The anomaly-induced effective action and natural inflation

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Abstract: The anomaly-induced inflation (modified Starobinsky model) is based on the application of the effective quantum field theory approach to the Early Universe. We present a brief general review of the model and show that it does not require a fine-tuning for the parameters of the theory or initial data, gives a real chance to meet a graceful exit to the FRW phase and also has positive features with respect to the metric perturbations.

1. Introduction

The possibility of observing phenomena occuring in the early universe, and in particular during inflation [1, 2] give a chance to learn new information about the high energy physics. One of the options is to consider the model of inflation which contains smaller phenomenological input compared to the conventional inflaton models (see, e.g. [3]) and can be directly deduces from the results of quantum field theory (QFT) in curved space-time. The remarkable example of such model is based on the vacuum quantum effects, in the simplest case on the effects of massless fields. In this case the leading quantum phenomenon is conformal anomaly. The original version of the anomaly-induced inflation [4, 5, 6, 7] has been developed in 80-ties is the cosmological model which takes into account the vacuum quantum effects of the free, massless and conformally coupled to metric matter fields [8]. The quantum correction to the Einstein equation

$$R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu} \,=\, 8\pi G\, < T_{\mu\nu} > -\,\Lambda \tag{1.1}$$

(where we added the cosmological constant (CC) for the sake of generality) produces a non-trivial effect because the anomalous trace of the stress tensor

$$T \;= < T^{\mu}_{\mu} > = \,-\,(wC^2 + bE + c\nabla^2 R)\,, \tag{1.2}$$

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where

$$w = \frac{1}{(4\pi)^2}\Big(\frac{N_0}{120}+\frac{N_{1/2}}{20}+\frac{N_1}{10}\Big)\,,$$
$$b = -\frac{1}{(4\pi)^2}\Big(\frac{N_0}{360}+\frac{11\,N_{1/2}}{360}+\frac{31\,N_1}{180}\Big)\,,$$
$$c = \frac{1}{(4\pi)^2}\Big(\frac{N_0}{180}+\frac{N_{1/2}}{30}-\frac{N_1}{10}\Big)\,. \tag{1.3}$$

In the absence of matter, one can obtain the cosmological solution in two distinct ways: using the (00)-component [4, 5] or via the anomaly-induced effective action [9, 10]. Indeed, the last option is completely equivalent to taking the trace of (1.1). The resulting equation has, for $k=0$ FRW metric, the following form (since the cases $k=\pm 1$ are quite similar [11] we will not consider them here):

$$\frac{\dddot{a}}{a}+\frac{3\,\dot{a}\dddot{a}}{a^2}+\frac{\ddot{a}^2}{a^2}-\left(5+\frac{4b}{c}\right)\frac{\ddot{a}\dot{a}^2}{a^3}-\frac{M_P^2}{8\pi c}\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}-\frac{2\Lambda}{3}\right)=0\,, \tag{1.4}$$

where $M_P = G^{-1/2}$ is a Planck mass. The equation above has remarkable particular solution

$$a(t) = a_0\cdot \exp(Ht) \tag{1.5}$$

where (motivated by the recent supernova data [12]), we consider only positive CC in the low-energy regime

$$H = \frac{M_P}{\sqrt{-32\pi b}}\left(1\pm\sqrt{1+\frac{64\pi b}{3}\frac{\Lambda}{M_P^2}}\right)^{1/2}. \tag{1.6}$$

As far as $\Lambda \ll M_P^2$, we meet two very different solutions

$$H_c = \sqrt{\frac{\Lambda}{3}} \tag{1.7}$$

$$\text{and}\qquad H_S = \frac{M_P}{\sqrt{-16\pi b}}\,. \tag{1.8}$$

The first solution (1.7) is exactly the one in the theory without quantum corrections, while the second solution H_S is the inflationary solution of Starobinsky. We suppose that the first solution corresponds, approximately, to the present-day universe and the second one to the beginning of the inflation. Hence, our purpose will be to find a natural interpolation between them.

Let us consider the initial phase of inflation, where the CC plays no much role. The equation for the (00) component, equivalent to the Eq. (1.4), has been completely studied by Starobinsky [5]. The phase portrait of the theory may look quite different depending on the sign of the coefficient c [11]. The inflationary solution with H_S is stable for $c>0$ and unstable for $c<0$. The phase diagram for $c>0$ is presented at the Fig. 1. It is easy to see that there is only one (inflationary) attractor, therefore stable inflation does not depend on the initial conditions (except the need to start from the homogeneous and isotropic metric).

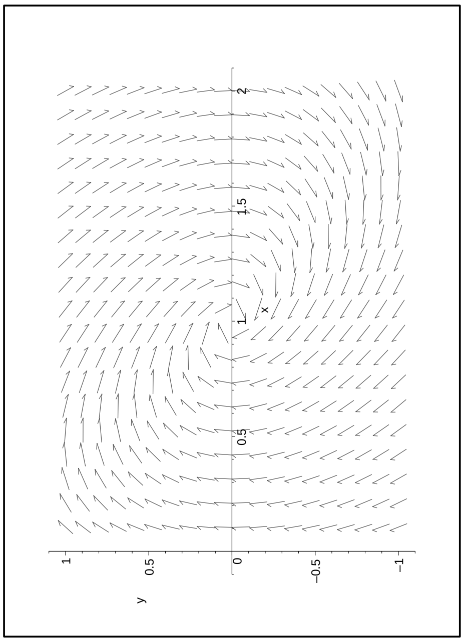

Figure 1: *The phase diagram for the stable version of the Starobinsky model.*

In the unstable case the phase diagram can be found in [5]. There are several distinct attractors, one of which corresponds to the FRW evolution [5] and others to the unphysical, run-away type solutions [13]. The original Starobinsky model deals only with the unstable solution and implies that the initial conditions must be chosen in a special way: 1) very close to the exact exponential solution (1.5), such that the inflation lasts long enough. 2) this choice must provide that, after the inflationary phase ends, the Universe will approach the attractor corresponding to the FRW solution. All the matter content of the Universe is created after the inflation ends through the decay of the massive degree of freedom induced by anomaly [5, 6]. Unfortunately, despite this model is based on the QFT results and does not need a special inflaton field, the amount of the fine-tuning for the initial conditions is at least the same as for the inflaton-based models.

In the recent works [14, 15, 11] we have developed an alternative version of the Starobinsky model, which does not require a fine-tuning for the initial data, for it interpolates, naturally, between stable and unstable regimes. In the rest of this article, we shall present a brief exposition of our model.

2. Inflation and SUSY, simple tests

First of all, we rewrite the condition of stability $c > 0$ in terms of the field content of the underlying QFT. We assume that the theory includes N_0 scalars, $N_{\frac{1}{2}}$ fermions and N_1 vectors. The numbers N_0, $N_{\frac{1}{2}}$, N_1 reflect a particle content of the theory, and have nothing to do with the real matter which might fill the Universe. Using a standard result (1.2), we obtain [13]

$$c > 0 \quad \Longleftrightarrow \quad N_1 < \frac{1}{3}\,N_{1/2} + \frac{1}{18}\,N_0\,. \tag{2.1}$$

The last inequality does not hold for the Standard Model, where $N_{1,\frac{1}{2},0} = (12, 24, 4)$. However, it is satisfied for its minimal supersymmetric extension (MSSM) with $N_{1,\frac{1}{2},0} = (12, 32, 104)$. The same is true for any realistic supersymmetric model, because the supersymmetrization procedure implies introducing fermion and scalar superpartners (sparticles) while the fundamental interactions (corresponding to the content of vector fields) are kept the same. We can see that the transition between stable and unstable inflation can be associated with the SUSY breaking. Usually, the SUSY breaking implies the special form of the mass spectrum, when all sparticles are very heavy compared to observable particles (soft breaking). Therefore it is clear that inflation becomes unstable when its energy scale becomes less than the masses of the most of the sparticles and these sparticles decouple.

All the time we will be concerned by the Feynman diagrams including the loops of the matter fields with external gravitational tails. For this reason, the typical energy for us is the energy of gravitons. In the cosmological setting, we shall associate it with the magnitude of the Hubble parameter H. Let us introduce the notation M_* for the energy scale where the inequality (2.1) changes its sign to the opposite. The anomaly-induced inflation model assumes that the value of H is decreasing with time $\dot{H} < 0$ and that H_S is just an initial value of H. The stable inflation becomes unstable at the instant t_f which is defined as a solution of the equation $H(t_f) = M_*$.

It is worth mentioning two relevant results of QFT in curved space-time. 1) The decoupling of the loops of massive fields in curved space-time really takes place [16, 17]. In particular, one can observe the smooth and monotone evolution of the coefficient c with scale and also the change of its sign from positive to negative due to the decoupling of the sparticles [17]. 2) The coefficient c in Eq. (1.2) contains an arbitrariness related, primarily, with the possibility to add $\int \sqrt{-g}R^2$-term to the classical action of vacuum. However, this arbitrariness does not contradict our version of the anomaly-induced inflation (see [18] for the details) because it can be always fixed by the renormalization condition. It is important that the ambiguity does not impose new constraints on the model and does not require a fine-tuning in the mentioned renormalization condition.

Before we proceed with inflation, let us perform two simple tests of the model.

First test. Consider the late FRW Universe with $k = \Lambda = 0$: The typical value of the Hubble parameter is $H_0 \sim 10^{-42}\,GeV$. Then, all massive particles decouple and the unique contribution to the anomaly comes from photon $N_{1,\frac{1}{2},0} = (1,0,0)$. It is easy to see that $c < 0$ and the "fast" inflation is unstable. Consider $a(t) \sim t^{2/3}$ in the equation (1.4) without CC and inserting the dust-like source $\rho_m/(a^3\, c)$ in the $r.h.s$

$$\frac{\dddot{a}}{a} + \frac{3\,\dot{a}\dddot{a}}{a^2} + \frac{\ddot{a}^2}{a^2} - \left(5 + \frac{4b}{c}\right)\frac{\ddot{a}\dot{a}^2}{a^3} - 2k\left(1 + \frac{2b}{c}\right)\frac{\ddot{a}}{a^3} = \frac{M_P^2}{8\pi c}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) - \frac{\rho_m}{ca^3}\,. \quad (2.2)$$

For the large values of time the "classical" (Einstein and the dust sourse) terms in the $r.h.s$ behave like $1/t^2$, while the fourth-derivative quantum corrections in the $l.h.s$ behave as $1/t^4$. Then, the quantum corrections are becoming irrelevant at $t \to \infty$.

Second test. Consider the same physical situation as above but this time without matter and with a small positive CC. In this case, according to (1.8), $H = H_c = \sqrt{\Lambda/3}$. We will now check whether this solution is stable with respect to perturbations of H. Consider $H \to H_c + \text{const} \times e^{\lambda t}$. The characteristic equation for λ has the form

$$\lambda^3 + 7H_c\lambda^2 + \left[\frac{(3c-b)4{H_c}^2}{c} - \frac{M_P^2}{8\pi c}\right]\lambda - \frac{32\pi b{H_c}^3 + M_P^2 H_c}{2\pi c} = 0\,.$$

It is remarkable that all the solutions for the positive CC have negative real part

$$\lambda_1 = -4H_c\,, \qquad \lambda_{2/3} = -\frac{3}{2}\,H_c \pm \frac{M_P}{\sqrt{8\pi|c|}}\,i\,,$$

while for the zero CC these solutions have zero real part and one would need to study the nonlinear approximation. Thus, $\Lambda > 0$ stabilizes our universe with respect to the dangerous quantum corrections.

3. Vacuum effects of massive fields

At that point we can conclude that our choice of the overall $\int \sqrt{-g}R^2$-term does not lead to problems in the IR regime and can safely proceed in the study of the UV energies, where we expect to meet natural inflation due to the quantum effects of matter fields. The inflation starts in the stable phase because the particle content $(N_0, N_{1/2}, N_1)$ is supersymmetric, and then becomes unstable due to the SUSY breaking and the decoupling of the sparticles. The above story looks very appealing because it does not involve any sort of fine-tuning, links inflation with SUSY and also links the SUSY breaking with the graceful exit. However, until now there was an obvious loophole in this story. We are considering the inflation derived from anomaly-induced action, that is the action that results from the quantum effects of massless conformal fields. At the same time, in order to use the notion of decoupling one has to evaluate the effects of massive fields. Furthermore we expect that, for some reason, the value of the Hubble parameter H will decrease during inflation. But this would never happen if we have only massless fields, for in this case the inflation is exponential and $H = H_S$ is a constant. Hence, our main hope is that taking masses of the fields into account we will really see the inflation slowing down.

In general, the problem of deriving the effective action of vacuum for the massive fields is not solved yet. The existing regular methods, like covariant Schwinger-DeWitt method, correspond to the expansion in the series in curvatures and their derivatives, divided by the corresponding powers of the particle masses. Therefore, these methods are efficient only for the limit of large masses or, contrary to that, in the massless limit where the effective action can be obtained through integrating the conformal anomaly. In our case, the effective action of vacuum should be calculated in the small-mass limit where the mentioned regular methods are not applicable. That is why the calculations Anzats for this case has been developed in [15] and generalized in [11]. Let us discuss it here in some details.

The idea is very simple: we formulate massive fields as massless and conformal, by introducing a new scalar field χ and a new massive parameter M. The conformization of the Einstein-Hilbert action has been known for a time [19] and for the massive matter fields it was known at the level of a dilatation symmetry within the cosmon model [20]. The conformal symmetry involving χ is reflected by the new Noether identity. This identity is anomalous, and integrating anomaly one arrives at the effective action. After that the new degree of freedom χ is frozen and we arrive at the effective action of massive fields.

The first step is to introduce the conformal representation of the massive fields. This can be achieved by replacing

$$m_{s,f} \to \frac{m_{s,f}}{M}\,\chi\,, \qquad \frac{1}{16\pi G}\,R \to \frac{M_P^2}{16\pi M^2}\,\left[\,R\chi^2 + 6\,(\partial\chi)^2\,\right]\,, \qquad \Lambda \to \frac{\Lambda}{M^2}\,\chi^2\,, \quad (3.1)$$

where $m_{s,f}$ are scalar and fermion masses.

The divergences of the theory in the conformal representation have the form

$$\Gamma^{(1)}_{div} = \frac{\mu^{n-4}}{4-n}\int d^n x\sqrt{|g|}\Big\{wC^2 + bE + c\nabla^2 R + \frac{\tilde{f} M_P^2}{4\pi M^2}[R\chi^2 + 6(\partial\chi)^2] + \frac{\tilde{g} M_P^4 \chi^4}{4\pi M^4}\Big\}, \tag{3.2}$$

where

$$\tilde{f} = \frac{1}{3\pi}\sum_f \frac{N_f m_f^2}{M_P^2}\,, \qquad \text{and} \qquad \tilde{g} = \frac{1}{4\pi}\sum_s \frac{N_s m_s^4}{M_P^2\Lambda} - \frac{1}{\pi}\sum_f \frac{N_f m_f^4}{M_P^2\Lambda}\,. \tag{3.3}$$

Here the sums are taken over all species of fermions and scalars with masses m_f, m_s and multiplicities N_f, N_s.

The classical Noether identity for the vacuum part of the effective action has the form

$$\mathcal{T} \,=\, -\,\frac{2}{\sqrt{-g}}g_{\mu\nu}\,\frac{\delta S_{vac}}{\delta g_{\mu\nu}} + \frac{1}{\sqrt{-g}}\,\chi\,\frac{\delta S_{vac}}{\delta \chi} \,=\, 0\,. \tag{3.4}$$

The identity above differs from the usual conformal Noether identity $< T^\mu_\mu >= 0$ due to the presence of χ. Correspondingly, the conformal anomaly means $< \mathcal{T} > \neq 0$ instead of usual $< T^\mu_\mu > \neq 0$. Simple calculations give [15, 11]

$$< \mathcal{T} > = -\Big\{\, wC^2 \,+\, bE \,+\, c\nabla^2 R \,+\, \frac{\tilde{f}\, M_P^2}{4\pi M^2}\,[R\chi^2 + 6(\partial\chi)^2] \,+\, \frac{\tilde{g}\, M_P^4}{4\pi M^4}\,\chi^4 \,\Big\}. \tag{3.5}$$

After deriving the anomaly-induced effective action and fixing the conformal unitary gauge $\chi = \bar{\chi}\, e^{-\sigma} = M$, the one-loop effective action becomes

$$\bar{\Gamma}^{(1)} = \int d^4x\sqrt{|\bar{g}|}\{w\bar{C}^2\sigma + b(\bar{E} - \frac{2}{3}\bar{\nabla}^2\bar{R})\sigma + 2b\sigma\bar{\Delta}\sigma\} - \frac{3c+2b}{36}\int d^4x\sqrt{-g}R^2$$

$$-\int d^4x\sqrt{|\bar{g}|}\Big\{\frac{e^{2\sigma}}{16\pi G}[\bar{R} + 6(\bar{\nabla}\sigma)^2][1-\tilde{f}\sigma] - \frac{\Lambda\, e^{4\sigma}}{8\pi G}\,[1-\tilde{g}\sigma]\Big\} + S_c[g_{\mu\nu}, M]\,, \tag{3.6}$$

where $\sigma = \ln a$ and $S_c[g_{\mu\nu}, M]$ is the unknown functional which is a constant of integration for the anomaly-induced effective action. In the case of massive field this functional is not conformal invariant (it would be if we do not replace $\chi \to M$), and therefore the formula above is just an approximation. The comparison with the renormalization group corrected classical action of vacuum shows that (3.6) is direct generalization of it, with the usual scaling parameter t (see, e.g. [10]) substituted by the local quantity σ. Therefore, (3.6) must be a reliable approximation for the small-mass limit, exactly in the region where the Schwinger-DeWitt expansion is not efficient.

According to the Eq. (3.6), the leading effect of the particle masses is that $1/G$ and the CC are replaced by the variable expressions

$$M_P^2 \;\to\; M_P^2\,(1 - \tilde{f}\ln a)\,, \tag{3.7}$$

$$\Lambda\, M_P^2 \;\to\; \Lambda\, M_P^2\,(1 - \tilde{g}\ln a)\,. \tag{3.8}$$

The last formulas show that, in principle, the effect of masses is slowly accumulating when the inflation goes on. The reason is that, the dependence is logarithmic and moreover the quantities (3.3) are very small for any GUT, and incredibly small for, e.g. MSSM.

The equation for the conformal factor following from the action (3.6) has the form of (1.4) with the replacement (3.7), plus some non-essential terms [11] with an additional factor of $\tilde{f}$. Therefore, we can expect that the procedure (3.7) should also provide an approximate solution on the basis of the exact stable inflationary solution (1.8). There is, however, a strong constraint related to the value of $\tilde{g}$. At the beginning of inflation we assume a small $\Lambda \ll M_P^2$, that is why the solution (1.8) does not depend on CC. But, when the inflation evolves, the Hubble parameter (1.8) will decrease according to (3.7) and the absolute value of the CC will increase very fast according to (3.8). Then, instead of the graceful exit to the approximate FRW with the small value of the CC, the universe will end up with the new phase of inflation, driven by the $\Lambda M_P^2 \tilde{g} \sigma$ term. Indeed, the region when this terms becomes large, is close to the limit of validity of the approximation behind (3.6), but in order to perform the preliminary analysis it is better to impose a constraint on the particle spectrum of the SUSY model and request that the β-function for the CC equals zero. Then, $\tilde{g} = 0$ too, and the analysis gets simplified.

The relation (3.7) can be easily rewritten as a differential equation for the conformal factor $H = \dot{\sigma} = H_S \cdot (1 - \tilde{f}\sigma)$, and the last can be solved immediately to give the following approximate analytical solution

$$\sigma(t) \,=\, H_0\, t \,-\, \frac{H_0^2}{4}\, \tilde{f}\, t^2 \,. \qquad (3.9)$$

It is interesting that the numerical analysis confirms the parabolic dependence (3.9) with enormous precision [11].

The relation (3.9) can be used to evaluate the total number of the inflationary e-folds for different models of the SUSY breaking. The first option is MSSM with the value $M_* \propto 1\,TeV$. It is easy to see that in this case $\tilde{f} \propto (M_*/M_P)^2 = 10^{-32}$ and therefore the total amount of the e-folds is 10^{32}. The expected temperature of the Universe after the end of inflation can be evaluated from Einstein equation in a usual way $T \propto \sqrt{M_*\, M_P} = 10^{11}\, GeV$, which is a standard estimate for the inflaton-based models. Alternatively, one may consider the SUSY breaking at the GUT scale. Suppose $M_* \propto 10^{14}\, GeV$. Then the total amount of e-folds is about 10^{10} and the expected temperature after the end of inflation is high $T \propto 10^{16}\, GeV$. In this case the inflation does not solve the monopole problem of GUT's. Hence, the anomaly-induced inflation really favors low-energy SUSY. Indeed, the intermediate versions (like, e.g. the Pati-Salam model) with the SUSY breaking at $10^{10}\, GeV$ are also possible. In this case we obtain $T \propto 10^{14}\, GeV$ which is better with respect to the monopole problem.

4. Problems of stability

The stability of the inflationary solution from the initial stage until the graceful exit represents one more consistency test of the model. At the beginning the role of the masses of the quantum fields is negligible and the criterion of stability is given by (2.1). Consider the later phase of inflation, when the quantum effects of massive fields temper exponential behavior. In this case we can use the approximate analytic method and also numerical simulations. The results of both methods are the same [11]. Let us briefly describe the analytic method. The stability or instability with respect to the small perturbations depends on the behavior of $\sigma(t)$ at the relatively small intervals of time, where the Hubble parameter H can be treated as a constant. Of course, when we move from one such interval to another, H changes providing a source for the perturbations. The direct calculations give the following equation for the perturbations $\sigma \to \sigma + y(\tau)$, where we used "renormalized" time variable $\tau = t/H$, $\quad H = const$:

$$\ddddot{y} + 7\,\dddot{y} + 2\Big(6 - \frac{b}{c}\Big)\,\ddot{y} - \frac{8b}{c}\,\dot{y} - \frac{4b}{c}\,\tilde{f}\,y = 0 \tag{4.1}$$

At this point we assumed, as before, a relatively small value of the cosmological constant. The last equation has a very special form, because all the coefficients are constants and all but the last have the magnitude of the order one. The last coefficient is extremely small because of the factor $\tilde{f} \ll 10^{-9}$. The stability of equation with constant coefficients may be explored, e.g. using the Routh-Hurwitz (RH) conditions. A priory the RH determinants may have an arbitrary sign, but in our case they all turn out to be positive, such that the stability of the solution for tempered inflation (with respect to the perturbations of the ginal region $H \approx M_*$.

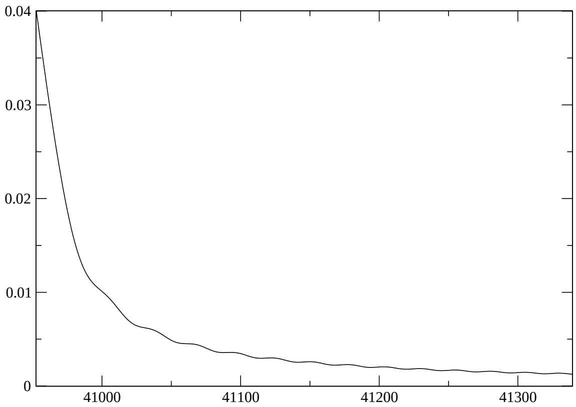

Figure 2: *Oscillations of $H(t)$ at the last stage of the stable inflation. Illustrative plot for $\tilde{f} = 10^{-5}$.*

The numerical analysis shows that, at the final stage of the stable anomaly-induced inflation, when the value of Hubble parameter is approaching M_*, this parameter starts to oscillate (see Fig. 2). Indeed, these oscillations are due to the perturbations introduced by the change of one "constant" value of H to another.

The last testing of the model which has been performed so far is the stability with respect to tensor perturbations of the metric. In the covariant formalism (see, e.g., [21]) the evolution of the tensor degree of freedom is described by the coordinate-dependent scalar factor $h(t,\vec{r})$ of the tensor mode. The dynamical equation for $h(t)$ is very complicated [22, 13], even for the theory of massless fields. Moreover, this equation contains an ambiguity due to the conformal functional $S_c[g_{\mu\nu}]$ [9, 13]. One can partially fix this ambiguity by choosing the proper vacuum for the perturbations [13]. As a result we meet an almost flat spectrum of the perturbations, however their amplitude may increase very fast. At the same time the amplification of the amplitudes is performing much slower than the expansion of the conformal factor. As a result the total metric becomes more and more homogeneous and isotropic. This is a situation at the initial stage of inflation.

At the last stage, e.g. in the last 65 e-folds, the equation for $h(t)$ is greatly simplified due to the enormous number of the total e-folds between the beginning and the end of inflation. The typical value for σ depends on the model but, as we discussed above, it varies between 10^{10} and 10^{32}. In the last 65 inflationary e-folds the σ itself may be treated as a big number. This feature greatly simplifies the equation for $h \equiv h(t, \vec{x})$, which can be presented as follows

$$b_0 \ddddot{h} + b_1 \dddot{h} + b_2 \ddot{h} + b_3 \dot{h} + b_4 h +$$

$$+ n_1 e^{-2\sigma} \nabla^2 \dot{h} + n_2 e^{-2\sigma} \nabla^2 \ddot{h} + n_3 e^{-4\sigma} \nabla^4 h \;=\; 0\,, \tag{4.2}$$

where

$$b_0 = b_0(t) = a_1 + w \cdot \sigma(t)\,, \qquad b_1 = 6H\,b_0 + 2wH\,,$$
$$b_2 = 11H^2\,b_0 + H^2(c - b/2 + 7w)\,,$$
$$b_3 = 6H^3\,b_0 + H^3(3c - 3b/2 + 5w)\,, \qquad b_4 = -12H^4 b\,, \tag{4.3}$$

$$n_1 = -2H\,b_0\,, \qquad n_2 = -2\,b_0\,, \qquad n_3 = \,b_0\,. \tag{4.4}$$

It is worth noticing that in the general case, without the approximation of constant H and without treating σ as a big number, the equation for h is much more complicated [22, 13, 23]. But in the physical situation of interest the equation can be simplified even further. The terms with space derivatives $\nabla h(t, \vec{r})$ are suppressed by the factors of $\exp(-2\sigma)$ and therefore are negligible. Furthermore, we can divide the equation by σ and see that all the coefficients become constants with accuracy of $1/\sigma$. In particular, in the last terms we can safely replace $1/\sigma$ by $1/\sigma_f = \tilde{f}$. This value $\sigma_f = \tilde{f}^{-1}$ corresponds to the point of transition from stable to unstable inflation. The most important difference is that, in the limit of large σ we do not meet an arbitrariness related to the conformal functional $S_c[g_{\mu\nu}]$ and to the choice of the classical action of vacuum. In fact, the equation for $h(t, \vec{r})$ is completely defined by the universal β-functions w, b, c for the vacuum parameters. In particular, the difference between the equations of [22] and [13] (it is due to the distinct choices of $S_c[g_{\mu\nu}]$) disappears in this approximation.

The remaining equation for $h(t)$ has the form

$$\ddddot{h} + 6\,\dddot{h} + 11\,\ddot{h} + 6\,\dot{h} - \frac{12b}{w\sigma_f}\,h \;=\; 0\,. \tag{4.5}$$

It is remarkable that the general structure of the Eq. (4.5) is quite similar to the one of the Eq. (4.1) for the perturbations of $\sigma(t)$. But it is even more remarkable that the solution of (4.5) does not have growing modes. One of the roots of the characteristic equation has the magnitude of the order of $\tilde{f}$ and others of the order of one, but all of them have negative real parts. For the physically reasonable choice of the initial data the amplitude of the perturbations almost remains constant. The significant amplification of the tensor perturbations takes place only for the waves with the energies close to the Planck one, where all our semiclassical approach is not consistent.

We conclude that the stability of the inflationary solution with respect to the perturbations of the conformal factor and the tensor mode of the metric holds from the initial stage (when the quantum fields may be approximately considered massless) until the scale M_*, when most of the sparticles decouple and the inflation becomes unstable[1]. Indeed, the stability of the model may be jeopardized in the transition period, where we expect to meet rapid oscillations of the conformal factor which should lead to reheating. The main line in the further development of the model must be related with the quantitative model description of the decoupling and the transition period.

5. Concluding remarks

We have briefly described the model of inflation based on the effective action of vacuum. This effective action follows from the quantum corrections of the massive fields. Indeed, our approach to the effective action of the massive fields is based on the special Anzats [15, 11] which is reliable at the beginning of inflation, when masses are small compared to H. Due to the unbroken supersymmetry, at this scale the coefficient $\tilde{g} = 0$ and the CC term is irrelevant. In this case the formula (3.9) describes the evolution of the Universe, and the Hubble parameter is decreasing linearly with time. If we continue this evolution until the point $H = M_*$, the sparticles should decouple and the universe starts a new unstable phase of the evolution. We assume that this stage ends in the FRW phase or in the present-day state with the small positive CC (1.8). However, it is easy to see an inconsistency in the consideration presented above. In fact, our Anzats becomes not reliable when the value of the Hubble parameter H is approaching $H = M_*$, because this is the scale comparable to the masses of many sparticles. Similarly, at this scale the standard curvature expansions (e.g. Schwinger-DeWitt) are also not applicable, because the masses of the particles and the Hubble parameter H are of the same order of magnitude in this region. All in all, what we have at the moment is the description of the asymptotic regimes. It is not clear, however, how to achieve the qualitative description of the region $H \propto M_*$, which is indeed the most interesting phenomenologically. If we get such a description, this can open the way to the investigation of the reheating and density perturbations (which were actually considered in [24] for the original Starobinsky model).

Finally, despite the anomaly-induced inflation is not as developed as inflaton models, it represents an attractive alternative to them. In particular, even at the present state of knowledge we have some obvious advantages, such as the possibility to avoid a standard fine-tuning in the choice of initial data, a good chance to achieve a natural graceful exit and also to control the amplitude of the gravitational perturbations. Only further theoretical and phenomenological study of this model and comparison with experimental/observational data may eventually confirm or rule out this model of inflation.

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[1] The credibility of the approximation of $H = \text{const}$ for the tensor perturbations is not a trivial question, but its verification requires significant new calculations and therefore will be reported elsewhere.

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