the proton decay will be observed in the near future, because possible parameter values are in critical ranges for the proton decay in order to explain the quark and lepton (charged lepton and neutrino) masses and mixings.

The present model leads to a radiatively-induced neutrino mass matrix M_ν^{rad} which is given by sum of two rank-1 matrices as shown in Eq. (3.16). The "two" is originated in the two contributions from charged lepton loop and down-quark loop. This can show a good hierarchical form and hence promising.

The present model will be worth noticing. In the present model, the coupling constants λ_{ijk} of $\nu_{Li}e_{Lj}e^c_{Rk}$ and $\nu_{Li}d^c_{Rj}d_{Lk}$ are proportional to the mass matrices $(M_e^*)_{jk}$ and $(M_d^\dagger)_{jk}$, respectively. The model will give fruitful phenomenology in flavor violating processes.

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