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Spontaneous CP in a Susy theory of flavour

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Abstract: We present a model based on an $SU(3)$ family symmetry providing a full description of quark and lepton masses and mixing angles. CP is spontaneously broken in the flavour sector reproducing the observed results of the Jarlskog invariant in the CKM mixing matrix. Moreover, our model predicts the amount of CP violation to be expected in the neutrino sector. Furthermore, this approach solves the Supersymmetric CP and FCNC problems in a gravity mediation scenario. The symmetry predicts the structure of the squark and slepton mass matrices and this will be checked in future experiments.

The flavour sector of the SM remains the big unknown in high energy physics. In the SM Flavour (Yukawa) couplings are not fixed by symmetry and so they are merely unrelated parameters to be fixed by experiment. In our search for a more fundamental theory we need to improve our understanding of the flavour physics, and in analogy to the gauge sector, one might expect an spontaneously broken familly symmetry which determines the different flavour parameters [1]. The appearance of CP violation in the SM is equally mysterious. We do not know why there are complex couplings in the SM giving rise to a violation of the CP symmetry in neutral meson systems. In fact, these two problems are deeply related in the SM as the Yukawa couplings are the only source for both the flavour structures and the CP violation phenomena, although this need not be so in other extensions of the SM where new sources of CP violation different from the Yukawa couplings are present.

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Here, we address both problems together and build a supersymmetric flavour theory which determines all the flavour structures in the theory as well as the different phases breaking the CP symmetry. In the supersymmetric context where the lack of understanding of flavour and CP is especially severe, this analysis is particularly relevant. The Minimal Supersymmetric extension of the SM (MSSM) suffers the so-called "SUSY flavour problem" and "SUSY CP problem". The simplest solution to these problems consists of arbitrarily assuming that all the new flavour structures are proportional to the identity and that all new phases vanish. However, this approach cannot be justified without some (so far unspecified) symmetry reason, especially taking into account that the Yukawa couplings of the SM do not share these features. Thus we will study both CP violation and the origin of flavour structures in the framework of a supersymmetric $SU(3)$ family symmetry model which has been shown [2, 3, 4] to provide a correct texture for the Yukawa matrices in agreement with an accurate phenomenological fit [5]. However, the improvement on the determination of CKM mixing angles and CP asymmetries requires the presence of phases in the elements of the Yukawa matrices. In our analysis we assume that CP is an exact symmetry of the theory of unbroken flavour $SU(3)$ and is only broken by complex vacuum expectation values of the flavon fields that determine the Yukawa structures. We show that this approach can accomodate successfully the observed masses, mixings and CP violation effects. Moreover, we obtain predictions on the structure of the sfermion mass matrices that will be checked with experiment after the discovery of Supersymmetric partners in the future experiments. Finally we provide relations for the CP phases in the quark, lepton and the sfermion sectors.

One reason why it is difficult to construct a theory of flavour is that measurement in the quark sector of the eigenvalues (quark masses) and the CKM mixing matrix is all the information we can extract using SM interactions about the full quark Yukawa matrices. However, in most extensions of the Standard Model, and in particular in supersymmetric extensions, the new non SM interactions can provide new information on the physics of flavour. At the moment, we still must rely on simplifying assumptions to try to disentangle the complicated structure of masses and mixing angles. A recent phenomenological analysis [5] under these assumptions shows that the following symmetric textures give an excellent fit to quark masses and mixing angles,

$$Y_d \propto \begin{pmatrix} 0 & b\,\bar{\varepsilon}^3 & c\,\bar{\varepsilon}^3 \\ . & \bar{\varepsilon}^2 & a\,\bar{\varepsilon}^2 \\ . & . & 1 \end{pmatrix}, \qquad Y_u \propto \begin{pmatrix} 0 & b'\varepsilon^3 & c'\varepsilon^3 \\ . & \varepsilon^2 & a'\varepsilon^2 \\ . & . & 1 \end{pmatrix} \tag{1}$$

with $\bar{\varepsilon} \simeq 0.15$, $\varepsilon \simeq 0.05$, $b = 1.5$, $a = 1.3$, $c = 0.4$ and a', b', c' are poorly fixed from experimental data. We adopt this basic structure as our starting point.

As shown in [2, 4] these structures can be successfully reproduced from an spontaneously broken $SU(3)$ family symmetry. Under this symmetry all left handed fermions (ψ_i and ψ_i^c) are triplets. Furthermore, we have several new scalar fields which are either triplets ($\overline{\theta}_3$, $\overline{\theta}_{23}$, $\overline{\theta}_2$) or antitriplets (θ_3, θ_{23}). We assume that $SU(3)_{fl}$ is broken in two steps. The first step occurs when θ_3 gets a large vev breaking $SU(3)$ to $SU(2)$. Subsequently a smaller vev of θ_{23} breaks the remaining symmetry. After this breaking we obtain the effec-

Field	ψ	ψ^c	λ	$\overline{\lambda}$	H	Σ	θ_3	θ_{23}	$\overline{\theta_3}$	$\overline{\theta_{23}}$	$\overline{\theta_2}$
$\mathbf{SU(3)}$	3	3	3	3	1	1	$\overline{3}$	$\overline{3}$	3	3	3
R	1	1	-2	0	2	0	-3	-3	3	-6	12
$\mathbf{Z_{15}}$	0	0	0	0	0	8	0	-4	1	6	-2

Table 1: Transformation of the matter superfields under the family symmetries.

tive Yukawa couplings at low energies through the Frogatt-Nielsen mechanism integrating out heavy fields. In fact, the large third generation Yukawa couplings require θ_3 (and $\bar{\theta}_3$) vev of the order of the mediator scale, M, while θ_{23}/M (and $\bar{\theta}_{23}/M$) have vevs of order ε in the up sector and $\bar{\varepsilon}$ in the down sector. To generate the correct Yukawa texture in Eq. (1) we require that the fields θ_{23} and $\bar{\theta}_{23}$ get equal vevs in the second and third components. Furthermore, following [4] we assume that θ_3 and $\overline{\theta}_3$ transform as $\mathbf{3} \oplus \mathbf{1}$ under $SU(2)_R$ to accommodate up and down type Yukawas. Moreover, in the context of an $SO(10)$ grand unified theory it is possible simultaneously to describe quark and lepton Yukawa matrices. The main ingredient to reach this goal, is a new field Σ, which is a $\overline{\mathbf{45}}$ of $SU(5)$ with vev along the $(B - L + \kappa T_3)$ direction generating the usual Georgi-Jarlskog factors that correct the difference between quark and lepton Yukawas.

These symmetries are not enough to reproduce the textures in Eq. (1) and we must impose some additional symmetries to forbid unwanted terms in the effective superpotential. The choice of these symmetries is not uniquely defined, and here we present a possible example. However the quark Yukawa textures consistent with the requirement of $SO(10)$ unification strongly constrain the Dirac Yukawa couplings superpotential and many of the features discussed here are common to any $SU(3)_{fl}$ model. In this example we need two additional global symmetries to obtain the correct Yukawa textures. In principle we have a continuous R-symmetry which will play the role of the Peccei-Quinn symmetry to solve the strong CP problem and a discrete symmetry Z_{15}, under which the fields transform as shown in Table 1. Using these charges we can build the effective Yukawa couplings as,

$$\begin{aligned} W_Y = H\psi_i\psi_j^c \Big[\theta_3^i\theta_3^j \,+\, \theta_{23}^i\theta_{23}^j\Sigma \,+\, \left(\epsilon^{ikl}\overline{\theta}_{23,k}\overline{\theta}_{3,l}\theta_{23}^j \,+\, \epsilon^{jkl}\overline{\theta}_{23,k}\overline{\theta}_{3,l}\theta_{23}^i\right)\left(\theta_{23}\overline{\theta_3}\right) \\ \epsilon^{ijk}\overline{\theta}_{23,k}\left(\theta_{23}\overline{\theta_3}\right)^2 \,+\, \epsilon^{ijk}\overline{\theta}_{3,k}\left(\theta_{23}\overline{\theta_3}\right)\left(\theta_{23}\overline{\theta_{23}}\right) \,+\, \theta_3^i\theta_{23}^j\left(\theta_3\overline{\theta_3}\right)^4\Big] \end{aligned} \tag{2}$$

In a similar way we can calculate the structure of the Majorana matrix.

$$\begin{aligned} W_M = \frac{\psi_i^c\psi_j^c}{M}\Big[\lambda^i\lambda^j \,+\, \theta_3^i\theta_{23}^j\left(\lambda\overline{\theta_3}\right)^2\left(\theta_3\overline{\theta_3}\right)^2 \,+\, \theta_{23}^i\theta_{23}^j\left(\lambda\overline{\theta_3}\right)^2\left(\theta_3\overline{\theta_3}\right)^6 \\ +\,\epsilon^{ikl}\overline{\theta}_{23,k}\overline{\theta}_{3,l}\theta_3^j\left(\lambda\overline{\theta_3}\right)^2\left(\theta_3\overline{\theta_3}\right)^6 \,+\, \epsilon^{ikl}\overline{\theta}_{23,k}\overline{\theta}_{3,l}\theta_{23}^j\left(\lambda\overline{\theta_3}\right)^2\left(\theta_3\overline{\theta_3}\right)\left(\theta_{23}\overline{\theta_3}\right)^2\Big] \end{aligned} \tag{3}$$

After minimisation of the scalar potential and using the $SU(3)$ symmetry to choose the basis and phases, we obtain the following vevs,

$$\theta_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} a_3^u & 0 \\ 0 & a_3^d\, e^{i\chi} \end{pmatrix};\; \bar{\theta}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} a_3^u\, e^{i\alpha_u} & 0 \\ 0 & a_3^d\, e^{i\alpha_d} \end{pmatrix};$$

$$\theta_{23} = \begin{pmatrix} 0 \\ b_{23} \\ b_{23}\ e^{i\beta_3} \end{pmatrix}; \qquad \bar{\theta}_{23} = \begin{pmatrix} 0 \\ b_{23}\ e^{i\beta_2'} \\ b_{23}\ e^{i(\beta_2'-\beta_3)} \end{pmatrix}; \tag{4}$$

And these vevs are such that, $\frac{a_3^u}{M_u} = \frac{a_3^d}{M_d} = \varepsilon^{1/4}$, $\frac{b_{23}}{M_u} = \varepsilon$ and $\frac{b_{23}}{M_d} = \bar{\varepsilon}$, with $\bar{\varepsilon} \simeq 0.15$ and $\varepsilon \simeq 0.05$ at the symmetry breaking scale that we take approximately equal to M_{GUT}. Then we have the following Yukawa textures,

$$Y^f = \begin{pmatrix} 0 & \varepsilon^3 e^{i\delta}\, X_1 & \varepsilon^3 e^{i(\delta+\beta_3)}\, X_2 \\ \ldots & \varepsilon^2\, \frac{\Sigma}{|a_3|^2} & \varepsilon^2 e^{i\beta_3}\, \frac{\Sigma}{|a_3|^2} \\ \ldots & \ldots & e^{2i\chi} \end{pmatrix} \frac{|a_3|^2}{M^2}, \tag{5}$$

where X_1 and X_2 account for the different contributions to the $(1,2)$ and $(1,3)$ elements from the last three terms in Eq. (2) and $\delta^{u,d} = (2\alpha_3^{u,d} + \beta_3 + \beta_2')$.

In the same way at M_{Planck} both $SU(3)_{fl}$ and CP are exact symmetries of the theory and then the Kähler potential is invariant under these symmetries. After the flavour symmetry is spontaneously broken we can calculate the effective Kähler potential,

$$\begin{aligned} K = \psi_i^\dagger \psi_j \Big(\delta^{ij}(c_0 + d_0 X X^\dagger) + \frac{1}{M^2}[\theta_3^{i\dagger}\theta_3^j(c_1 + d_1 X X^\dagger) + \\ \theta_{23}^{i\dagger}\theta_{23}^j(c_2 + d_2 X X^\dagger)] + \frac{1}{M^4}[\Big(\theta_3^{i\dagger}\theta_{23}^j(\theta_3\theta_{23}^\dagger) + \theta_3^i\theta_{23}^{j\dagger}(\theta_3^\dagger\theta_{23})\Big)(c_3 + d_3 X X^\dagger) \\ +(\epsilon^{ikl}\overline{\theta}_{3,k}\overline{\theta}_{23,l})^\dagger(\epsilon^{jmn}\overline{\theta}_{3,m}\overline{\theta}_{23,n})(c_4 + d_4 X X^\dagger)]\Big) \end{aligned} \tag{6}$$

where c_i, d_i are $\mathcal{O}(1)$ coefficients in the different terms and X is a possible hidden sector with non-vanishing F-term breaking SUSY.

The general structure of the Yukawa matrix and the soft breaking mass matrices in the $SU(3)$ family symmetry models is given by Eqs. (2,3). This structure is fixed mainly by the quark masses and mixing angles and in the fit of the quark textures the presence of phases plays an important role. For instance we can fix the phase Φ_1 by using the Gatto-Sartori-Tonin [6] relation,

$$V_{us} \approx |s_{12}^d - e^{-i\Phi_1} s_{12}^u| = \left| \sqrt{\frac{m_d}{m_s}} - e^{-i\Phi_1} \sqrt{\frac{m_u}{m_c}} \right|, \tag{7}$$

where we have used that $s_{12}^u = \sqrt{\frac{m_u}{m_c}}$ and $s_{12}^d = \sqrt{\frac{m_d}{m_s}}$, which applies for the structure of the mass matrix in Eq. (5). The ratios of masses are $\sqrt{\frac{m_d}{m_s}} = 0.228 \pm 0.005$, $\sqrt{\frac{m_u}{m_c}} = 0.06 \pm 0.01$. Comparing Eq. (7) with the experimental value of $V_{us} = 0.2196 \pm 0.0026$, we obtain $\Phi_1 = -1.29 \pm 0.14$ $(-74^0 \pm 8^0)$.

We also have to calculate the Jarlskog invariant in the CKM matrix from

$$\begin{aligned} J_{\rm CP} = \mathrm{Im}\{V_{32}^* V_{21}^* V_{22} V_{31}\} \simeq \mathrm{Im}\{s_{23}^Q \left[s_{12}^d s_{12}^u s_{23}^Q e^{i\Phi_1} - s_{12}^d s_{13}^d e^{i\Phi_2} \right]\} \\ s_{23}^Q e^{-i\xi_s} = s_{23}^d e^{-i(\gamma_{23}^d - \gamma_{23}^u)} - s_{23}^u \end{aligned} \tag{8}$$

where V is the CKM matrix, which depends on $\sin^f(\theta_{ij}) \equiv s_{ij}^f$.

In the PDG "standard" parametrization [7] the Jarlskog invariant is

$$J_{\rm CP} = s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2 \sin\delta, \tag{9}$$

with c_{ij}, s_{ij} the cosinus or sinus of a rotation in the (i,j) plane. Using the measured values of the mixing angles at the electroweak scale we obtain $J_{\rm CP} = 3.3\times 10^{-5}\ \sin\delta \simeq (2.8\pm 0.4)\times 10^{-5}$, with $\delta = 1.02 \pm 0.22$. In our model the Jarlskog invariant in Eq. (8) must reproduce this value up to a correction in the phase due to the relatively small SUSY contributions to the unitarity triangle fit. We must remember that Φ_1 is nearly maximal, while Φ_2 is only order $\bar\varepsilon^2$ and this implies that the Jarlskog invariant will be dominated by the first contribution in Eq. (8). If we take into account the RGE evolution from M_{GUT} to M_W we obtain,

$$J_{\rm CP}|_{M_W} \simeq \chi^{-2}\ J_{\rm CP}|_{M_{GUT}} \simeq \chi^{-2} a^2\ b\ b^\prime \bar\varepsilon^5 \varepsilon \sin\Phi_1 \simeq 2.9\times 10^{-5} \sin\Phi_1 \tag{10}$$

where $\chi = (M_{GUT}/M_Z)^{-Y_t^2/(16\pi^2)} \simeq 0.7$ [8, 5]. Therefore, from the Jarlskog invariant we conclude that Φ_1 must be also near maximal.

The neutrino Majorana matrix has the form,

$$M_R = \begin{pmatrix} 0 & \lambda\varepsilon^{5.75} e^{i\delta^\prime} & \varepsilon^5 e^{i(\delta^\prime + 3\alpha_3 - 2\beta_3)} \\ ... & \eta_2 \varepsilon^{5.5} e^{i(8\alpha_3)} & \eta_1 \varepsilon^{2.75} e^{i(4\alpha_3)} \\ ... & ... & 1 \end{pmatrix} \frac{c^2}{M}, \tag{11}$$

with $\delta^\prime = (6\alpha_3 + 2\beta_3 + \beta_2^\prime)$ and λ, η_1, η_2, $\mathcal{O}(1)$ coefficients. Similarly, the Yukawa matrix is given by Eq. (5) with $\Sigma_\nu = 0$ and the additional contribution from the new term mixing θ_3 and θ_{23},

$$Y^\nu = \begin{pmatrix} 0 & \varepsilon^3 e^{i\delta^u}\, X_1 & \varepsilon^3 e^{i(\delta^u + \beta_3)}\, X_2 \\ ... & 0 & \eta\varepsilon^{2.75} e^{i4\alpha_u} \\ ... & ... & 1 \end{pmatrix} \frac{|a_3^u|^2}{M_u^2}, \tag{12}$$

with η an order 1 coefficient. Now, using the see-saw formula, $\chi = Y^\nu M_R^{-1} (Y^\nu)^T$, we obtain the low energy neutrino mass matrix and the corresponding neutrino masses and mixing angles. In fact, we can easily obtain the observed masses and mixing angles. For the paramenters $X_1 = 1.4$, $X_2 = 2$, $\eta = 1.5$, $\lambda = 0.34$, $\eta_2 = 0.75$ and $\eta_1 = 1/3$ and neglecting charged lepton mixings, we obtain for the neutrino mixings,

$$R^\nu = \begin{pmatrix} -0.781084 & 0.584879 & 0.218687 \\ -0.524308 & -0.424105 & -0.738401 \\ 0.339129 & 0.691413 & -0.637918 \end{pmatrix} \tag{13}$$

and for the eigenvalues $m_1 \simeq 0.007 \frac{v_2^2}{M_3}$, $m_2 \simeq 0.12 \frac{v_2^2}{M_3}$ and $m_1 \simeq 1.02 \frac{v_2^2}{M_3}$.

The MNS phase can be obtained from the invariant Im $(H^\nu_{12} H^\nu_{23} H^\nu_{31})$ in the basis in which the charged leptons are diagonal [1] where $H^\nu_{ij} = \sum_k \chi^\prime_{ik} \chi^{\prime *}_{jk}$. Neglecting all charged

[1]Note that after some simple algebra and using the unitarity of the MNS mixing matrix Im $(H^\nu_{12} H^\nu_{23} H^\nu_{31}) = (m^2_{\nu_3} - m^2_{\nu_1})(m^2_{\nu_3} - m^2_{\nu_2})(m^2_{\nu_2} - m^2_{\nu_1})$ Im $(V_{22} V^*_{23} V_{33} V^*_{32})$.

fermion mixings except the rotation in the $(1,2)$ sector, we obtain,

$$\chi' \simeq \begin{pmatrix} \chi_{11} - 2\chi_{21} s^l_{12} e^{i\varphi_L} & \chi_{12} - \chi_{22} e^{i\varphi_L} s^l_{12} & \chi_{13} - \chi_{23} s^l_{12} e^{i\varphi_L} \\ \cdots & \chi_{22} + 2\chi_{12} s^l_{12} e^{-i\varphi_L} & \chi_{23} + \chi_{13} s^l_{12} e^{-i\varphi_L} \\ \cdots & \cdots & \chi_{33} \end{pmatrix}, \tag{14}$$

for $s^l_{12} \approx \bar{\varepsilon} X_1/\Sigma_2$ and $\varphi_L = -2\alpha^d_3 - 2\alpha^u_3 + \beta_3$. Then

$$\begin{aligned} H^\nu_{12} H^\nu_{23} H^\nu_{31} &\approx A_{12} A_{31} A_{23} + s^l_{12} \left[A_{12} (A^2_{31} - A^2_{23}) e^{-i\varphi_L} + \right. \\ & \left. A_{31} A_{23} (|\chi_{11}|^2 - |\chi_{22}|^2 + |\chi_{13}|^2 - |\chi_{33}|^2) e^{i\varphi_L} \right] \\ &\simeq s^l_{12} \sin\varphi_L \left[\chi_{12} \chi^2_{23} \left(\chi_{22}\chi_{33}(\chi_{22} - \chi_{33}) + \chi^2_{23}(\chi_{22} + \chi_{33}) \right) + \right. \\ & \left. \chi_{13}\chi_{23}\chi_{22} \left(\chi^2_{23}(\chi_{33} - \chi_{22}) + \chi_{22}\chi_{33}(\chi_{22} + \chi_{33}) \right) \right] \end{aligned} \tag{15}$$

where we have defined $A_{ij} = \sum_{a=1,3} \chi_{ia}\chi^*_{ja}$ and we have used that A_{ij} are real up to order $\varepsilon^{2.75}$ and that $\chi_{1i} << \chi_{22}, \chi_{23}, \chi_{32}, \chi_{33}$.

In general, this imaginary part does not vanish except in special points of the parameter space, and therefore the observable MNS phase is given by $\varphi_L = -2\alpha^d_3 - 2\alpha^u_3 + \beta_3$, i.e., it depends both on the neutrino and charged lepton phases (α_d).

Then the leptogenesis CP asymmetry can be obtained as,

$$\epsilon_1 = -\frac{3}{16\pi (Y^{\nu\dagger} Y^\nu)_{11}} \sum_{i \neq 1} \mathrm{Im}\left[(Y^{\nu\dagger} Y^\nu)^2_{1i} \right] \frac{M_1}{M_i} \tag{16}$$

where the neutrino Yukawa matrices are given in the basis of diagonal Majorana masses. In this case, it is easy to see that any left handed rotation or rephasing cancels, for instance the phases coming from charged leptons can not play any role here. Then we have that $\varphi = 4\alpha^u_3 + \beta_3$ is the only relevant phase for leptogenesis.

Finally the neutrinoless double beta decay phase, which is simply the relative phase among the two dominant contributions of χ'_{11} in the basis of diagonal charged lepton masses. This time from Eq. (14) we obtain,

$$(\chi')_{11} = \tilde{P}^2_1 \left(\chi_{11} - 2\bar{\varepsilon} \frac{X_1}{\Sigma} \chi_{12} e^{-i\varphi_L} \right) \tag{17}$$

and if we remember that χ_{ij} are real up to order $\varepsilon^{2.75}$ we obtain that the neutrinoless double beta decay phase is also φ_L and it coincides with the MNS phase. Notice that the relevance of charged lepton phases is generic in any model with offdiagonal charged lepton textures.

The structure of soft mass matrices relevant for FCNC and CP experiments [9] is also obtained from Eq. (3). For instance, the right handed down squark mass matrix in the SCKM matrix,

$$\frac{(M^2_{\tilde{D}_R})}{m^2_0} \simeq \begin{pmatrix} 1 + \bar{\varepsilon}^3 & -\bar{\varepsilon}^3 e^{-i\omega} \frac{|a_3|^2}{\Sigma_d} X_1 & -\bar{\varepsilon}^3 e^{-i\omega} \frac{|a_3|^2}{\Sigma_d} X_1 \\ \bar{\varepsilon}^3 e^{i\omega} \frac{|a_3|^2}{\Sigma_d} X_1 & 1 + \bar{\varepsilon}^2 & \bar{\varepsilon}^2 - \bar{\varepsilon}^3 \Sigma_d e^{-i2(\beta-\chi)} \\ -\bar{\varepsilon}^3 e^{i\omega} \frac{|a_3|^2}{\Sigma_d} X_1 & \bar{\varepsilon}^2 - \bar{\varepsilon}^3 \Sigma_d e^{i2(\beta-\chi)} & 1 + \bar{\varepsilon} \end{pmatrix} \tag{18}$$

with $\omega = \text{Arg}\left(1+\bar{\varepsilon}\frac{\Sigma_d}{\Sigma_u}e^{i\Phi_1}\right)$. Then, we obtain,

$$\text{Im}(M^2_{\tilde{D}_R})_{12}\Big|_{M_W} \simeq \bar{\varepsilon}^4\frac{|a_3|^2}{5\Sigma_u}X_1\sin\Phi_1 m_0^2 \simeq 2.3\times 10^{-4}m_0^2 \qquad (19)$$

where we taken $\Phi_1 = 90^0$ and $|a_3|^2/\Sigma_u \simeq 1.5$ and we have approximated the RGE effects from M_{GUT} to M_W by a factor 1/5. This must be compared with the phenomenological MI bounds [10] for an average squark mass of 500 GeV, $\sqrt{|\text{Im}(\delta^d_{RR})^2_{12}|} = 3.2\times 10^{-3}$. So, the presence of offdiagonal entries in these squark mass matrices can have sizeable effects in several low energy observables and they can reach a 20% of the measured value of ε_K. However, if we consider simultaneously the RR and LL mass insertions the contribution could be even larger.

In the case of δ_{13} MI that could contribute to $B_d\bar{B}_d$ mixing and the $J/\psi K_S$ CP asymmetry, we can see from Eq. (18) that these MI are exactly of the same order as the corresponding δ_{12} MI. However, the values for the MI required to saturate these observables are now much larger and the phenomenological bounds are, $\sqrt{|\text{Im}(\delta^d_{RR})^2_{13}|} \leq 0.3$ [11]. Thus no sizeable effects are possible here.

Finally, we would like to comment on possible effects in lepton flavour violation, as $\mu\to e\gamma$ and $\tau\to\mu\gamma$ decays. In this case, the slepton mass matrices are exactly identical to Eq. (18), with the only replacement of $\Sigma_d\to\Sigma_e$. In this case, the bounds on the leptonic MI are more difficult to obtain as they depend on other parameters $(\mu, \tan\beta, M_{1/2}, m_0^2)$. However, there some bounds for fixed values of $\tan\beta$ [12]. The most sensitive precess is $\mu\to e\gamma$ where for $\tan\beta = 10$ and average slepton mass of 300 GeV we get $(\delta^e_{LL})_{12} \leq 3\times 10^{-4}$ while for the RR mass insertion the bound is much worse due to a possible cancelation among diagrams. Now we have for the LL MI from Eq. (18), $(\delta^e_{LL})_{12} \leq 2.5\times 10^{-4}$ and therefore a $\mu\to e\gamma$ decay close to the experimental bound is indeed possible. On the other hand it is important to notice that the μ term and the trilinear couplings in this model are real apart from sufficiently small corrections proportional to flavon vevs. Then flavour diagonal CP violation as electric dipole moments are under control [13].

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