**R-Parity violation in a SUSY GUT model and radiative neutrino masses**

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**Abstract:** Within the framework of an SU(5) SUSY GUT model, a mechanism which effectively induces R-parity-violating terms below the unification energy scale $M_X$ is proposed. The model has matter fields $\mathbf{5}_{L(+)} + \mathbf{10}_{L(-)}$ and Higgs fields $H(-)$ and $\overline{H}(+)$ in addition to the ordinary Higgs fields $H(+)$ and $\overline{H}(-)$ which contribute to the Yukawa interactions, where $(\pm)$ denote the transformation properties under a discrete symmetry $Z_2$. The $Z_2$ symmetry is only broken by the $\mu$-term $\overline{H}(+)H(-)$ softly, so that the $\mathbf{5}_{L(+)} \leftrightarrow \overline{H}(+) \leftrightarrow \overline{H}(-)$ mixing appears at $\mu < m_{SB}$, and R-parity violating terms $\mathbf{5}_L\mathbf{5}_L\mathbf{10}_L$ are effectively induced from the Yukawa interactions $\overline{H}(+)\mathbf{5}_L\mathbf{10}_L$, i.e. the effective coupling constants $\lambda_{ijk}$ of $\nu_{L_i} e_{L_j} e_{L_k}$ and $\nu_{L_i} d_{R_j} d_{L_k}$ are proportional to the mass matrices $(M^*_e)_{jk}$ and $(M^*_d)_{jk}$, respectively. The parameter regions which are harmless for the proton decay are investigated. Possible forms of the radiatively induced neutrino mass matrix are also investigated.

**1. introduction**

The origin of the neutrino mass generation is still a mysterious problem in the unified understanding of the quarks and leptons. As the origin, from the standpoint of a grand unification theory (GUT), currently, the idea of the so-called seesaw mechanism is influential. On the other hand, an alternative idea that the neutrino masses are radiatively induced is still attractive. As an example of such a model, the Zee model is well known. Regrettably, the original Zee model is not on the framework of GUT. A possible idea to embed the Zee model into GUTs is to identify the Zee scalar $h^+$ as the slepton $\tilde{e}_R$ in an R-parity-violating supersymmetric (SUSY) model. However, usually, it is accepted that SUSY models with R-parity violation are incompatible with a GUT scenario, because the

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R-parity-violating interactions induce proton decay [8, 12]. By the way, there is another problem in a GUT scenario, i.e. how to give doublet-triplet splitting in SU(5) 5-plet Higgs fields. There are many ideas to solve this problem [1]. Although these mechanisms are very attractive, in the present paper, we will take another choice, that is, fine tuning of parameters: we consider a possibility that a mechanism which provides the doublet-triplet splitting gives a suppression of the R-parity violating terms with baryon number violation while it gives visible contributions of the doublet component to the low energy phenomena (neutrino masses, lepton flavor processes, and so on) [5]. In the present paper, we will try to give an example of such a scenario.

In the present paper, in order to suppress the dim-5 proton decay, a discrete symmetry $Z_2$ is introduced. The essential idea is as follows: we consider matter fields $\bar{\Phi}_{L(+)}^5 + 10_{L(-)}$ of SU(5) and two types of SU(5) 5-plet and 5-plet Higgs fields $H_{(\pm)}$ and $\bar{H}_{(\pm)}$, where $(\pm)$ denote the transformation properties under a discrete symmetry $Z_2$ (we will call it “$Z_2$-parity” hereafter). The superpotential in the present model is given by

$$W = W_Y + W_H + W_{mix},$$

(1.1)

where $W_Y$ denotes Yukawa interactions

$$W_Y = \sum_{i,j} (Y_u)_{ij} H_{(+)} (10_{L(-)})_i 10_{L(-)}_j + \sum_{i,j} (Y_d)_{ij} \bar{H}_{(-)} (10_{L(-)})_i 10_{L(-)}_j.$$  

(1.2)

Under the discrete symmetry $Z_2$, R-parity violating terms $\bar{\Phi}_{L(+)}^5 \bar{\Phi}_{L(+)}^{10}_{(-)}$ are exactly forbidden. The discrete symmetry $Z_2$ is softly violated only by the following $\mu$-terms

$$W_H = \bar{H}_{(+)} (m_++g_+\Phi) H_{(+)} + \bar{H}_{(-)} (m_-+g_-\Phi) H_{(-)} + m_{SB} \bar{H}_{(+)} H_{(-)}.$$  

(1.3)

where $\Phi$ is an SU(5) 24-plet Higgs field with the vacuum expectation value (VEV) $\langle \Phi \rangle = \nu_{24}\text{diag}(2,2,2,-3,-3)$, and it has been introduced in order to give doublet–triplet splittings in the SU(5) 5- and 5-plets Higgs fields at an energy scale $\mu < M_X$ ($M_X$ is an SU(5) unification scale). The $Z_2$-parity is violated only by the term $\bar{H}_{(+)} H_{(-)}$. Note that $H_{(-)}$ and $\bar{H}_{(+)}$ in the $m_{SB}$-term do not contribute to the Yukawa interaction (1.2) directly, so that proton decay via the dimension-5 operator is suppressed in the limit of $m_{SB} \to 0$. (A similar idea, but without $Z_2$ symmetry, has been proposed by Babu and Barr [8].) The terms $W_{mix}$ have been introduced in order to bring the $\bar{H}_{(+)} \leftrightarrow \bar{H}_{(-)}$ mixing:

$$W_{mix} = \sum_i \bar{\Phi}_{L(+)}^5 \left( b_i m_5 + c_i g_5 \Phi \right) H_{(+)}.$$  

(1.4)

where $\sum_i |b_i|^2 = \sum_i |c_i|^2 = 1$. At the energy scale $\mu < M_X$, the terms $W_H + W_{mix}$ are effectively given by

$$W_H + W_{mix} = \sum_{a=2,3} m_+^{(a)} \left[ \bar{H}_{(+)}^{(a)} \cos \alpha^{(a)} + \sum_i d_i \bar{\Phi}_{L(+)}^5 \sin \alpha^{(a)} \right] H_{(+)}^{(a)} + \sum_{a=2,3} m_-^{(a)} \bar{H}_{(-)}^{(a)} H_{(-)}^{(a)} + m_{SB} \sum_{a=2,3} \bar{H}_{(+)}^{(a)} H_{(-)}^{(a)},$$

(1.5)

1The $Z_2$ symmetry can softly violated not only by the term $\bar{H}_{(+)} H_{(-)}$, but also by terms $\bar{H}_{(+)} H_{(+)}$ and $\bar{\Phi}_{L(+)}^5 H_{(-)}$. However, in the present scenario, the existence of $\bar{H}_{(+)} H_{(-)}$ is essential. The details are discussed in Appendix A of [8].
In order to suppress the proton decay, we want to take $m_+^{(2)} \sim M_W$ with a sizable $\alpha^{(2)}$, but $m_+^{(3)} \sim M_X$ with a negligibly small $\alpha^{(3)}$. However, from the relations (1.6) and (1.7), we obtain the relation

$$d_i \tan \alpha^{(3)} = \frac{m_+^{(2)} \sin \alpha^{(2)} d_i + 5g_5v_{24}c_i}{m_+^{(2)} \cos \alpha^{(2)} + 5g_5v_{24}}.$$  (2.1)

The requirement $|\alpha^{(3)}| \leq M_W/M_X$ leads to the constraint $|g_5| \leq M_W/M_X$ for $|g_+| \sim 1$. We do not like to introduce such a small dimensionless parameter $g_5$. Therefore, for simplicity, we will put $g_5 = 0$ hereafter. Then, without loss of generality, we can put

$$\mathbf{\tilde{\nu}_L}^{(+)} = \sum_i b_i \mathbf{\tilde{\nu}_L}^{(+)i}$$  (2.2)

where $\mathbf{\tilde{\nu}_L}^{(+)1}$ does not mean the observed first generation particle. (Hereafter, for convenience, we denote $\mathbf{\tilde{\nu}_L}^{(+)1}$ as $\mathbf{\tilde{\nu}_L}^{(+)1}$ simply. The effective parameters $m_+^{(a)}$, $m_-^{(a)}$ and $\alpha^{(a)}$
are given as follows:

\[ m_{\pm}^{(2)} = \sqrt{(m_+ - 3g_v v_{24})^2 + m_5^2}, \quad m_{\pm}^{(3)} = \sqrt{(m_+ + 2g_v v_{24})^2 + m_5^2}, \]
\[ m_{-}^{(2)} = m_+ - 3g_v v_{24}, \quad m_{-}^{(3)} = m_+ + 2g_v v_{24}, \]  
\[ \tan \alpha^{(2)} = \frac{m_5}{m_+ - 3g_v v_{24}} \sim \frac{m_5}{m_+^{(2)}}, \quad \tan \alpha^{(3)} = \frac{m_5}{m_+ + 2g_v v_{24}} \sim \frac{m_5}{m_+^{(3)}}. \]  

(2.3)

We will take

\[ m_{\pm}^{(2)} \sim M_W, \quad m_{\pm}^{(3)} \sim M_X, \]
\[ m_{-}^{(2)} \sim M_I, \quad m_{-}^{(3)} \sim M_X, \]  
\[ \tan \alpha^{(2)} \sim \frac{m_5}{m_W}, \quad \tan \alpha^{(3)} \sim \frac{m_5}{m_X}, \]  

(2.4)

where \( M_I \sim 10^{14} \) GeV and \( m_5 \sim 10^1 \) GeV as we state later. The mass matrix in the basis of \((H^+_(-), H^+_(+), \tilde{\psi}^L_{(+1)})\) and \((H^+_+, H^+_(-))\) is given by

\[ M = \begin{pmatrix} 0 & m_- \\ m_+ \cos \alpha & m_{SB} \\ m_+ \sin \alpha & 0 \end{pmatrix}. \]  

(2.5)

Here and hereafter, for simplicity, we drop the index \((a)\). The mass matrix \((2.3)\) is diagonalized as

\[ \bar{U}^T M U = D \equiv \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}. \]  

(2.6)

Note that the matter field \( \tilde{\psi}^L_{1} \) is still massless, and also note that it is not in the eigenstate of the \( Z_2 \) parity.

The mixing matrix \( U \) is easily obtained from the diagonalization of

\[ M^T M = \begin{pmatrix} |m_+|^2 & m_{SB} m_+^* \cos \alpha \\ m_{SB}^* m_+ \cos \alpha & |m_{SB}|^2 + |m_-|^2 \end{pmatrix}. \]  

(2.7)

For real \( m_1, m_{SB} \) and \( m_\pm \), we obtain

\[ U = \begin{pmatrix} \cos \theta_u & \sin \theta_u \\ -\sin \theta_u & \cos \theta_u \end{pmatrix}, \]  

(2.8)

\[ \tan 2\theta_u = \frac{2m_{SB} m_+ \cos \alpha}{m_{SB}^2 + m_+^2 - m_5^2}, \]  

(2.9)

\[ m_1^2 = \frac{1}{2} \left( m_{SB}^2 + m_+^2 + m_-^2 \right) - \frac{1}{2} Q, \]  

(2.10)

\[ m_2^2 = \frac{1}{2} \left( m_{SB}^2 + m_+^2 + m_-^2 \right) + \frac{1}{2} Q, \]  

(2.11)

where

\[ Q = (m_{SB}^2 - m_+^2 + m_-^2) \cos 2\theta_u + 2m_{SB} m_+ \cos \alpha \sin 2\theta_u. \]  

(2.12)
When we define
\[ A \equiv m_{SB}^2 - m_+^2 + m_-^2, \quad B \equiv 2m_{SB}m_+ \cos \alpha, \] (2.13)
\[ \cos 2\theta_u = \frac{A}{\sqrt{A^2 + B^2}}, \quad \sin 2\theta_u = \frac{B}{\sqrt{A^2 + B^2}}, \] (2.14)
the quantity \( Q \) is given by
\[ Q = \sqrt{A^2 + B^2} = \sqrt{[m_{SB}^2 + (m_+ - m_-)^2][m_{SB}^2 + (m_+ + m_-)^2] - 4m_{SB}^2m_+^2 \sin^2 \alpha}. \] (2.15)
The rotation \( \mathcal{U} \) is also obtained from the diagonalization of
\[ M M^\dagger = \begin{pmatrix} m_+ & m_{SB}m_- & 0 \\ m_{SB}m_- & m_{SB}^2 + m_+^2 \cos^2 \alpha & m_+^2 \cos \alpha \sin \alpha \\ 0 & m_+^2 \cos \alpha \sin \alpha & m_+^2 \sin^2 \alpha \end{pmatrix}. \] (2.16)
The mixing matrix elements \( U_{i3} \) are easily obtained as follows:
\[ U_{13} = \frac{1}{N_3} m_{SB} \sin \alpha, \quad U_{23} = -\frac{1}{N_3} m_- \sin \alpha, \quad U_{33} = \frac{1}{N_3} m_- \cos \alpha, \] (2.17)
where
\[ N_3^2 = -m_+^2 + m_{SB}^2 \sin^2 \alpha. \] (2.18)
Other matrix elements are obtained as follows: We express the mixing matrix \( \mathcal{U} \) as
\[ \mathcal{U} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13} & c_{23}c_{12} - s_{23}s_{12}s_{13} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13} & -s_{23}c_{12} - c_{23}s_{12}s_{13} & c_{23}c_{13} \end{pmatrix}, \] (2.19)
where \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \cos \theta_{ij} \). Then, by comparing (2.19) with (2.17), we obtain
\[ s_{13} = \frac{m_{SB} \sin \alpha}{\sqrt{m_+^2 + m_{SB}^2 \sin^2 \alpha}}, \quad c_{13} = \frac{1}{\sqrt{1 + (m_{SB}/m_-)^2 \sin^2 \alpha}}, \] (2.20)
\[ s_{23} = \frac{\mathcal{U}_{23}}{c_{13}} = -\sin \alpha, \quad c_{23} = \cos \alpha. \] (2.21)
By using the relation \( (M^\dagger M)_{11} = \mathcal{U}_{11} \mathcal{U}_{11}^\dagger (m_1')^2 + \mathcal{U}_{12} \mathcal{U}_{12}^\dagger (m_2')^2 \), the mixing angle \( \theta_{12} \) is obtained as follows:
\[ \cos 2\theta_{12} = \frac{1}{2m_+^2 - m_+^2} \left[ m_1^2 + m_2^2 - 2 \frac{m_+^2}{c_{13}^2} \right] = \frac{1}{Q} \left( m_{SB}^2 + m_+^2 - m_-^2 - 2m_{SB}^2 \sin^2 \alpha \right) \]
\[ = -\frac{m_+^2 - m_-^2 \cos 2\alpha - m_+^2}{\sqrt{m_1^4 + 2(m_2^2 - m_2^2)m_2^2 + m_2^4 + 2m_2^2m_+^2 \cos 2\alpha + m_+^4}}. \] (2.22)
Note that \( \cos 2\theta_{12} \simeq -1 \) for \( m_- \gg m_{SB}, m_+ \), so that \( \theta_{12} \simeq \pi/2 \).
Since the physical fields \((\mathcal{H}_1, \mathcal{H}_2, \mathcal{L}_{L1}, \mathcal{L}_{L2}, \mathcal{L}_{L3})\) are given by
\[
\begin{pmatrix}
\mathcal{H}_{(-)} \\\n\mathcal{H}_{(+)} \\\n\mathcal{L}_{L(+)}^1 \\\n\mathcal{L}_{L(+)}^2 \\\n\mathcal{L}_{L(+)}^3
\end{pmatrix} = \begin{pmatrix}
U_{11} & U_{12} & U_{13} & 0 & 0 \\
U_{21} & U_{22} & U_{23} & 0 & 0 \\
U_{31} & U_{32} & U_{33} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0
\end{pmatrix} \begin{pmatrix}
\mathcal{H}_1 \\
\mathcal{H}_2 \\
\mathcal{L}_{L1} \\
\mathcal{L}_{L2} \\
\mathcal{L}_{L3}
\end{pmatrix},
\]
the Yukawa interactions \(\mathcal{H}_{(-)} \mathcal{L}_{L(+)} \mathcal{L}_{L(-)}\) lead to the effective Yukawa interactions at a low energy scale
\[
(Y_d)_{ij} \mathcal{H}_{(-)} \left[ \delta_{i1}(U_{11} U_{33} - U_{13} U_{31}) \mathcal{L}_{L1} + U_{11}(\delta_{i2} \mathcal{L}_{L2} + \delta_{i3} \mathcal{L}_{L3}) \right] 10_{L(-)}j ,
\]
and the induced \(R\)-parity violating terms
\[
(Y_d)_{ij} \mathcal{L}_{L1} \left( \delta_{i1}(U_{13} U_{33} \mathcal{L}_{L1} + \delta_{i2} \mathcal{L}_{L2} + \delta_{i3} \mathcal{L}_{L3}) \right) 10_{L(-)}j ,
\]
where we have assumed that \(|m_1| \ll |m_2|\), i.e. the Higgs field surviving at a low energy scale is not \(\mathcal{H}_2\), but \(\mathcal{H}_1\).

The effective Yukawa interactions \((2.23)\) give the fermion mass matrices
\[
(M^e_d)_{ij} = \begin{cases} 
(U_{11} U_{33} - U_{13} U_{31}) (Y_d)_{ij} v_d & \text{for } i = 1 , \\
(U_{12} Y_d)_{ij} v_d & \text{for } i = 2, 3 ,
\end{cases}
\]
\[
(M^d_d)_{ij} = \begin{cases} 
(U_{11} U_{33} - U_{13} U_{31}) (Y_d)_{ij} v_d & \text{for } i = 1 , \\
(U_{12} Y_d)_{ij} v_d & \text{for } i = 2, 3 ,
\end{cases}
\]
where \(v_d = (\mathcal{H}_1)\), and, in \((2.26)\), we have used the general formula \(U_{ik} U_{jl} - U_{il} U_{jk} = \varepsilon_{ijk} \delta_{kl}\) for an arbitrary \(3 \times 3\) unitary matrix \(U\). Note that in the present model, the relation \(M_d = M^e_d\) does not hold.

From the \(R\)-parity violating terms \((2.24)\), we obtain coefficients \(\lambda_{ij}^{(2,2)}\), \(\lambda_{ij}^{(2,3)}\), \(\lambda_{ij}^{(3,2)}\)
and \(\lambda_{ij}^{(3,3)}\), which are the coefficients of the interactions \((\nu_{L1}\varepsilon_{Li} - \nu_{L1\nu_{Li}} e^c_{Rj} - \varepsilon_{L1\nu_{Li}} d_{Rj} d_{Lj})\), \((d_{Ri} e_{Li} u_{Lj} - d_{Ri} u_{Lj}^\dagger d_{Lj})\), and \(\varepsilon_{\alpha\beta\gamma} \bar{d}_{R1}^{\alpha} \bar{d}_{R1}^{\beta} \bar{c}_{R1}^{\gamma}\), respectively, as follows:
\[
\lambda_{ij}^{(2,2)} = 0 , \\
\lambda_{ij}^{(2,3)} = \kappa (M^e_d)_{ij} / v_d \quad (i = 2, 3) , \\
\lambda_{ij}^{(3,2)} = \xi \kappa (M^e_d)_{ij} / v_d \quad (i = 2, 3) , \\
\lambda_{ij}^{(3,3)} = 0 , \\
\lambda_{ij}^{(3,3)} = \xi \kappa (M^e_d)_{ij} / v_d \quad (i = 2, 3),
\]
where
\[
\kappa = \frac{U_{12}^{(2)}}{U_{11}^{(2)}} , \\
\xi = \frac{U_{13}^{(3)}}{U_{12}^{(3)}}
\]
Note that the proton decay due to the exchange of squarks $\tilde{d}_i$ is forbidden in the limit of $\xi \to 0$, while the radiatively-induced neutrino masses do not become zero even if $\xi \to 0$. To suppress the proton decay due to these couplings, $\kappa$ should be small while to have neutrino masses $\kappa$ should have a sufficient strength. In this respect we can show that $\kappa \sim O(0.1)$ is welcome. Also to suppress the proton decay through dimension 5 operators it is necessary that $m_{SB}$ must be small, though it is also bounded from below to make yukawa couplings be sufficiently large. [26].

3. Radiatively induced neutrino mass matrix

In a SUSY GUT scenario, there are many origins of the neutrino mass generations [10]. For example, the sneutrinos $\tilde{\nu}_{iL}$ can have VEVs $\langle \tilde{\nu} \rangle \neq 0$, and the neutrinos $\nu_{Li}$ acquire their masses thereby [11]. In the present model, there is a $R$-parity violating bilinear term $\overline{\nu}_{L(+)H_{(+)}}$, while there is no $\overline{H}_{(-)}H_{(+)}$ term (the so-called $\mu$-term). In the physical field basis (the basis on which the mass matrix (3.2) is diagonal), the so-called $\mu$-term, $m_1 \overline{H}_1 H_1$, appears, while the $\overline{H}_1 H_1$ term is absent. Therefore, in the present model, the sneutrinos cannot have VEVs $\langle \tilde{\nu}_{i} \rangle$ at tree level. The VEVs $\langle \tilde{\nu}_{i} \rangle \neq 0$ will appear only through the renormalization group equation (RGE) effect. The contribution highly depends on an explicit model of the SUSY symmetry breaking. Since the purpose of the present paper is to investigate a general structure of the radiative neutrino masses, for the moment, we confine ourselves to discussing possible forms of the radiative neutrino mass matrix.

The radiative neutrino mass matrix $M_{\nu}^{rad}$ is given by

$$M_{\nu}^{rad} = M_{\nu}^e + M_{\nu}^d,$$

where $M_{\nu}^e$ is generated by the interactions $\nu_L e_L \tilde{e}_R$ and $\nu_L \tilde{e}_R e_R$ (i.e. by the charged lepton loop) and $M_{\nu}^d$ is generated by $\nu_L d_R \tilde{d}_L$ and $\nu_L \tilde{d}_R d_L$ (i.e. by the down-quark loop). We assume that the contributions from Zee-type diagrams due to $\overline{H}_1 H_1 \rightarrow \tilde{e}_R \tilde{e}_R$ mixing is negligibly small because the term $\overline{H}_1 H_1 10_L$ must be not $\overline{H}_1 H_1 10_L$, but $\overline{H}_1 H_1 210_L$ (recall that only the field $\overline{H}_1$ has the VEV in the present model).

We consider the radiative diagram with $(\nu_{Li})_j \rightarrow (e_R)_l + (\tilde{e}_L)_m$ and $(e_L)_k + (\tilde{e}_L)_m \rightarrow (\nu_{Li})_i$. The contributions $(M_{\nu}^e)_{ij}$ from the charged lepton loop are, except for the common

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Radiative generation of neutrino Majorana mass}
\end{figure}
factors, given as follows:

\[(M^e_{\nu})_{ij} = (\lambda_{1km}\delta_{1i} - \lambda_{1im}\delta_{k1})(\lambda_{1jl}\delta_{n1} - \lambda_{1nl}\delta_{j1})(M_{e})_{kl}(\widetilde{M}^2_{eLR})_{nm} + (i \leftrightarrow j) , \quad (3.2)\]

where \(M_{e}\) and \(\widetilde{M}^2_{eLR}\) are charged-lepton and charged-slepton-LR mass matrices, respectively. Here and hereafter, we will drop the common factor in \((M^\text{rad}_{\nu})_{ij}\), because we have an interest only in the relative structure of the matrix elements \((M^\text{rad}_{\nu})_{ij}\). Since, as usual, we assume that the structure of \(\widetilde{M}^2_{eLR}\) is proportional to that of \(M_{e}\), we obtain

\[(M^e_{\nu})_{ij} = \lambda_{im}\lambda_{jml}(M_{e})_{im}(M_{e})_{kl}(\widetilde{M}^2_{eLR})_{nm} \quad (3.3)\]

Since \(\lambda^e_{1ij} \equiv \lambda^{(2,2)}_{1ij} = \kappa(1 - \delta_{i1})(M^l_{e})_{ji}\) from the expression (2.29), we obtain the contribution from the charged lepton loop:

\[M^e_{\nu} = H_e^T S_1 H_e - S_1 H_e H_e - H_e^T H_e^T S_1 + S_1 \text{Tr}(H_e H_e) \quad (3.4)\]

where we have dropped the common factor \(\kappa\), and the Hermitian matrix \(H_e\) and the rank-1 matrix \(S_1\) are defined by

\[H_e = M_{e} M^T_{e} , \quad S_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (3.5)\]

Similarly, we can obtain the contributions from the down-quark loop. From the expression (2.30), we denote \(\lambda^d_{1ij} \equiv \lambda^{(2,3)}_{1ij}\) as

\[\lambda^d_{1ij} = \kappa \rho \delta_{1i} + (1 - \delta_{1i})(M^l_{d})_{ij} \quad (3.7)\]

where

\[\rho = \frac{1}{1 - \xi \kappa U^T_{31} / U^T_{33}} \simeq 1 \quad (3.8)\]

Then, we obtain

\[M^d_{\nu} = S_1 \text{Tr}(H_d H_d) , \quad (3.9)\]

where

\[H_d = M^T_d M_d \quad (3.10)\]

Note that the result (3.9) is independent of the value of \(\rho\).

The field \(\tilde{\nu}_{L(+)\nu}\) defined in Eq. (2.22) does not mean the observed first-generation field \((d^c, \nu, e)_{L}\) (and its SUSY partner). The forms of \(M^e_{\nu}\) and \(M^d_{\nu}\) on the general basis are given by

\[M^e_{\nu} = H_e^T S H_e - S H_e H_e - H_e^T H_e^T S + S \text{Tr}(H_e H_e) \quad (3.11)\]

\[M^d_{\nu} = S \text{Tr}(H_d H_d) \quad (3.12)\]

where \(S\) is an arbitrary rank-1 matrix \(S = U_5^T S_1 U_5\), which is given by the rebasing \(\tilde{5}_i \rightarrow \tilde{5}'_i = (U_5^T \tilde{5})_i\).
It is convenient to investigate the form $M^\nu_{\text{rad}}$ on the basis on which the charged lepton mass matrix $M_e$ is diagonal: $H_e = D_e^2 = \text{diag}(m^2_{e1}, m^2_{e2}, m^2_{e3}) \equiv \text{diag}(m^2_e, m^2_\mu, m^2_\tau)$. Then, the matrix $M^e_\nu$ is given by

$$M^e_\nu = \begin{pmatrix} S_{11}(m^4_{e2} + m^4_{e3}) & S_{12}(m^4_{e3} + m^2_{e1}m^2_{e2}) & S_{13}(m^4_{e3} + m^2_{e1}m^2_{e3}) \\ S_{21}(m^4_{e3} + m^2_{e1}m^2_{e2}) & S_{22}(m^4_{e3} + m^4_{e1}) & S_{23}(m^4_{e3} + m^2_{e1}m^2_{e3}) \\ S_{31}(m^4_{e3} + m^2_{e1}m^2_{e2}) & S_{32}(m^4_{e3} + m^2_{e2}m^2_{e3}) & S_{33}(m^4_{e1} + m^2_{e2}m^2_{e3}) \end{pmatrix}$$ (3.13)

where $S_{ij}$ is the value of $S_{ij}$.

$$S = \text{diag}(1, 1, 0) \rightleftharpoons m^4_\tau \text{PSP} ,$$ (3.14)

$$P = \text{diag}(1, 1, 0) .$$ (3.15)

Therefore, we can express the neutrino mass matrix $M^\nu_{\text{rad}}$ as the following form:

$$M^\nu_{\text{rad}} = m_0 \left( \text{PSP} + k^2 S \right) ,$$ (3.16)

where $k$ is given by $k \simeq (m_b/m_s)^2$ and $m_0$ will be given later [in Eq. (3.18)]. The matrix $S$ is a rank-1 matrix, so that PSP is also rank-1 matrix. In other words, the radiative neutrino mass matrix $M^\nu_{\text{rad}}$ has the following form by assuming $(U_3)_1 \sim O(\epsilon), (U_3)_2 \sim O(\epsilon)$ : $M^\nu_{\text{rad}} = m_0 \left\{ \left( \begin{array}{ccc} \epsilon^2 & \epsilon & 1 \\ \epsilon & \epsilon & 1 \\ 1 & 1 & 1 \end{array} \right) + k \left( \begin{array}{ccc} \epsilon^2 & \epsilon & 1 \\ \epsilon & \epsilon & 1 \\ 1 & 1 & 1 \end{array} \right) \right\}$ (3.17)

It shows a good hierarchical pattern [15].

So far, we have not discussed the absolute magnitude of the neutrino mass matrix $M^\nu_{\text{rad}}$. When we assume $m^2(\tilde{\nu}_R) \equiv m^2(\tilde{\nu}_R3) \simeq m^2(\tilde{\nu}_R2) \simeq m^2(\tilde{\nu}_R1)$ and $m^2(\tilde{\nu}_L) \equiv m^2(\tilde{\nu}_L3) \simeq m^2(\tilde{\nu}_L2) \simeq m^2(\tilde{\nu}_L1)$ and the rank-1 matrix $S$ is normalized as $\text{Tr}(SS^\dagger) = 1$, the coefficient $m_0$ in the expression (3.16) is given by

$$m_0 = \frac{1}{16\pi^2} \kappa^2 \frac{m^{(2)}_{m_\tau}}{v^2} F(m^2(\tilde{\nu}_R), m^2(\tilde{\nu}_L)) ,$$ (3.18)

where

$$F(m^2_R, m^2_L) = \frac{1}{m^2_R - m^2_L} \ln \frac{m^2_R}{m^2_L} .$$ (3.19)

If $F(m^2(\tilde{\nu}_R), m^2(\tilde{\nu}_L)) \simeq F(m^2(\tilde{\nu}_R), m^2(\tilde{\nu}_L))$, the factor $k$ is given by $k \simeq (m_b/m_s)^4 = 8.6$. However, in the present paper, we regard $k$ as a free parameter. By using $1/16\pi^2 = 6.33 \times 10^{-3}$, $m^{(2)}_1 \equiv m(B^{(2)}_1) = 2 \times 10^2$ GeV, $m_\tau(m_Z) = 1.75$ GeV, $v = 174$ GeV and $\tan \beta = 1.5$, we obtain

$$m_0 \simeq 1.9\kappa^2 F \text{ eV} ,$$ (3.20)

where $F$ is the value of $F(m^2(\tilde{\nu}_R), m^2(\tilde{\nu}_L))$ in the unit of TeV. If the neutrino mass matrix $M_\nu$ is dominated by the radiative mass terms $M^\nu_{\text{rad}}$ and we wish that the largest one of $m_{\nu i}$ is of the order of $\sqrt{\Delta m^2_{\text{atm}}} \simeq 0.05$ eV, the value $\kappa \sim 10^{-1}$ is favorable.
4. Summary

In conclusion, within the framework of an SU(5) SUSY GUT model, we have proposed a mechanism which effectively induces $R$-parity-violating terms at $\mu < m_{SB}$. In our model, those terms with lepton number violation are large enough to generate neutrino Majorana masses while those with baryon number violation are strongly suppressed so that the experimental bound of proton decay is evaded. This is related with doublet-triplet splitting. We have matter fields $\mathbf{5}_L(+) + 10_L(-)$ and two types of Higgs fields $H(\pm)$ and $\mathbf{10}_L(\pm)$, where $(\pm)$ denote the transformation properties under a discrete symmetry $Z_2$. The Higgs fields $H(\pm)$ and $H(\pm)$ couple to $10_{L(-)}10_{L(-)}$ and $\mathbf{5}_L(+)10_{L(-)}$, respectively, to make the Yukawa interactions. The $Z_2$ symmetry is only broken by the $\mu$-term, $m_{SB}H(+)H(-)$, so that the $H(+) \leftrightarrow \mathbf{5}(+)$ mixing is effectively induced at $\mu < m_{SB}$. Because of the heaviness of the color triplet components of the Higgs fields, the mixing is sizable in the $SU(2)_L$ doublet sector, while it is negligibly small in the $SU(3)_c$ triplet sector.

Whether the model is harmless or not for proton decay is highly sensitive to the choice of the parameter values, especially, $m_{SB}$ and $m_5$. A smaller value of $m_{SB}$ gives a lighter mass for the massive Higgs fields $H_2$ (another one, $H_1$, corresponds to the Higgs field in the conventional model), so that the case spoils the unification of the gauge coupling constants at $\mu = M_X$. On the other hand, a large value of $m_{SB}$ induces the proton decay due to the dimension-5 operator. We have taken $m_{SB} \sim 10^{14}$ GeV. Also, a large value of $m_5$ induce the proton decay due to the exchange of squark $\tilde{d}$. We have taken $m_5 \sim 10^4$ GeV. Those parameter values can give a reasonable magnitude of the neutrino mass. However, the choice of such a small $m_5$ gives a small mixing between $\mathbf{10}_L(+)\leftrightarrow \mathbf{5}(+)$, so that the case gives $\mathbf{10}_L(+)\leftrightarrow \mathbf{5}(+)$, so that the case gives $\mathbf{10}_L(+)\leftrightarrow \mathbf{5}(+)$. Therefore, the case with $|\alpha(2)| \ll 1$ cannot give a sizable deviation from $M_\nu^2 = M_e$. However, this is critical for each parameter value. The details are dependent on the explicit model, i.e. on the choice of the forms $S$ and $U \equiv U_R^d$. A further careful study based on an explicit model will be required.

Anyhow, if the present scenario is working, the proton decay will be observed in the near future, because possible parameter values are in critical ranges for the proton decay in order to explain the quark and lepton (charged lepton and neutrino) masses and mixings.

The present model leads to a radiatively-induced neutrino mass matrix $M_\nu^\text{rad}$ which is given by sum of two rank-1 matrices as shown in Eq. (3.16). The “two” is originated in the two contributions from charged lepton loop and down-quark loop. This can show a good hierarchical form and hence promising.

The present model will be worth noticing. In the present model, the coupling constants $\lambda_{ijk}$ of $\nu_{Li}e_L^c R_{ik}$ and $\nu_{Li}d_R^{cL} d_{Lj}^c$ are proportional to the mass matrices $(M_\nu^r)_{jk}$ and $(M_\nu^l)_{jk}$, respectively. The model will give fruitful phenomenology in flavor violating processes.

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