Testing θ_{13}, matter effects and leptonic CP violation

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Abstract: The prospects for improved limits or measurements of the small neutrino mixing parameter θ_{13}, matter effects and leptonic CP-violation with future long baseline accelerator and future reactor neutrino oscillation experiments are discussed. It will be shown that there exists a healthy physics program with a remarkable potential for very precise measurements of mass splittings and mixing angles which should ultimately allow measurements of leptonic CP violation. This program is very interesting. It will allow interesting insights into flavour physics, neutrino mass models and it is rather likely connected to the baryon asymmetry of the universe.

1. Introduction

Neutrino physics is in the midst of a discovery phase which was initiated by the evidence for atmospheric neutrino oscillations in the Super Kamiokande experiment in 1998[1]. This evidence has by now become a very solid proof of neutrino flavour conversions and of the L/E dependence as required by oscillations. The long standing solar neutrino problem has also been resolved in the last years by neutrino oscillations. The SNO experiment provided very solid evidence for the corresponding neutrino flavour transitions [2] and there exists altogether good evidence for the L/E dependence of oscillations. Interpreting the solar neutrino deficit by oscillations allowed initially different solutions with vastly different parameters. This situation was clarified by the KamLAND experiment which demonstrated finally with reactor anti-neutrinos [3] that the so-called LMA-solution is correct. The initial ambiguities in the determination of oscillation parameters, as well as alternative explanations, are therefore now eliminated. Ignoring the disputed LSND result [4], the existing experimental results fit rather nicely into a picture with three massive neutrinos, which corresponds to the simplest scenario for three generations. Neutrino oscillations involve then two mass-squared differences ($\Delta m^2_{12} \simeq \Delta m^2_{sol.}$ and $\Delta m^2_{23} \simeq \Delta m^2_{atm.}$), three mixing angles ($\theta_{12}$, θ_{23}, and θ_{13}), and a CP-violating phase (δ). Atmospheric neutrino

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data [5] and the first results from the K2K long-baseline accelerator experiment [6] determine $\Delta m^2_{23} = (2^{+1.2}_{-0.9}) \times 10^{-3}$ eV^2 (errors at 3σ) and $\theta_{23} \approx 45°$ [5], whereas solar neutrino data [7, 8], combined with the results from the KamLAND reactor experiment [3] lead to $\Delta m^2_{12} = (6.9^{+2.6}_{-1.5}) \times 10^{-5}$ eV^2 and $\sin^2\theta_{12} = 0.3^{+0.09}_{-0.07}$ at 3σ [9, 10, 11]. Altogether the progress in the determination of the neutrino oscillation parameters has been tremendous. Both the solar and atmospheric Δm^2 values are roughly now known within a factor of two, while the range spanned initially many orders of magnitude. The results can now approximatively be summarized by two independent two flavour oscillations. The mixing angle θ_{13} parameterizes three flavour effects in neutrino oscillations and it is so far only known to be small from the Chooz [12, 13] and Palo Verde [14] experiments. The current bound for θ_{13} depends on the true value of the atmospheric mass squared difference and it gets rather weak for $\Delta m^2_{31} \lesssim 2 \times 10^{-3}$ eV^2. However, in that region an additional constraint on θ_{13} from global solar neutrino data becomes important [10]. At the current best fit value of $\Delta m^2_{31} = 2 \times 10^{-3}$ eV^2 we have the bounds at 90% (3σ) CL for 1 *dof*

$$\sin^2 2\theta_{13} \leq 0.16\ (0.25)\,, \quad \sin^2\theta_{13} \leq 0.053\ (0.066)\,, \quad \theta_{13} \leq 10.8°\ (14.9°)\,. \qquad (1.1)$$

It may appear as if the future is now a less interesting period where the parameters are only improved. However, this is not the case sind future precision measurements of the mass splittings and the mixings offer exciting possibilities. First, unlike the quark sector, neutrino and charged lepton parameters are not obscured by hadronic uncertainties. The precision to which the underlying flavour information is determined is therefore only limited by the ultimate experimental precision. Precision measurements will thus act as very sensitive tests of flavour models. Secondly, future precision measurements will be able to test the three-flavourdness of neutrino oscillations. This is conceptually interesting, especially since it tests also the unitarity of the three flavours rather precisely. Genuine three flavour oscillation effects occur only for a finite value of θ_{13} and establishing a finite value of θ_{13} is therefore one of the next milestones in neutrino physics. The third important possibility has to do with the fact that nature chose the LMA-solution. Leptonic CP-violation effects are in this case sizable, such that they may be detected in future experiments. Leptonic CP violation is also a three flavour effect, and it can thus only be tested if θ_{13} is finite. In the usual see-saw [15] scenario there exist also Majorana CP phases in the light neutrino sector, as well as further CP phases in the heavy Majorana sector, which are involved in leptogenesis. In general the heavy and light CP phases are not connected, but most flavor models create relations between these two sectors, relating thus low energy leptonic CP violation to leptogenesis and mass models. Precision measurements of neutrino oscillations allow therefore to address very interesting questions of particle physics. There is thus a very strong motivation to establish first in the next generation of experiments a finite value of θ_{13} in order to aim in the long run at a measurement of leptonic CP violation (see e.g. [16, 17, 18, 19, 20]).

2. Three flavour oscillation framework in matter

Most existing results on neutrino oscillations can so far be understood in an effective two neutrino framework with the well known oscillation probability for a baseline L and neutrino energy E_ν

$$P(\nu_{f_1} \to \nu_{f_2}) = |VEV\nu_{f_1}(t)|\nu_{f_2}(t=0)|^2 = \sin^2 2\theta \cdot \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right) , \qquad (2.1)$$

where θ is the mixing angle between the two flavour eigenstates f_1 and f_2 and where $\Delta m^2 = m_2^2 - m_1^2$ is the difference between the mass eigenvalues. An effective two neutrino description is no longer sufficient for future oscillation experiments and matter effects must be included in addition. The generalization of the oscillation formulae in vacuum to N neutrinos leads to the probabilities for flavour transitions $\nu_{f_l} \to \nu_{f_m}$ given by

$$P(\nu_{f_l} \to \nu_{f_m}) = \underbrace{\delta_{lm} - 4\sum_{i>j} \mathrm{Re} J_{ij}^{f_l f_m} \sin^2 \Delta_{ij}}_{P_{CP}} \underbrace{-2\sum_{i>j} \mathrm{Im} J_{ij}^{f_l f_m} \sin 2\Delta_{ij}}_{P_{\not{C}\not{P}}} \qquad (2.2)$$

where the shorthands $J_{ij}^{f_l f_m} := U_{li}U_{lj}^* U_{mi}^* U_{mj}$ and $\Delta_{ij} := \frac{\Delta m_{ij}^2 L}{4E}$ have been used. These generalized vacuum transition probabilities depend on all combinations of quadratic mass differences $\Delta m_{ij}^2 = m_i^2 - m_j^2$ as well as on different products of elements of the leptonic mixing matrix U. We will assume for the rest of this article a three neutrino framework such that $1 \le i, j \le 3$ and U is a 3×3 mixing matrix parameterized in the standard way

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} , \qquad (2.3)$$

where $c_{ij} = \cos(\theta_{ij})$ and $s_{ij} = \sin(\theta_{ij})$. U contains three leptonic mixing angles and one Dirac-like leptonic CP phase δ. Note that the most general mixing matrix for three Majorana neutrinos contains two further Majorana-like CP phases, but it can easily be seen that these two extra diagonal Majorana phases do not enter in the above oscillation formulae. Disappearance probabilities, *i.e.* the transitions $\nu_{f_l} \to \nu_{f_l}$, do not even depend on δ, since $J_{ij}^{f_l f_l}$ is only a function of the modulus of elements of U. Appearance probabilities, like $\nu_e \to \nu_\mu$ are therefore the place where leptonic CP violation can be studied. Eq. (2.2) contains a CP conserving part P_{CP} and a CP violating part $P_{\not{C}\not{P}}$, and both terms depend on the CP phase δ. An obvious extraction strategy for CP-violation would thus be to look at CP asymmetries [21]. Note, however, that the beams of a LBL experiment traverse the Earth on a certain path and the presence of matter violates by itself CP, which modifies eq. (2.2) and which makes a measurement of leptonic CP violation more involved.

The general oscillation formulae in vacuum, eq. (2.2), lead to well known, but rather lengthy trigonometric expressions for the oscillation probabilities in vacuum. These expressions become even longer and do not exist in closed form when arbitrary matter corrections are taken into account. For effectively constant matter densities, which is often a good assumption, the problem simplifies somewhat, but the general oscillation probabilities are still very lengthy. The Hamiltonian describing three neutrino oscillation in matter can then be written in flavour basis as

$$H = \frac{1}{2E_\nu} U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^T + \frac{1}{2E_\nu} \begin{pmatrix} A + A' & 0 & 0 \\ 0 & A' & 0 \\ 0 & 0 & A' \end{pmatrix} . \tag{2.4}$$

The first term describes oscillations in vacuum in flavour basis. The quantities A and A' in the second term are given by the charged current and neutral current contributions to coherent forward scattering in matter. The charged current contribution is given by

$$A = \pm \frac{2\sqrt{2} G_F Y \rho E_\nu}{m_n} = 2VE_\nu \ , \tag{2.5}$$

where G_F is Fermi's constant, Y is the number of electrons per nucleon, m_n is the nucleon mass and ρ is the matter density. A is positive for neutrinos in matter and anti-neutrinos in anti-matter, while it is negative for anti-neutrinos in matter and neutrinos in anti-matter. The flavour universal neutral current contributions A' lead to an overall phase which does not enter the transition probabilities. The over-all neutrino mass scale m_1^2 can be written as a term proportional to the unit matrix and can similarly be removed, such that only Δm_{21}^2 and Δm_{31}^2 remain in the first term of eq. (2.4). After re-diagonalization of the Hamiltonian in constant matter density one finds that matter effects lead in a very good approximation to an A-dependent parameter mapping in the 1-3 subspace which can be written as

$$\sin^2 2\theta_{13,m} = \frac{\sin^2 2\theta_{13}}{C_\pm^2} \ , \tag{2.6}$$

$$\Delta m_{31,m}^2 = \Delta m_{31}^2 C_\pm \ , \tag{2.7}$$

$$\Delta m_{32,m}^2 = \frac{\Delta m_{31}^2 \ (C_\pm + 1) + A}{2} \ , \tag{2.8}$$

$$\Delta m_{21,m}^2 = \frac{\Delta m_{31}^2 \ (C_\pm - 1) - A}{2} \ . \tag{2.9}$$

The index m denotes effective quantities in matter and

$$C_\pm^2 = \left(\frac{A}{\Delta m_{31}^2} - \cos 2\theta \right)^2 + \sin^2 2\theta \ . \tag{2.10}$$

Note that A in $C_\pm$ can change its sign and the mappings for neutrinos and anti-neutrinos are therefore different, resulting in different effective mixings and masses. This is an important effect, which will allow detailed tests of coherent forward scattering of neutrinos in matter at future LBL experiments. Note that oscillations in matter depend unlike vacuum oscillations via $C_\pm$ on the sign of Δm_{31}^2. This is very interesting, since it opens the possibility to extract the $sign(\Delta m_{31}^2)$ via matter effects.

Inserting the parameter mappings eqs. (2.6)-(2.9) into the full oscillation formulae leads still to quite lengthy expressions for the oscillation probabilities in matter, where it is not easy to oversee all effects. It is therefore instructive to simplify the problem further to a point, where a qualitative analytic understanding of all effects becomes possible, while quantitative statements should be evaluated numerically with the full expressions. The key for further simplification is to expand the oscillation probabilities in small quantities. These expansion parameters are $\alpha = \Delta m^2_{21}/\Delta m^2_{31} \simeq 10^{-2}$ and $\sin^2 2\theta_{13} \leq 0.16$. The matter effects can be parameterized by the dimensionless quantity $\hat{A} = A/\Delta m^2_{31} = 2VE/\Delta m^2_{31}$, where $V = \sqrt{2}G_F n_e$. Using $\Delta \equiv \Delta_{31}$, the leading terms in this expansion are, for example, for $P(\nu_\mu \to \nu_\mu)$ and $P(\nu_e \to \nu_\mu)$ [22, 23, 17]

$$P(\nu_\mu \to \nu_\mu) \approx \\ 1 - \cos^2\theta_{13}\sin^2 2\theta_{23}\sin^2\Delta + 2\ \alpha\ \cos^2\theta_{13}\cos^2\theta_{12}\sin^2 2\theta_{23}\Delta\cos\Delta, \tag{2.11}$$

$$\begin{aligned} P(\nu_e \to \nu_\mu) \approx{}& \sin^2 2\theta_{13}\sin^2\theta_{23}\ \frac{\sin^2((1-\hat{A})\Delta)}{(1-\hat{A})^2} \\ &\pm \sin\delta\cdot\sin 2\theta_{13}\ \alpha\ \sin 2\theta_{12}\cos\theta_{13}\sin 2\theta_{23}\sin(\Delta)\frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\ &+\cos\delta\cdot\sin 2\theta_{13}\ \alpha\ \sin 2\theta_{12}\cos\theta_{13}\sin 2\theta_{23}\cos(\Delta)\frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\ &+\alpha^2\ \sin^2 2\theta_{12}\cos^2\theta_{23}\frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}\ , \end{aligned} \tag{2.12}$$

where in eq. (2.12) "+" stands for neutrinos and "−" for anti-neutrinos. The most important feature of eq. (2.12) is that all interesting effects in the $\nu_e \to \nu_\mu$ transition depend crucially on θ_{13}. The size of $\sin^2 2\theta_{13}$ determines thus if the total transition rate, matter effects, effects due to the sign of Δm^2_{31} and CP violating effects are measurable. One of the most important questions for future oscillation experiments is therefore how far experiments can push the θ_{13} limit below the current CHOOZ bound of approximately $\sin^2 2\theta_{13} < 0.16$.

Before we discuss in some detail some features of eqs. (2.11) and (2.12), we would like to comment on the underlying assumptions and the reliability of these equations. First eqs. (2.11) and (2.12) are an expansion in terms of the small quantities α and $\sin 2\theta_{13}$. Higher order terms are suppressed at least by another power of one of these small parameters and these corrections are thus typically at the percent level. Note that the expansion in α is actually an expansion in the solar and not the atmospheric frequency. The expansion does therefore not break down before the first atmospheric oscillation maximum, i.e. at $\Delta \simeq 1$, but at much larger baselines before the first (sub-dominant) solar oscillation maximum, i.e. at $\alpha\Delta \simeq 1$. The latter condition gives an upper bound for the baseline where eqs. (2.11) and (2.12) are very good approximations

$$L \lesssim 8000\,\mathrm{km}\left(\frac{E_\nu}{\mathrm{GeV}}\right)\left(\frac{10^{-4}\mathrm{e}V^2}{\Delta m^2_{21}}\right)\ , \tag{2.13}$$

while the first oscillation maximum sits at $\alpha\cdot L \simeq L/30$. Eqs. (2.11) and (2.12) are therefore excellent approximations at and well beyond the first oscillation maximum of long baseline experiments. The matter corrections in eqs. (2.11) and (2.12) are derived for constant average matter density. Numerical test have shown that this approximation works quite well as long as the matter profile is reasonably smooth.

Note that all quantitative results which will be presented are based on numerical simulations of the full problem in matter. These results do therefore not depend on any approximation. Eqs. (2.11) and (2.12) will only be used to understand the problem analytically, which is extremely helpful in order to oversee the multi-dimensional parameter space. The full numerical analysis and eqs. (2.11) and (2.12) rest, however, on the assumption of a standard three neutrino scenario. It is thus assumed that the LSND signal [4] will not be confirmed by the MiniBooNE experiment [24].

3. Correlations and Degeneracies

Eqs. (2.11) and (2.12) exhibit certain parameter correlations and degeneracies, which play an important role in the analysis of LBL experiments, and which would be hard to understand in a purely numerical analysis of the high dimensional parameter space. The most important properties are:

- Eqs. (2.11) and (2.12) depend only on the product $\alpha \cdot \sin 2\theta_{12}$ or equivalently $\Delta m^2_{21} \cdot \sin 2\theta_{12}$. This are the parameters related to solar oscillations which will be taken as external input. The fact that only the product enters, implies that it may be better determined than the product of the measurements of Δm^2_{21} and $\sin 2\theta_{12}$.

- Next we observe in eq. (2.12) that the second and third term contain both a factor $\sin(\hat{A}\Delta)$, while the last term contains a factor $\sin^2(\hat{A}\Delta)$. Since $\hat{A}\Delta = 2VL$, we find that these factors depend only on L, resulting in a "magic baseline" when $2VL_{magic} = \pi/4V$, where $\sin(\hat{A}\Delta)$ vanishes. At this magic baseline only the first term in eq. (2.12) survives and $P(\nu_e \to \nu_\mu)$ does no longer depend on δ, α and $\sin 2\theta_{12}$. This is in principle very important, since it implies that $\sin^2 2\theta_{13}$ can be determined at the magic baseline from the first term of eq. (2.12) whatever the values and errors of δ, α and $\sin 2\theta_{12}$ are. For the given matter density of the Earth we find

$$L_{magic} = \pi/4V \simeq 8100 \text{ km} , \tag{3.1}$$

  which fits nicely into the Earth. This is quite amazing, since V depends on completely unrelated constants of nature like G_F such that L_{magic} could be very different.

- Next we observe that only the second and third term of eq. (2.12) depend on the CP phase δ, and both terms contain a factor $\sin 2\theta_{13} \cdot \alpha$, while the first and fourth term of eq. (2.12) do not depend on the CP phase δ and contain factors of $\sin^2 2\theta_{13}$ and α^2, respectively. The extraction of CP violation is thus always suppressed by the product $\sin 2\theta_{13} \cdot \alpha$ and the CP violating terms are obscured by large CP independent terms if either $\sin^2 2\theta_{13} \ll \alpha^2$ or $\sin^2 2\theta_{13} \gg \alpha^2$. The determination of the CP phase δ is thus best possible if $\sin^2 2\theta_{13} \simeq 4\theta^2_{13} \simeq \alpha^2$.

- Another observation is that the last term in eq. (2.12), which is proportional to $\alpha^2 = (\Delta m^2_{21})^2/(\Delta m^2_{31})^2$, dominates in the limit of tiny $\sin^2 2\theta_{13}$. The error of Δm^2_{21} limits therefore for small $\sin^2 2\theta_{13}$ the parameter extraction. This last term implies a finite transition probability even for $\theta_{13} = 0$. Observing $\nu_e \to \nu_\mu$ or $\nu_\mu \to \nu_e$ appearance transitions does therefore not necessarily establish a finite value of $\theta_{13} = 0$ in a three flavor framework.

- Eqs. (2.11) and (2.12) have a structure which suggests that transformations exist, which leave these equations invariant. We expect therefore degeneracies, *i.e.* for given L/E parameter sets with identical oscillation probabilities. An example of such an invariance is given by a simultaneous replacement of neutrinos by anti-neutrinos and $\Delta m^2_{31} \to -\Delta m^2_{31}$. This is equivalent to changing the sign of the second term of eq. (2.12) and replacing $\alpha \to -\alpha$ and $\Delta \to -\Delta$, while $\hat{A} \to \hat{A}$. It is easy to see that eqs. (2.11) and (2.12) are unchanged, but this constitutes no degeneracy, if we can distinguish neutrinos and anti-neutrinos experimentally.

- The first real degeneracy [25] can be seen in the disappearance probability eq. (2.11), which is invariant under the replacement $\theta_{23} \to \pi/2 - \theta_{23}$. Note that the second and third term in eq. (2.12) are not invariant under this transformation, but this change in the sub-leading appearance probability can approximately be compensated by small parameter shifts. This implies that the degeneracy can in principle be lifted with precision measurements in the disappearance channels.

- The second degeneracy can be found in the appearance probability eq. (2.12) in the $(\delta - \theta_{13})$-plane [26]. In terms of θ_{13} (which is small) and δ the four terms of eq. (2.12) have the structure

$$P(\nu_e \to \nu_\mu) \approx \theta_{13}^2 \cdot F_1 + \theta_{13} \cdot (\pm \, \sin\delta F_2 + \cos\delta F_3) + F_4 \ , \tag{3.2}$$

where the quantities F_i, $i = 1, .., 4$ contain all the other parameters. The requirement $P(\nu_e \to \nu_\mu) = const.$ leads for both neutrinos and anti-neutrinos to parameter manifolds of degenerate or correlated solutions. Having both neutrino and anti-neutrino beams, the two channels can be used independently, which is equivalent to considering simultaneously eq. (3.2) for $F_2 \equiv 0$ and $F_3 \equiv 0$. The requirement that these probabilities are now independently constant, *i.e.* $P(\nu_e \to \nu_\mu) = const.$ for $F_2 \equiv 0$ and $F_3 \equiv 0$, leads to more constraint manifolds in the $(\delta - \theta_{13})$-plane, but some degeneracies still survive.

- The third degeneracy [27] is given by the fact that a change in sign of Δm^2_{31} can essentially be compensated by an offset in δ. Therefore we note again that the transformation $\Delta m^2_{31} \to -\Delta m^2_{31}$ leads to $\alpha \to -\alpha$, $\Delta \to -\Delta$ and $\hat{A} \to -\hat{A}$. All terms of the disappearance probability, eq. (2.11), are invariant under this transformation. The first and fourth term in the appearance probability eq. (2.11), which do not depend on the CP phase δ, are also invariant. The second and third term of eq. (2.11) depend on the CP phase and change by the transformation $\Delta m^2_{31} \to -\Delta m^2_{31}$. The fact that these changes can be compensated by an offset in the CP phase δ is the third degeneracy.

- Altogether there exists thus an eight-fold degeneracy [25], as long as only the $\nu_\mu \to \nu_\mu$, $\bar{\nu}_\mu \to \bar{\nu}_\mu$, $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ channels and one fixed L/E are considered. However, the structure of eqs. (2.11) and (2.12) makes clear that the degeneracies can be broken by using in a suitable way information from different L/E values. This can be achieved in total event rates by changing L or E [28, 29], but it can in principle also be done by using information in the event rate spectrum of a single baseline L, which requires detectors with very good energy resolution [17]. Another strategy to break the degeneracies is to include further oscillation channels in the analysis ("silver channels") [30, 28].

The discussion of this section shows the strength of the analytic approximations, which allow to understand the complicated parameter interdependence. It also helps to optimally plan experimental setups and to find strategies to resolve the degeneracies.

4. Simulations of future oscillation experiments

The potential of future experiments depends on event rates which depend only indirectly on the above oscillation probabilities. This requires simulations of these experiments for which we use the package GLoBES [31]: This package contains all relevant experimental and theoretical aspects. Sensitivities and precision are defined from fits to the simulated event rates for certain physics parameters which were used as input for the simulation. Every event can be classified by the information on the flavor of the detected neutrino and the type of interaction. The particles detected in an experiment are produced by neutral current (NC), inelastic charged current (CC) or quasi–elastic charged current (QE) interactions. The contribution to each mode depends on a number of factors, like detector type, the neutrino energy and flavour. In order to calculate realistic event rates we compute first for each neutrino flavor and energy bin the number of events for each type of interaction in the fiducial mass of an ideal detector. Next the deficiencies of a real detector are included, like limited event reconstruction capabilities. The combined description leads to the differential event rate spectrum for each flavor and interaction mode as it would be seen by a detector which is able to separate all these channels. Finally different channels must be combined, since they can not be observed separately. This can be due to physics, *e.g.*, due to the flavor–blindness of NC interactions, or it can be a consequence of detector properties, *e.g.*, due to charge misidentification. In order to include backgrounds, the channels are grouped in an experiment specific way into pairs of signal and background. The considered backgrounds are NC–events which are misidentified as CC–events and CC–events identified with the wrong flavor or charge. For superbeams we include furthermore the background of CC–events coming from an intrinsic contamination of the beam. Finally all available signal channels are combined in the analysis and a global fit is performed to extract the physics parameters in an optimal way for certain experimental parameters and certain external physics parameters, as well as their errors. The relevant channels are for a neutrino factory for each polarity of the beam the ν_μ–CC channel (disappearance) and $\bar{\nu}_\mu$–CC channel (appearance) event rate spectra. The backgrounds for these signals are NC events for all flavours and misidentified ν_μ–CC events. For conventional and superbeam experiments the signal is for each polarity of the beam given by the ν_μ–QE channel (disappearance) and ν_e–CC channel (appearance). The backgrounds are here NC events for all flavors, misidentified ν_μ–CC events, and, for the ν_e–CC channel, the ν_e–CC beam contamination. For further details see [20, 31].

5. Accelerator based long baseline experiments

Future accelerator based long baseline experiments can be grouped according their time scale of operation. The K2K experiment is already running and tests the leading atmospheric oscillation already now. Then there are the MINOS and CNGS projects which are under construction. The parameters of these experiments are essentially fixed and we call them therefore 'current projects'. These experiments aim at testing the leading oscillation at the 10 % level (Δm^2_{31} and θ_{23}). An interesting question is how well these experiments will be able to test the three flavourdness of the oscillation. The key parameter for that is θ_{13} and the expected sensitivities are shown in fig. 1, where it can be seen that these experiments have the potential for modest improvements of the existing θ_{13} limit. After

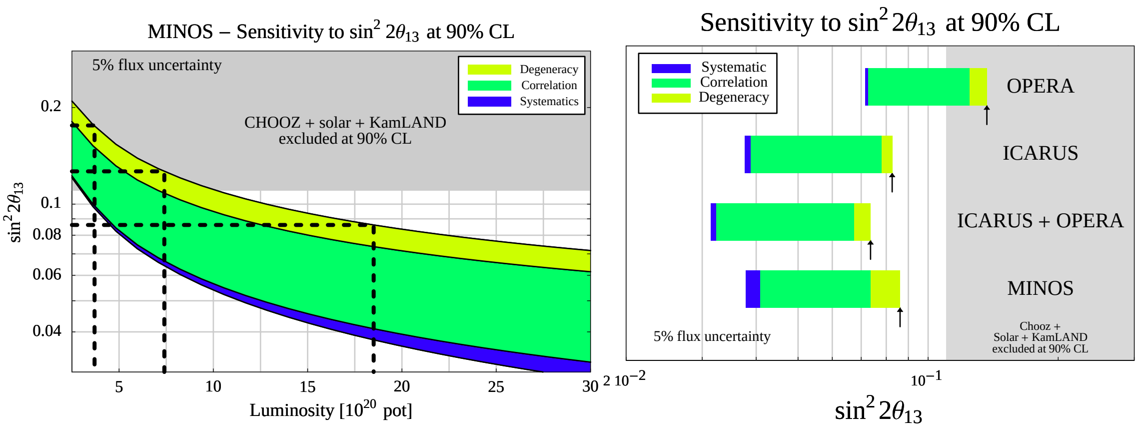

Figure 1: Left plot: The sensitivity of the MINOS experiment to θ_{13} as a function of the protons on target (pot) assuming a 5% flux uncertainty. The grey(colored) bands show the reduction of sensitivity from a purely statistical limit (lower end of the dark grey/blue band) by systematics (dark grey/blue), correlations (medium grey/green) and degeneracies (light grey/yellow). The upper end of the light grey(yellow) band represents the final 90%CL limit. The dashed lines represent what 1,2 and 5 years of operation might achieve (from left to right). Right plot: Comparison of 5 years of operation for the MINOS and CNGS experiments. The grey area for large $\sin^2 2\theta_{13}$ indicates in all cases the current limit from the Chooz experiment. Further details can be found in a forthcoming paper [32].

that there is a 'next generation' of accelerator based long baseline oscillation experiments. Two well known projects which might be realized first are JHF-SuperKamiokande (now called JPARC) and a NuMI off-axis experiments. Both experiments aim at a precision of a few percent for the leading oscillation parameters Δm^2_{31} and θ_{23}. These experiments would further be able to significantly improve the limit on $\sin^2 2\theta_{13}$ down to a few times 10^{-2}. On even longer time scales the JHF-HyperKamiokande experiment or a neutrino factory may be feasible. The sensitivity reach of these experiments is shown in comparison to MINOS and CNGS in figure 2.

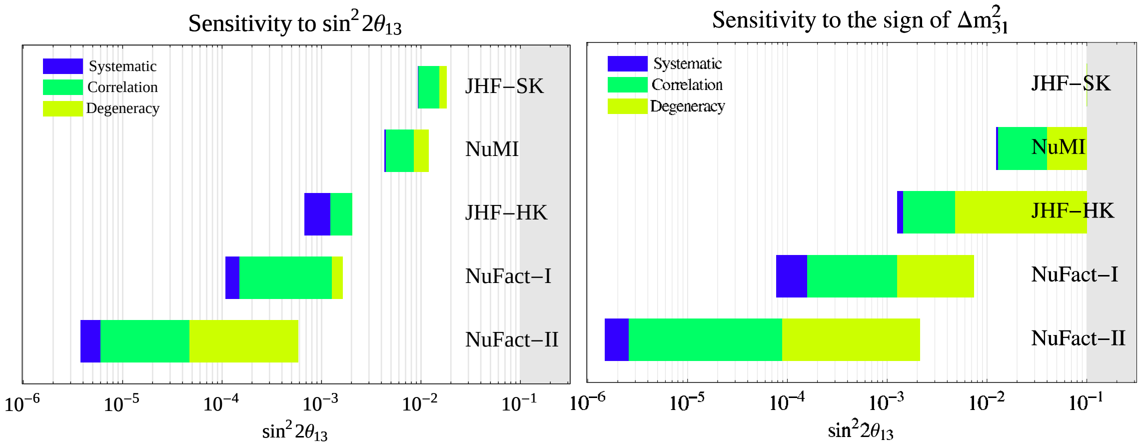

Figure 2: Left plot: The θ_{13} sensitivity of different future accelerator based neutrino oscillation experiments [20]. Right plot: The θ_{13} values for which sensitivity to matter effects, i.e. $sign(\Delta m^2_{31})$ exists. The shown bands are again the reduction of sensitivity from a purely statistical limit (left end of the dark grey/blue range) by systematics (right end of dark grey/blue), correlations (medium grey/green) and degeneracies (light grey/yellow). The right end of the light grey(yellow) band represents the final 90%CL limit. The grey area for large $\sin^2 2\theta_{13}$ indicates the current limit from the Chooz experiment.

6. Synergies

We discussed so far the potential of different individual oscillation experiments. The philosophy was to use the (expected) knowledge of physics parameters as input and to ask how well each proposed or planned experiment could measure quantities like θ_{13}. We saw that there exist competing plans with similar sensitivities which might be realized at the same time scale. This allows to improve the results by simply combining the statistics of two such similar experiments. However, it is possible to utilize potentially synergies between experiments which are much more than the simple addition of statistics. The point is that individual experiments measures only a certain parameter combination which may moreover exhibit the discussed degeneracies and correlations. Combinations of experiments with similar sensitivities may then be able to separate these parameter combinations partly or fully. An example of such a discussion is given by combining the JHF-SuperKamiokande (JHF-SK) and NuMI off-axis experiments for a fixed time of operation in the best possible way. The JHF-SK baseline is assumed to be fixed, while the NuMI baseline could still be chosen. For the energies of these experiments JHF-SK is essentially insensitive to matter effects, while matter effects play already some role for the longer NuMI baselines. Both experiments can run partly with neutrino and partly with anti-neutrino beams. The cross-sections for anti-neutrinos are, however, smaller, leading to fewer events for the same running period. An anti-neutrino running is moreover in many aspects like a different experiment, but it is clear that anti-neutrino information is crucial in order to resolve the parameters. It is therefore natural to ask how the two experiments could be combined in an optimal way. This is shown in figure 3, where it is shown how the θ_{13} sensitivities change for different ways of operation.

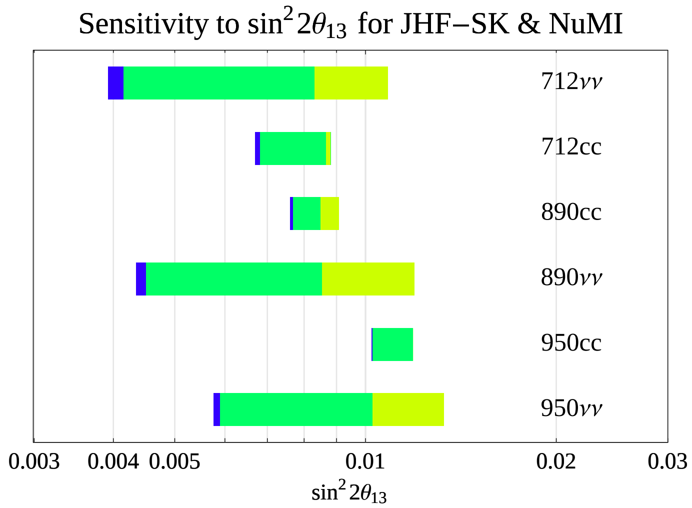

Figure 3: The θ_{13} sensitivity of different combinations of the JHF-SuperKamiokande and NuMI off-axis experiments. The labels indicate for different scenarios the NuMI baseline in km and the type of beam for JHF-SK (first character) and NuMI (second character). ν stands for neutrino beams only, while c stands for combined neutrino and anti-neutrino running. The color coding is as defined in the caption of fig. 2 and the right edge of the bars corresponds to the sensitivity limit of each setup. For further details see [18].

7. Adding reactor experiments

Another type of experiment which might be realized during the next years is a new generation of reactor experiments. A near detector is used to eliminate many common systematical errors and the far detector is located typically at a baseline of a few kilometer. For these short baselines matter effects can be ignored and one finds to second order in the small quantities for the oscillation probability $\sin 2\theta_{13}$ and α,

$$1 - P_{\bar{e}\bar{e}} = \sin^2 2\theta_{13}\, \sin^2 \Delta_{31} + \alpha^2\, \Delta_{31}^2\, \cos^4 \theta_{13}\, \sin^2 2\theta_{12}\,. \tag{7.1}$$

At the first atmospheric oscillation maximum, Δ_{31} is approximately $\pi/2$ and $\sin^2 \Delta_{31}$ is close to one, which means that the second term on the right-hand side of this equation can be neglected for $\sin^2 2\theta_{13} \gtrsim 10^{-3}$. The reactor measurement is dominated in this case at short baselines by the product of $\sin^2 2\theta_{13}$ and $\sin^2 \Delta_{31}$, which must be measured as deviation from one. Eq. (7.1) implies that correlations and degeneracies play essentially no role in reactor experiments. The behavior in the $\sin^2 2\theta_{13}$-Δm^2_{21}-plane will also be different since eq. (7.1) is essentially independent of Δm^2_{21}. A reactor experiment helps in two ways. First, a reactor experiments would provide a direct, essentially uncorrelated and clean measurement for θ_{13} [33] which can be used to disentangle the accelerator results. Secondly, the reactor measurement can replace the cross-section suppressed anti-neutrino running of the accelerator experiments, leading to statistical improvements in the neutrino measurements [34].

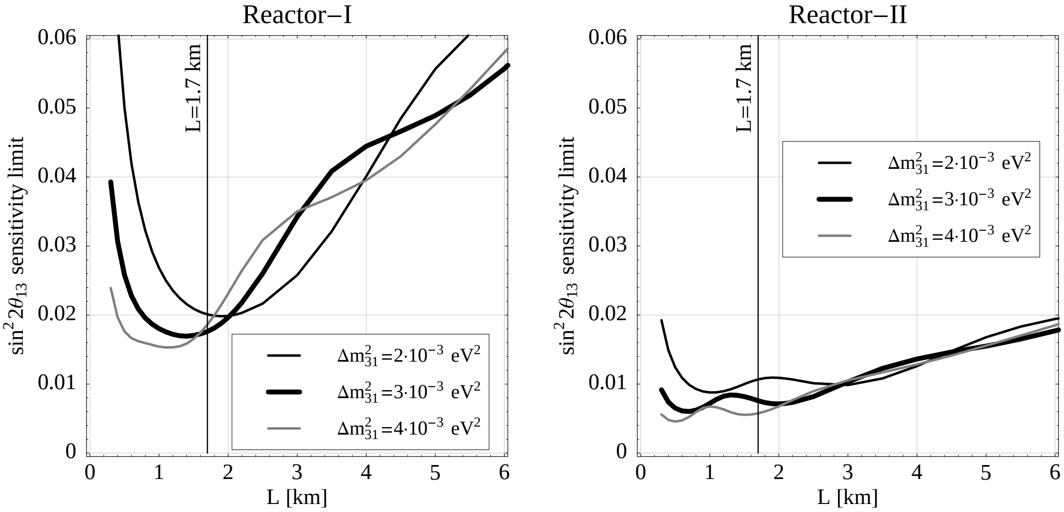

Figure 4: The $\sin^2 2\theta_{13}$ sensitivity of reactor experiments as a function of the distance of the far detector. Shown are two scenarios. Reactor-I assumes for the product of detector mass, reactor thermal power and running time 400t GW y. Reactor-II corresponds to 8000t GW y. Reactor-I could, for example, be realized by a 10t detector, 8GW thermal power and 5 years of data taking. For further details see [34].

The $\sin^2 2\theta_{13}$-sensitivity of a reactor experiment is shown in figure 4 as a function of the distance of the far detector. The limits which can be achieved are similar to those discussed above for the next generation of accelerator experiments. A comparison is shown in figure 5. The precise information on $\sin^2 2\theta_{13}$ from a reactor experiment can be used to break the degeneracies at least partly. The precise $\sin^2 2\theta_{13}$ information from a reactor experiment can , for example, be used to optimally search for $sign(\Delta m^2_{31})$ and for leptonic CP violation. Figure 6 shows the synergies between JHF-SK, NuMI and a new reactor experiment for these quantities. Figure 6 demonstrates clearly that there are significant improvements when the three experiments are optimally combined.

7.1 Theoretical Motivation for non-zero θ_{13}

One may ask if there exist theoretical reasons why θ_{13} should be within the reach of a new next generation experiment, with a sensitivity down to $\sin^2 2\theta_{13} \simeq 0.01$. This question is of course connected to the origin of neutrino masses. For example, there exist apparent regularities in the fermionic field content which make it very tempting to introduce right-handed neutrino fields leading to both Dirac and Majorana mass terms for neutrinos. Diagonalization of the resulting mass matrices yields Majorana mass eigenstates and due

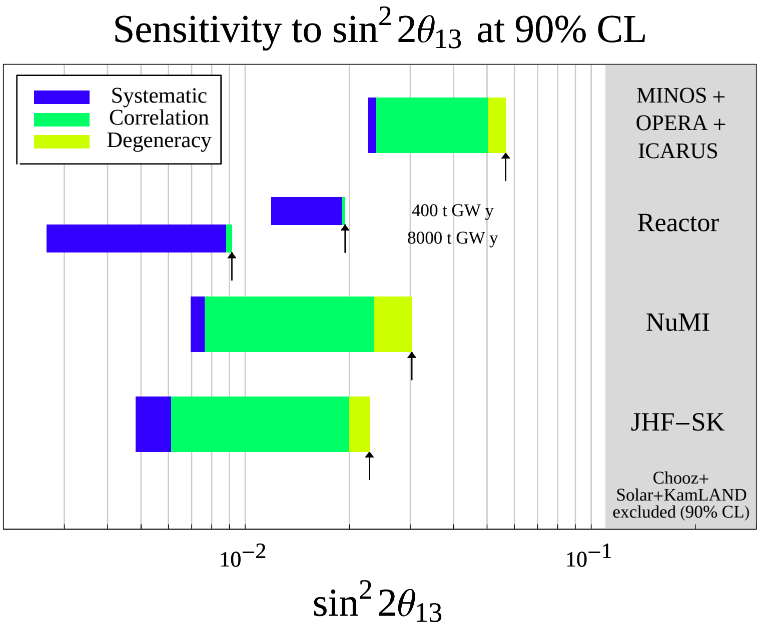

Figure 5: The θ_{13} sensitivity of potential reactor experiments compared to accelerator experiments which could be built on similar time scales. MINOS+OPERA+ICARUS is what can be obtained from the combination of these experiments. The results for the scenarios Reactor-I and Reactor-II are significantly better and they cut into the correlation and degeneracy range of NuMI and JHF-SK. This shows that there is synergy between a new reactor experiment and JHF-SK and NuMI. See figure 2 for the colour coding.

to the see-saw mechanism [15] very small neutrino masses. This can be nicely realized in embeddings of the SM into larger gauge symmetries, such as SO(10).

A reason for expecting a particular value of θ_{13} does clearly not exist as long as one extends the SM only minimally to accommodate neutrino masses. θ_{13} is then simply some unknown parameter which could take an arbitrarily small value, including zero. The situation changes in models of neutrino masses. Even then one should acknowledge that in principle any value of θ_{13} can be accommodated. Indeed, before the discovery of large leptonic mixing, many theorists who did consider lepton mixing expected it to be similar to quark mixing, characterized by small mixing angles. Experiment led theory in showing the striking results that $\sin^2 2\theta_{23} \simeq 1$ and $\tan^2\theta_{12} \simeq 0.44$, while θ_{13} is small. Indeed, the most remarkable property of leptonic mixing is that two angles are large. Therefore, today there is no particular reason to expect the third angle, θ_{13}, to be extremely small or even zero. This can be seen in neutrino mass models which are able to predict a large θ_{12} and θ_{23}. They often have a tendency to predict also a sizable value of θ_{13}. This is both the case for models in the framework of Grand Unified Theories and for models using flavour symmetries. There exist also many different texture models of neutrino masses and mixings, which accommodate existing data and try to predict the missing information by assuming certain elements of the mass matrix to be either zero or equal. Again one finds typically a value for θ_{13} which is not too far from current experimental bounds. A similar behavior is found in so-called “anarchic mass matrices”. Starting essentially with random

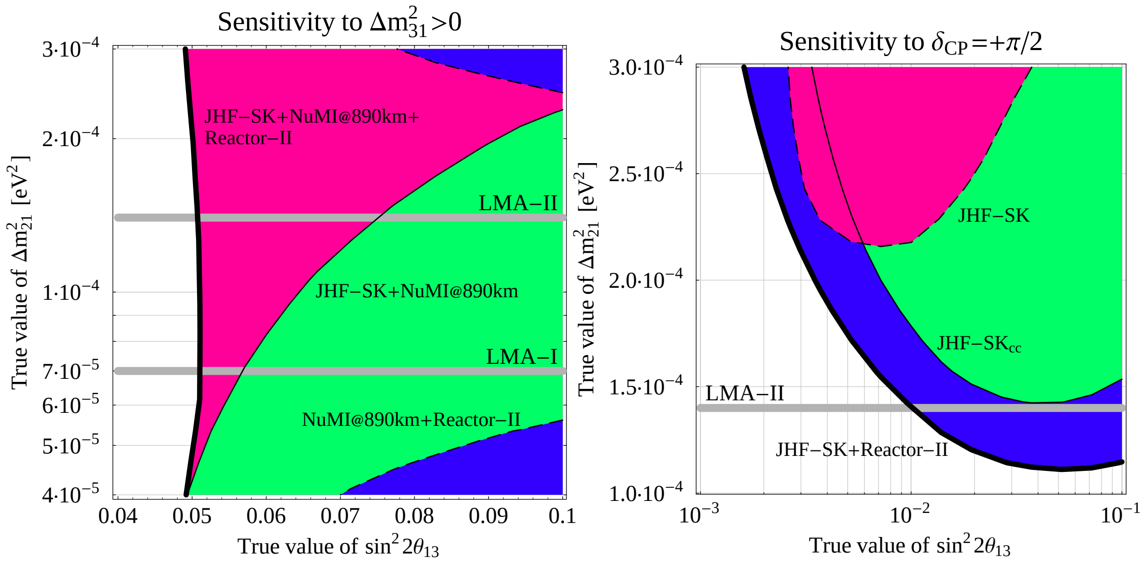

Figure 6: Synergies from the combination of JHF-SK, NuMI (at 890 km) and a Reactor-II experiment. The left plot shows the parameter space in the Δm^2_{31}-$\sin^2 2\theta_{13}$ plane where $sign(\Delta m^2_{31})$ can be determined. It can be seen that the combination is sensitive to $sign(\Delta m^2_{31})$ independently of Δm^2_{31} for $\sin^2 2\theta_{13} > 0.05$. The right plot shows the parameter space in the Δm^2_{31}-$\sin^2 2\theta_{13}$ plane where a CP phase $\delta = \pi/2$ could be determined. The improvement comes here mostly from the fact that the reactor measurement allows the beams to run fully with neutrinos. For details see [34].

neutrino mass matrix elements one finds that large mixings are actually quite natural.

An overview of various predictions is given in table 7.1. For more extensive reviews, see for example [35, 36, 37, 38]. The conclusion from all these considerations about neutrino mass models is that a value of θ_{13} close to the CHOOZ bound would be quite natural, while smaller values become harder and harder to understand as the limit on θ_{13} is improved. Besides, neutrino masses and mixing parameters are subject to quantum corrections between low scales, where measurements are performed, and high scales where some theory predicts θ_{13}. Even in the “worst case” scenario, where θ_{13} is predicted to be exactly zero, they cause θ_{13} to run to a finite value at low energy. Strictly speaking, $\theta_{13} = 0$ cannot be excluded completely by this argument, as the high-energy value could be just as large as the change due to running and of opposite sign. However, a severe cancellation of this kind would be unnatural, since the physics generating the value at high energy are not related to those responsible for the quantum corrections. The strength of the running of θ_{13} depends on the neutrino mass spectrum and whether or not supersymmetry is realized. For the Minimal Supersymmetric Standard Model one finds a shift $\Delta \sin^2 2\theta_{13} > 0.01$ for a considerable parameter range, i.e. one would expect to measure a finite value of θ_{13} [39]. Conversely, limits on model parameters would be obtained if an experiment were to set an upper bound on $\sin^2 2\theta_{13}$ in the range of 0.01. In any case, it should be clear that a precision of the order of quantum corrections to neutrino masses and mixings is very interesting in a number of ways.

Reference	$\sin\theta_{13}$	$\sin^2 2\theta_{13}$
SO(10)		
Goh, Mohapatra, Ng [40]	0.18	0.13
Orbifold SO(10)		
Asaka, Buchmüller, Covi [41]	0.1	0.04
SO(10) + flavour symmetry		
Babu, Pati, Wilczek [42]	$5.5 \cdot 10^{-4}$	$1.2 \cdot 10^{-6}$
Blazek, Raby, Tobe [43]	0.05	0.01
Kitano, Mimura [44]	0.22	0.18
Albright, Barr [45]	0.014	$7.8 \cdot 10^{-4}$
Maekawa [46]	0.22	0.18
Ross, Velasco-Sevilla [47]	0.07	0.02
Chen, Mahanthappa [48]	0.15	0.09
Raby [49]	0.1	0.04
SO(10) + texture		
Buchmüller, Wyler [50]	0.1	0.04
Bando, Obara [51]	0.01 .. 0.06	$4 \cdot 10^{-4}$ .. 0.01
Flavour symmetries		
Grimus, Lavoura [52, 53]	0	0
Grimus, Lavoura [52]	0.3	0.3
Babu, Ma, Valle [54]	0.14	0.08
Kuchimanchi, Mohapatra [55]	0.08 .. 0.4	0.03 .. 0.5
Ohlsson, Seidl [56]	0.07 .. 0.14	0.02 .. 0.08
King, Ross [57]	0.2	0.15
Textures		
Honda, Kaneko, Tanimoto [58]	0.08 .. 0.20	0.03 .. 0.15
Lebed, Martin [59]	0.1	0.04
Bando, Kaneko, Obara, Tanimoto [60]	0.01 .. 0.05	$4 \cdot 10^{-4}$ .. 0.01
Ibarra, Ross [61]	0.2	0.15
3×2 *see-saw*		
Frampton, Glashow, Yanagida [62]	0.1	0.04
Mei, Xing [63] (normal hierarchy)	0.07	0.02
(inverted hierarchy)	> 0.006	$> 1.6 \cdot 10^{-4}$
Anarchy		
de Gouvêa, Murayama [64]	> 0.1	> 0.04
Renormalization group enhancement		
Mohapatra, Parida, Rajasekaran [65]	0.08 .. 0.1	0.03 .. 0.04

Table 7.1: Incomplete selection of predictions for θ_{13}. The numbers should be considered as order of magnitude statements.

8. Conclusions

In summary, future measurements of θ_{13} are promising and there exist very good reasons to push the sensitivity limit from the current CHOOZ value by an order of magnitude and to hope that a finite value of θ_{13} will be found. At this precision even a negative result would be very interesting, since it would test or rule out many neutrino mass models and restrict parameters relevant for quantum corrections to masses and mixings. From a larger point of view these experiments probe if a small value of θ_{13} is a numerical coincidence or the result of some underlying symmetry. The theoretically well motivated range can be reached by using both reactor neutrinos and accelerator neutrino beams. Reactor measurements allow to determine or limit θ_{13} without the ambiguities associated with matter effects and CP violation. Combination with measurements of $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations using accelerator neutrino beams at long baselines will improve the knowledge of θ_{13} and will allow to limit or see matter and CP violation effects. In order to optimally exploit the physics opportunities both reactor and long baseline accelerator measurements will be necessary.

Acknowledgments: I would like to thank P. Huber, J. Kersten and W. Winter for the collaboration in the studies on which much of this article is based upon. This work was supported by the "Sonderforschungsbereich 375 für Astro-Teilchenphysik" der Deutschen Forschungsgemeinschaft.

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