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Lepton MHD and magnetic field generation in the SM

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Abstract: We derive a total set of MHD equations in SM describing evolution of a dense plasma with neutrinos. First this is done for a hot pair plasma consisting from electrons and positrons, neutrinos and antineutrinos of all flavors in an isotropic medium like the early unverse plasma at the lepton stage. Then we find how axial vector currents violating parity in SM contribute to MHD for a slightly polarized (anisotropic) plasma where a new mechanism for the amplification of mean magnetic fields arises du e to the collective neutrino-plasma interactions instead of assumed asymmetry of fluid velocity vortices leading to the same effect of α^2-dynamo.

Dedicated to my granddaughter Kat

1. Introduction

It is well-known that the magnetohydrodynamic (MHD) or macroscopic description of a plasma is less detailed and much simpler than the kinetic one which corresponds to the microscopic description of the plasma evolution and therefore it is a more complicated approach.

The MHD equation system allows, in particular, to derive the Faradey (induction) equation for magnetic fields in the standard model of electroweak interactions (SM) including weak interaction terms. The main goal of this work is the detailed derivation of Faradey equation in SM that is important for the generation of primordial magnetic fields in cosmology and magnetic fields in a supernova protostar where powerful neutrino fluxes interact with the dense plasma.

We derive the full set of MHD equations using the standard method of moments [1] for Relativistic Kinetic Equations (RKE) written in the collisionless (Vlasov) approximation. There are other ways to derive MHD, e.g. using the Lagrangian formalism for relativistic multicomponent fluid [2] while we prefer the method of the quantum RKE for lepton plasma

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in SM [3, 4] that is more approppriate to describe both classical and spin properties of polarized plasmas permiated by an external magnetic field.

Note that neutrino RKE is a useful tool to describe many phenomena in astrophysics and cosmology. In particular, neutrinos play the most important role for a supernova (SN) burst or in the lepton asymmetry formation before the primordial nucleosynthesis in the early universe. The usual motivation to use the RKE approach for neutrino propagation in a dense matter is stipulated by the account of neutrino collisions: within a SN neutrinosphere or in the hot lepton plasma of the early universe before neutrino decoupling.

However, in addition to collision integrals there are self-consistent weak interaction terms in the neutrino RKE [3] that are *linear over the Fermi constant* $\sim G_F$ (see below section II) and analogous to the Lorentz force terms for charge particles in the standard Boltzman RKE which in turn are linear over the electric charge $\sim q$ ($q = -\mid e \mid$ for electrons).

In the standard kinetics these self-consistent electromagnetic fields are well-known to play a very crucial role. In collisionless, or Vlasov approximation, such kinetic equations describe, e.g. thermonuclear plasmas in laboratory and stars for which an energy exchange between electromagnetic waves (eigen modes) and charged particles proceeds faster than via the direct particle collisions with all following issues in collisionless plasma: instabilities, heating, etc.

One expects that the self-consistent weak interaction ($\sim G_F$) could lead for neutrinos to some analogous collective interaction effects, e.g. to neutrino driven streaming instability of plasma waves in an isotropic plasma [5] or instability of spin waves in a polarized medium [4], and to the generation of magnetic fields in hot plasma of early universe [2, 6].

2. Lepton MHD in Standard Model (SM) of electroweak interactions

In this Section we derive MHD equations using the method [1] of moments of kinetic equations , or integrating RKE's over momenta, $\int d^3p(...)$, $\int d^3p\mathbf{p} \times (...)$, $\int d^3p\varepsilon_p \times (...)$. We start from the simple case of unpolarized (isotropic) plasma and in the next subsection we derive MHD using RKE's in a magnetized plasma [4].

2.1 Lepton MHD in unpolarized medium

In an isotropic unpolarized plasma the collisionless RKE for electrons and positrons ($e = \pm \mid e \mid$ with upper sign for positron) derived in SM including weak forces takes the form [5]

$$\frac{\partial f^{(\pm)}(\mathbf{p},\mathbf{x},t)}{\partial t} + \mathbf{v}\frac{\partial f^{(\pm)}(\mathbf{p},\mathbf{x},t)}{\partial \mathbf{x}} \pm \mid e \mid (\mathbf{E}(\mathbf{x},t) + [\mathbf{v}\times\mathbf{B}(\mathbf{x},t)])\frac{\partial f^{(\pm)}(\mathbf{p},\mathbf{x},t)}{\partial \mathbf{p}} \pm$$
$$\mp G_F\sqrt{2}\sum_a c_V^{(a)}\Big[-\nabla[n_{\nu_a}(\mathbf{x},t) - n_{\bar{\nu}_a}(\mathbf{x},t)] - \frac{\partial \mathbf{j}_{\nu_a}(\mathbf{x},t) - \mathbf{j}_{\bar{\nu}_a}(\mathbf{x},t)}{\partial t} +$$
$$+\mathbf{v}\times\nabla\times(\mathbf{j}_{\nu_a}(\mathbf{x},t) - \mathbf{j}_{\bar{\nu}_a}(\mathbf{x},t))\Big]\frac{\partial f^{(\pm)}(\mathbf{p},\mathbf{x},t)}{\partial \mathbf{p}} = 0 \ , \qquad (2.1)$$

where $j_\mu^{(\nu_a,\bar{\nu}_a)}(\mathbf{x},t) = (n_{\nu_a,\bar{\nu}_a}(\mathbf{x},t),\mathbf{j}_{\nu_a,\bar{\nu}_a}(\mathbf{x},t)) = \int d^3k(k_\mu/E_k)f^{(\nu,\bar{\nu})}(\mathbf{k},\mathbf{x},t)/(2\pi)^3$ is the neutrino (antineutrino) four-current density; $c_V^{(a)} = 2\xi \pm 0.5$ is the vector coupling for $a = e,\mu,\tau$ neutrinos with the upper sign for the electron ones $a = e$.

We complete the system by the neutrino (antineutrino) collisionless RKE's:

$$\frac{\partial f^{(\nu_a)}(\mathbf{k},\mathbf{x},t)}{\partial t} + \mathbf{n}\frac{\partial f^{(\nu_a)}(\mathbf{k},\mathbf{x},t)}{\partial \mathbf{x}} + G_F\sqrt{2}c_V^{(a)}\Big[-\nabla\left(n^{(e)}(\mathbf{x},t) - n^{(\bar{e})}(\mathbf{x},t)\right) - $$
$$-\frac{\partial[\mathbf{j}^{(e)}(\mathbf{x},t) - \mathbf{j}^{(\bar{e})}(\mathbf{x},t)]}{\partial t} + \mathbf{n}\times\nabla\times\left(\mathbf{j}^{(e)}(\mathbf{x},t) - \mathbf{j}^{(\bar{e})}(\mathbf{x},t)\right)\Big]\frac{\partial f^{(\nu_a)}(\mathbf{k},\mathbf{x},t)}{\partial \mathbf{k}} = 0\ ,$$
$$\frac{\partial f^{(\bar{\nu}_a)}(\mathbf{k},\mathbf{x},t)}{\partial t} + \mathbf{n}\frac{\partial f^{(\bar{\nu}_a)}(\mathbf{k},\mathbf{x},t)}{\partial \mathbf{x}} - G_F\sqrt{2}c_V^{(a)}\Big[-\nabla\left(n^{(e)}(\mathbf{x},t) - n^{(\bar{e})}(\mathbf{x},t)\right) - $$
$$-\frac{\partial[\mathbf{j}^{(e)}(\mathbf{x},t) - \mathbf{j}^{(\bar{e})}(\mathbf{x},t)]}{\partial t} + \mathbf{n}\times\nabla\times\left(\mathbf{j}^{(e)}(\mathbf{x},t) - \mathbf{j}^{(\bar{e})}(\mathbf{x},t)\right)\Big]\frac{\partial f^{(\bar{\nu}_a)}(\mathbf{k},\mathbf{x},t)}{\partial \mathbf{k}} = 0\ , \tag{2.2}$$

where $j_\mu^{(e,\bar{e})}(\mathbf{x},t) = (n_{e,\bar{e}}(\mathbf{x},t),\mathbf{j}_{e,\bar{e}}(\mathbf{x},t)) = \int d^3p(p_\mu/E_p)f^{(e,\bar{e})}(\mathbf{p},\mathbf{x},t)/(2\pi)^3$ is the electron (positron) four-current density.

In 5-moment approximation of ideal hydrodynamics we neglect collisions and hence omit viscosity, heat flux terms while retaining self-consistent electroweak interactions between leptons. Thus we have to derive the particle density conservation (continuity) equation, the motion (Euler) equation (momentum conservation) and energy conservation equation.

Continuity equations

The weak interaction forces above have the Lorentz structure or enter RKE's as the electromagnetic Lorentz force in the third term of Eq. (2.1), $F_{j\mu}^{weak}(\mathbf{x},t)\times(p^\mu/\varepsilon_p)\partial f^{(a)}(\mathbf{p},\mathbf{x},t)/\partial p_j$. Hence they do not contribute to the continuity equations [5] which take the standard form $\partial j_\mu^{(a)}/\partial x_\mu = 0$ after integration of RKE's (2.1) and (2.2) over momenta d^3p, d^3k correspondingly, resulting in

$$\frac{\partial n_\pm(\mathbf{x},t)}{\partial t} + \frac{\partial[n_\pm(\mathbf{x},t))\mathbf{V}_\pm(\mathbf{x},t)]}{\partial \mathbf{x}} = 0\ , \tag{2.3}$$

for charged leptons and

$$\frac{\partial n_{\nu_a,\bar{\nu}_a}(\mathbf{x},t)}{\partial t} + \frac{\partial[n_{\nu_a,\bar{\nu}_a}(\mathbf{x},t))\mathbf{V}^{(\nu_a,\bar{\nu}_a)}(\mathbf{x},t))}{\partial \mathbf{x}} = 0\ , \tag{2.4}$$

for neutrinos (antineutrinos). Here $n_\pm = n'_\pm\gamma_\pm$, $n_{\nu_a,\bar{\nu}_a} = n'_{\nu_a,\bar{\nu}_a}\gamma_{\nu_a,\bar{\nu}_a}$ are the lepton densities in the laboratory reference frame. The four-currents $j_\mu^{(a)} = n'_aU_\mu^{(a)}$ are given by the Lorentz-invariant densities $n'_a = j_\mu^{(a)}U^{(a)\mu}$ where $U^{(a)\mu} = (\gamma_a,\gamma_a\mathbf{V}^{(a)})$ is the unit four-velocity of the plasma a-component, $U_\mu^{(a)}U^{(a)\mu} = 1$, $\gamma_a = (1 - V_a^2)^{-1/2}$, $a = \pm,\nu_a,\bar{\nu}_a$.

Considering the particular case of the hot pair plasma $T_- = T_+ = T \gg \mu$ where the fast $e^\pm\gamma$ interaction provides equilibrium leading to the zero chemical potentials $\mu_- = -\mu_+ = \mu = 0$ and introducing the small perturbations for comoving components $\mathbf{V}_\pm = \mathbf{V} + \delta\mathbf{V}_\pm$,

$\delta\mathbf{V}_\pm \ll \mathbf{V}$ that means $\gamma_- \approx \gamma_+ = \gamma$ we get the single continuity equation for the total charged lepton density $n' = n'_- + n'_+$ instead of the two ones in Eq. (2.3),

$$\frac{\partial\gamma n'(\mathbf{x},t)}{\partial t} + \frac{\partial[\gamma n'(\mathbf{x},t))\mathbf{V}(\mathbf{x},t)]}{\partial\mathbf{x}} = 0 \ . \tag{2.5}$$

Such hydrodynamical approximation means strong correlation in a dense plasma between opposite charges due to which the continuous lepton medium becomes *electroneutral* conducting liquid (electrons and positrons move with the same velocities as a whole) resulting in electric field vanishes while magnetic field exists (lepton MHD, see below Eq. (2.9)). Neglecting protons the electroneutrality condition means that the background densities $n'_{\pm 0}$ entering the total ones $n'_\pm = n'_{\pm 0} + \delta n'_\pm$ obey the equality

$$n'_{-0} = n'_{+0} = n_{0e} \ ,$$

where in the hot plasma $n_{0e} = 0.183T^3$.

Note that the fluid (mean) velocity $\mathbf{V}$ differs from the microscopic $\mathbf{n}$ that enters the RKE of massless particles (2.2), $\mid \mathbf{n} \mid = 1$. Of course, the Lorentz transformation with the unit vector $U_\mu^{(\nu_a)} = (\gamma_{\nu_a}, \gamma_{\nu_a}\mathbf{V}_{\nu_a})$ does not change the value of the microscopic four-momentum $k_\mu = (E_k, \mathbf{k})$, $E_k^2 - k^2 = 0$.

Motion equations

Multiplying the RKE (2.1) by the momentum $\mathbf{p}$ and integrating it over d^3p with the use of the standard definitions of the fluid velocity
$\mathbf{V}_\pm(\mathbf{x},t) = n_\pm^{-1}\int d^3p\mathbf{v}f^{(\pm)}(\mathbf{p},\mathbf{x},t)/(2\pi)^3$, the positron (electron) density
$n_\pm = \int d^3pf^{(\pm)}(\mathbf{p},\mathbf{x},t)/(2\pi)^3$, and the generalized momentum of the lepton fluid $\mathbf{P}_\pm = w_\pm\gamma_\pm\mathbf{V}_\pm = n_\pm^{-1}\int d^3p\mathbf{p}f^{(\pm)}(\mathbf{p},\mathbf{x},t)/(2\pi)^3$, one obtains the Euler equation that coincides with Eq. (4.6) derived in [2] using another (relativistic Lagrangian) approach for multi-component fluid,

$$\begin{aligned}(\partial_t + \mathbf{V}_\pm\cdot\nabla)\,\mathbf{P}_\pm &= -\frac{\nabla p'_\pm}{n_\pm} \pm \mid e \mid (\mathbf{E} + [\mathbf{V}_\pm\times\mathbf{B}]) + \\ &\mp G_F\sqrt{2}\sum_{\nu_a} c_V^a\left[-\nabla\delta n_{\nu_a}(\mathbf{x},t) - \frac{\partial\delta\mathbf{j}_{\nu_a}(\mathbf{x},t)}{\partial t} + \mathbf{V}_\pm\times\nabla\times\delta\mathbf{j}_{\nu_a}(\mathbf{x},t)\right] \ ,\end{aligned} \tag{2.6}$$

where $\delta n_{\nu_a} = n_{\nu_a} - n_{\bar{\nu}_a}$, $\delta\mathbf{j}_{\nu_a} = \mathbf{j}_{\nu_a} - \mathbf{j}_{\bar{\nu}_a}$ are the neutrino density and neutrino 3-current density asymmetries respectively; $w_\pm = e_\pm + p'_\pm/n'_\pm$ is the Lorentz-scalar enthalpy per one particle; $e_\pm$, $p'_\pm = n'_\pm T_\pm$ are the internal energy and the pressure correspondingly, $T_\pm$ is the Lorentz-invariant temperature. In particular, for the Jüttner equilibrium distribution $f_\pm^{eq}(p) = \exp[(\mu_\pm - p_\mu U^\mu)/T_\pm]$, where $\mu_\pm$ is the Lorentz-invariant chemical potential, the thermodynamical characteristics are also Lorentz-invariant,

$$w_\pm = m_e\frac{K_3(m_e/T_\pm)}{K_2(m_e/T_\pm)}, \quad e_\pm = w_\pm - T_\pm, \quad p'_\pm = 4\pi m_e^2T_\pm^2K_2(m_e/T_\pm)\exp(\mu_\pm/T_\pm) \ . \tag{2.7}$$

For equilibrium pair plasma $T_+ = T_- = T$ all these characteristics coincide, $w_+ = w_- = w_e$, $p'_+ = p'_- = p_e$, etc.

Summing Euler equations for electrons and positrons (2.6) one obtains the motion equation (electric field and neutrino density terms do not contribute)

$$\frac{\mathrm{d}(\mathbf{P}_+ + \mathbf{P}_-)}{\mathrm{dt}} = -\frac{\nabla(p'_+ + p'_-)}{\gamma n_{0e}} + \mid e \mid (\mathbf{V}_+ - \mathbf{V}_-)\times\mathbf{B} - \\ -G_F\sqrt{2}\sum_{\nu_a} c_V^{(a)}\left[\mathbf{V}_+ - \mathbf{V}_-\right)\times\nabla\times\delta\mathbf{j}_{\nu_a}(\mathbf{x},t)\right] \ , \tag{2.8}$$

where $\mathrm{d/dt} = \partial/\partial\mathrm{t} + \mathbf{V}\cdot\nabla$.

Then we use in (2.8) the Maxwell equation without displacement current ($\partial\mathbf{E}/\partial t$ is omitted in MHD), $\delta\mathbf{j}_e^{(em)} =\mid e\mid n_{0e}\gamma(\mathbf{V}_+ - \mathbf{V}_-) = 2\mid e\mid n_e\delta\mathbf{V} = \mathrm{rot}\ \mathbf{B}/4\pi$ where $n_e = \gamma n_{0e}$ is the plasma density in the laboratory reference frame. We put also the total pressure $p = p'_+ + p'_- = 2p_e$, the total enthalpy $w = w_+ + w_- = 2w_e$ introducing the total generalized momentum $\mathbf{P} = \mathbf{P}_+ + \mathbf{P}_- = w\gamma\mathbf{V}$.

Thus, we obtain finally the MHD motion equation for pairs generalized in SM with neutrinos,

$$\frac{\mathrm{d}\mathbf{P}}{\mathrm{dt}} = -\frac{\nabla p}{n_e} + \frac{\mathrm{rot}\mathbf{B}\times\mathbf{B}}{4\pi n_e} - \frac{G_F\sqrt{2}}{\mid e\mid 4\pi n_e}\sum_{\nu_a} c_V^{(a)}\left[\mathrm{rot}\mathbf{B}\times\nabla\times\delta\mathbf{j}_{\nu_a}(\mathbf{x},\mathrm{t})\right] \ , \tag{2.9}$$

The motion equations for neutrinos and antineutrinos are derived multiplying the RKE (2.2) by the momentum $\mathbf{k}$ and integrating over d^3k,

$$\frac{\mathrm{d}\mathbf{K}_{\nu_a}}{\mathrm{dt}} = -\frac{\nabla p'_{\nu_a}}{n_{\nu_a}} + \mathbf{f}_{\nu_a}, \\ \frac{\mathrm{d}\mathbf{K}_{\bar{\nu}_a}}{\mathrm{dt}} = -\frac{\nabla p'_{\bar{\nu}_a}}{n_{\bar{\nu}_a}} + \mathbf{f}_{\bar{\nu}_a}, \tag{2.10}$$

where the generalized momenta

$$\mathbf{K}_{\nu_a,\bar{\nu}_a} = \gamma_{\nu_a,\bar{\nu}_a} w_{\nu_a,\bar{\nu}_a}\mathbf{V}_{\nu_a,\bar{\nu}_a} = n^{-1}_{\nu_a,\bar{\nu}_a}\int d^3k\mathbf{k}f^{(\nu_a,\bar{\nu}_a)}(\mathbf{k},\mathbf{x},t)/(2\pi)^3 \ ,$$

are given by the Lorentz-invariant thermodynamical functions $w_{\nu_a,\bar{\nu}_a} = e_{\nu_a,\bar{\nu}_a} + p'_{\nu_a,\bar{\nu}_a}/n'_{\nu_a,\bar{\nu}_a}$; the weak forces $\mathbf{f}_\nu$ given by,

$$\mathbf{f}_{\nu_a} = +G_F\sqrt{2}c_V^{(a)}\Big[-\nabla\delta n^{(e)}(\mathbf{x},t) - \frac{\partial\delta\mathbf{j}^{(e)}(\mathbf{x},t)}{\partial t} + \mathbf{V}_{\nu_a}\times\nabla\times\delta\mathbf{j}^{(e)}(\mathbf{x},t)\Big], \\ \mathbf{f}_{\bar{\nu}_a} = -G_F\sqrt{2}c_V^{(a)}\Big[-\nabla\delta n^{(e)}(\mathbf{x},t) - \frac{\partial\delta\mathbf{j}^{(e)}(\mathbf{x},t)}{\partial t} + \mathbf{V}_{\bar{\nu}_a}\times\nabla\times\delta\mathbf{j}^{(e)}(\mathbf{x},t)\Big] \ , \tag{2.11}$$

have opposite signs and, in general, depend on *different* fluid velocities, $\mathbf{V}_{\nu_a}\neq\mathbf{V}_{\bar{\nu}_a}$. Here we input charged lepton density and 3-current density asymmetries, $\delta n^{(e)} = n_- - n_+$, $\delta\mathbf{j}^{(e)} = \mathbf{j}^{(e)} - \mathbf{j}^{(\bar{e})}$ which are small in hot plasma.

Since there are different fluid velocities as well as possible different thermodynamical functions, $w_{\nu_a} = e_{\nu_a} + T_{\nu_a} = 4T_{\nu_a}\neq w_{\bar{\nu}_a} = 4T_{\bar{\nu}_a}$ with the equation of state for massless neutrinos, $p_\nu = e_\nu/3$ (see (2.7) for massless particles $m_\nu = 0$), we consider different motion equations for neutrinos and antineutrinos (2.10). Note that the inequality for electron neutrino species, $T_{\nu_e} \neq T_{\bar{\nu}_e}$ can arise due to beta-processes and the CC-current interaction and leads to a temperature difference for electron and muon (tau) neutrino components. For the latter (ν_μ, $\bar{\nu}_\mu$) one expects same temperatures, however, we do not use this property to simplify the system (2.10).

Energy equations

Multiplying the RKE (2.1) by the energy E_p and integrating over d^3p one obtains the energy conservation law (upper sign for positrons)

$$\frac{\partial[\gamma_\pm^2 n'_\pm e_\pm + \gamma_\pm^2 \mathbf{V}_\pm^2 p'_\pm]}{\partial t} + \frac{\partial[\gamma_\pm^2 n'_\pm w_\pm \mathbf{V}_\pm]}{\partial \mathbf{x}} = \\ = \pm\gamma_\pm n'_\pm \mathbf{V}_\pm \cdot \left(\mid e \mid \mathbf{E} - \mathbf{f}_e^{weak}\right) \ , \qquad (2.12)$$

where the inner energy $e_\pm$, the enthalpy $w_\pm$, the pressure $p_\pm$ are given by Eq. (2.7); the weak force in the r.h.s. acting on charged leptons is given by

$$\mathbf{f}_e^{weak} = G_F\sqrt{2}\sum_{\nu_a} c_V^a \left[-\nabla\delta n_{\nu_a}(\mathbf{x},t) - \frac{\partial\delta\mathbf{j}_{\nu_a}(\mathbf{x},t)}{\partial t}\right] \ . \qquad (2.13)$$

Adding energy equations (2.12) and using the relation $\mathbf{E} = -\mathbf{V}\times\mathbf{B}$ that is valid for an ideal conducting medium one gets the MHD energy equation for pairs generalized here in SM including weak forces,

$$\frac{\partial[\gamma^2 n' e_e + \gamma^2\mathbf{V}^2 p]}{\partial t} + \frac{\partial[\gamma^2 n' w_e \mathbf{V}]}{\partial \mathbf{x}} = -\frac{(\text{rot }\mathbf{B})\cdot\left(\mid \mathrm{e} \mid [\mathbf{V}\times\mathbf{B}] + \mathbf{f}_\mathrm{e}^\mathrm{weak}\right)}{4\pi \mid e \mid} \ , \qquad (2.14)$$

where the force $\mathbf{f}_e^{weak}$ is given by (2.13), $n' = n'_+ + n'_-$, $\mathbf{V}_- = \mathbf{V}_+ \approx \mathbf{V}$, $\gamma_+ = \gamma_- \approx \gamma$ and meaning the equilibrium reached through the fast $e\gamma$-interaction, $T_+ = T_- = T$ we put $e_e = e_+ = e_-$, $w_e = w_+ = w_-$, $p = p'_+ + p'_-$ in the agreement with (2.7).

Analogously multiplying the neutrino (antineutrino) RKE (2.2) by the energy E_k and integrating over d^3k one obtains the energy equation (upper sign for neutrinos)

$$\frac{\partial[\gamma_{\nu_a,\bar{\nu}_a}^2 n'_{\nu_a,\bar{\nu}_a} e_{\nu_a,\bar{\nu}_a} + \gamma_{\nu_a,\bar{\nu}_a}^2 \mathbf{V}_{\nu_a,\bar{\nu}_a}^2 p'_{\nu_a,\bar{\nu}_a}]}{\partial t} + \frac{\partial[\gamma_{\nu_a,\bar{\nu}_a}^2 n'_{\nu_a,\bar{\nu}_a} w_{\nu_a,\bar{\nu}_a} \mathbf{V}_{\nu_a,\bar{\nu}_a}]}{\partial \mathbf{x}} = \\ = \pm\gamma_{\nu_a,\bar{\nu}_a} n'_{\nu_a,\bar{\nu}_a}(\mathbf{V}_{\nu_a,\bar{\nu}_a}\cdot\mathbf{f}_{\nu_a,\bar{\nu}_a}) \ , \qquad (2.15)$$

where the weak force acting on neutrinos from the pair plasma $\mathbf{f}_{\nu_a,\bar{\nu}_a}$ is given by Eq. (2.11).

The set of MHD equations: the continuity ones (2.4), (2.5), the motion ones (2.9), (2.10) and the energy ones, (2.14), (2.15) is completed by the Faradey equation for the magnetic field $\mathbf{B}$ generalized in SM due to weak interactions (see next section, Eq. (3.2)).

2.2 Lepton MHD in polarized medium

Analogously with the case of unpolarized medium we can derive MHD equations in the presence of a strong *large-scale* uniform magnetic field $\mathbf{B}_0$ which polarizes plasma populating partially the main Landau (non-degenerate) levels for free electrons and positrons.

Other levels being populated by leptons with opposite spin projections are degenerate (the factor Lande $g_e = 2$ doubles such states) and do not contribute to the medium polarization.

The lepton density at the main Landau level in *anisotropic medium* is given by

$$n_0^{(\pm)} = \int \frac{d^3p}{(2\pi)^3} S_0^{(\pm)}(\varepsilon_p) = \frac{|\, e \,|\, B_0}{2\pi^2} \int_0^\infty dp f_0^{(\pm)}(\varepsilon_p) \; , \tag{2.16}$$

where in the hot plasma $T \gg \mu$ one obtains $n_0^{(\pm)} \simeq |\, e \,|\, B_0 T \ln 2/2\pi^2$.

Now using the electron RKE Eq. (30) from [4] we can generalize the Euler equation for electrons and positrons (2.6) for the case of a polarized medium,

$$\begin{aligned}
&(\partial_t + \mathbf{V}_\pm \cdot \nabla)\,\mathbf{P}_\pm = -\frac{\nabla p'_\pm}{n_\pm} \pm |\, e \,|\, (\mathbf{E} + [\mathbf{V}_\pm \times \mathbf{B}]) + \\
&\mp G_F\sqrt{2} \sum_{\nu_a} c_V^a \left[-\nabla \delta n_{\nu_a}(\mathbf{x}, t) - \frac{\partial \delta \mathbf{j}_{\nu_a}(\mathbf{x}, t)}{\partial t} + \mathbf{V}_\pm \times \nabla \times \delta \mathbf{j}_{\nu_a}(\mathbf{x}, t) \right] \mp \\
&\mp \frac{G_F\sqrt{2}}{n_e} \sum_{\nu_a} c_A^{(a)} \left[n_0^{(\pm)} \hat{\mathbf{b}}^{(0)} \mathrm{div}\; \delta \mathbf{j}_{\nu_a}(\mathbf{x}, \mathrm{t}) - \mathrm{N}_0^{(\pm)} \nabla(\hat{\mathbf{b}}^{(0)} \cdot \delta \mathbf{j}_{\nu_a}(\mathbf{x}, \mathrm{t})) \right] \; ,
\end{aligned} \tag{2.17}$$

where in a non-relativistic (NR) plasma, the relativistic polarization density terms

$$N_0^{(\pm)} = \frac{n_0^\pm}{3} + \frac{|\, e \,|\, B_0 m_e}{18\pi^2} \int_0^\infty f_0^\pm(\varepsilon_p) dp \frac{\partial v(3 - v^2)}{\partial p}$$

coincide with the main Landau level contributions, $N_0^{(\pm)} \to n_0^{(\pm)}$, given by Eq. (2.16); $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}'$ is the total magnetic field.

Adding equations (2.17) we obtain finally the pair motion equation in polarized medium:

$$\begin{aligned}
&\frac{\mathrm{d}\mathbf{P}}{\mathrm{dt}} = -\frac{\nabla p}{n_e} + \frac{\mathrm{rot}\mathbf{B} \times \mathbf{B}}{4\pi n_e} - \frac{G_F\sqrt{2}}{|\, e \,|\, 4\pi n_e} \sum_{\nu_a} c_V^{(a)} \left[\mathrm{rot}\mathbf{B} \times \nabla \times \delta \mathbf{j}_{\nu_a}(\mathbf{x}, \mathrm{t}) \right] + \\
&+ \frac{G_F\sqrt{2}}{n_e} \sum_a c_A^{(a)} \Big[(n_0^{(-)} - n_0^{(+)}) \hat{\mathbf{b}}^{(0)} \mathrm{div}\; \delta \mathbf{j}_{\nu_a}(\mathbf{x}, \mathrm{t}) - \\
&- (N_0^{(-)} - N_0^{(+)}) \nabla(\hat{\mathbf{b}}^{(0)} \cdot \delta \mathbf{j}_{\nu_a}(\mathbf{x}, t)) \Big] \; .
\end{aligned} \tag{2.18}$$

Note that the polarization asymmetries $n_0^{(-)} - n_0^{(+)}$, $N_0^{(-)} - N_0^{(+)}$ are small in the hot relativistic plasma of early universe while in a degenerate electron gas of a magnetized supernova, $T \ll \mu$, these asymmetries can be large since $n_0^{(-)} = |\, e \,|\, B_0 \mu/2\pi^2 \gg n_0^{(+)} = (|\, e \,|\, B_0 T/2\pi^2) e^{-\mu/T}$.

Assuming $\mathbf{B}' \ll \mathbf{B}_0$ we can include the perturbative field $\mathbf{B}'(\mathbf{x}, t)$ into the polarization terms in the last lines of Eq. (2.18) with the change $\mathbf{B}_0 \to \mathbf{B}$, $\hat{\mathbf{b}}^{(0)} \to \hat{\mathbf{b}}$.

In general, one can consider the limit of strong magnetic fields (or diluted media) for which the main Landau level is populated only. E.g. a degenerate electron gas obeying the condition $eB \geq \mu^2/2$ would be fully polarized, or $n_e \approx n_0^{(-)}$ [7], that could lead to comparable contributions of pseudovector and vector terms in the pair motion equation (2.18).

The neutrino (antineutrino) motion equations take the form which is similar to Eq. (2.10) while in a polarized medium the vector force for neutrinos $\mathbf{f}_{\nu_a}$ (2.11) (and similarly $\mathbf{f}_{\bar{\nu}_a}$ for antineutrinos) is added with the additional axial vector force, $\mathbf{f}_{\nu_a} \to \mathbf{f}_{\nu_a} + \mathbf{f}^{(A)}_{\nu_a}$,

$$\frac{\mathrm{d}\mathbf{K}_{\nu_a}}{\mathrm{dt}} = -\frac{\nabla p'_{\nu_a}}{n_{\nu_a}} + \mathbf{f}_{\nu_a} + \mathbf{f}^{(A)}_{\nu_a},$$
$$\frac{\mathrm{d}\mathbf{K}_{\bar{\nu}_a}}{\mathrm{dt}} = -\frac{\nabla p'_{\bar{\nu}_a}}{n_{\bar{\nu}_a}} + \mathbf{f}_{\bar{\nu}_a} + \mathbf{f}^{(A)}_{\bar{\nu}_a}. \tag{2.19}$$

The latter term ($\mathbf{f}^{(A)}_{\nu_a}$ and similarly $\mathbf{f}^{(A)}_{\bar{\nu}_a}$ with the change of common sign and fluid velocity $\mathbf{V}_{\nu_a} \to \mathbf{V}_{\bar{\nu}_a}$),

$$\mathbf{f}^{(A)}_{\nu_a} = \frac{G_F c_A^{(a)}}{\sqrt{2}} \left[-\nabla \delta A_0(\mathbf{x},t) - \frac{\partial \delta \mathbf{A}(\mathbf{x},t)}{\partial t} + \mathbf{V}_{\nu_a} \times \nabla \times \delta \mathbf{A}(\mathbf{x},t) \right] , \tag{2.20}$$

depends on the spin density asymmetry $\delta A_\mu(\mathbf{x},t)$,

$$\delta A_\mu(\mathbf{x},t) = A^{(-)}_\mu(\mathbf{x},t) - A^{(+)}_\mu(\mathbf{x},t) \ ,$$
$$A^{(\pm)}_\mu(\mathbf{x},t) = m_e \int \frac{d^3p}{(2\pi)^3}\frac{1}{\varepsilon_p} \left(\frac{\mathbf{p}\mathbf{S}^{(\pm)}(\mathbf{p},\mathbf{x},t)}{m_e}; \mathbf{S}^{(\pm)}(\mathbf{p},\mathbf{x},t) + \frac{\mathbf{p}(\mathbf{p}\cdot\mathbf{S}^{(\pm)}(\mathbf{p},\mathbf{x},t))}{m_e(\varepsilon_p + m_e)} \right) , \tag{2.21}$$

that is given by the charged lepton spin distributions $\mathbf{S}^{(\pm)}(\mathbf{p},\mathbf{x},t)$ obeying the spin RKE like Eq. (12) in [4]. In NR plasma such electron spin distribution defines the well-known hydrodynamical characteristic - magnetization $\mathbf{m}(\mathbf{x},t) = \mid \mu_B \mid \int (d^3p/(2\pi)^3 \mathbf{S}^{(-)}(\mathbf{p},\mathbf{x},t) \approx \mid \mu_B \mid \mathbf{A}(\mathbf{x},t)$ which obeys the Bloch evolution equation and completes the system of MHD equations for fermions in a polarized medium [4].

We note here that the neutrino (antineutrino) currents $\mathbf{j}_{\nu_a,\bar{\nu}_a}(\mathbf{x},t)$ entering through weak forces the pair motion equation (2.18) are connected with the neutrino generalized momenta $\mathbf{K}_{\nu_a,\bar{\nu}_a}(\mathbf{x},t)$ via

$$\mathbf{j}_{\nu_a,\bar{\nu}_a}(\mathbf{x},t) = \frac{n'_{\nu_a,\bar{\nu}_a}}{w_{\nu_a,\bar{\nu}_a}} \mathbf{K}_{\nu_a,\bar{\nu}_a}(\mathbf{x},t). \tag{2.22}$$

Note also that continuity equations (2.3), (2.4) are fulfilled in polarized medium [4]. We do not consider here energy equations that are easily derived from RKE's in polarized medium analogously to Eqs. (2.12-2.15).

3. Faradey equation in SM

In order to derive Faradey equation let us multiply the electron (positron) hydrodynamical motion equation (2.17) by $- \mid e \mid$ (and $+\mid e \mid$) correspondingly and then sum them to obtain the auxiliary result for the electric field in a polarized plasma:

$$\mathbf{E} = -\frac{1}{2}\sum_{\sigma=\pm} \mathbf{V}_\sigma \times \mathbf{B} + \sum_{\sigma=\pm} \frac{e_\sigma}{2e^2} \left(\partial_t \mathbf{P}_\sigma + \nu_{em}\delta\mathbf{P}_\sigma + (\mathbf{V}_\sigma\nabla)\mathbf{P}_\sigma \mp (V_\sigma)_n \nabla (P_\sigma)_n\right) +$$

$$+\frac{G_F\sqrt{2}}{\mid e\mid}\sum_{\nu_a}c_V^{(a)}\left[-\nabla\delta n_{\nu_a}-\partial_t\delta\mathbf{j}_{\nu_a}+\frac{1}{2}\sum_{\sigma=\pm}\mathbf{V}_\sigma\times\nabla\times\delta\mathbf{j}_{\nu_a}\right]-$$
$$-\frac{G_F}{\sqrt{2}\mid e\mid n_e}\sum_{\nu_a}c_A^{(a)}\left[(n_0^{(-)}+n_0^{(+)})\hat{\mathbf{b}}\ \frac{\partial\delta n_{\nu_a}(\mathbf{x},t)}{\partial t}+\right.$$
$$\left.+(N_0^{(-)}+N_0^{(+)})\nabla(\hat{\mathbf{b}}\cdot\delta\mathbf{j}_{\nu_a}(\mathbf{x},t))\right]\ . \qquad (3.1)$$

Let us stress that instead of the *difference* of electron and positron contributions in axial vector terms entering the pair motion equation (2.18) and given by the polarized density asymmetries $\sim(n_0^{(-)}-n_0^{(+)})$ we obtained here the *sum* of them $\sim(n_0^{(-)}+n_0^{(+)})$ that can lead to an essential effect in hot plasma (see below section V).

Using for the last term at the first line of Eq. (3.1) the identity $(V_\sigma)_n\nabla(P_\sigma)_n-(\mathbf{V}_\sigma\nabla)\mathbf{P}_\sigma=\mathbf{V}_\sigma\times\nabla\times\mathbf{P}_\sigma$ and the thermodynamics relation for the work $dR_\sigma/dt=\mathbf{V}_\sigma d\mathbf{P}_\sigma/dt=-p_\sigma dv_\sigma/dt$, $(V_\sigma)_n\nabla(P_\sigma)_n=\nabla(\varepsilon_\sigma-T_\sigma S_\sigma)+S_\sigma\nabla T_\sigma$, where ε_σ ,S_σ are the internal energy and entropy per one particle (of the kind $\sigma=\pm$), p_σ, v_σ, T_σ are the pressure, the volume and the temperature correspondingly; then substituting Eq. (3.1) into the Maxwell equation $\partial_t\mathbf{B}=-\nabla\times\mathbf{E}$ we obtain the Faradey equation generalized in SM with neutrinos and antineutrinos:

$$\partial_t\mathbf{B}=\nabla\times\mathbf{V}\times\mathbf{B}-\nabla\times\eta\nabla\times\mathbf{B}+\sum_\sigma\left(\frac{e_\sigma}{2e^2}\right)\nabla T_\sigma\times\nabla S_\sigma-$$
$$-\sum_\sigma\left(\frac{e_\sigma}{2e^2}\right)\nabla\times(\partial_t\mathbf{P}_\sigma-\mathbf{V}_\sigma\times\nabla\times\mathbf{P}_\sigma)-$$
$$-\frac{G_F\sqrt{2}}{\mid e\mid}\sum_{\nu_a}c_V^{(a)}\nabla\times(\partial_t\delta\mathbf{j}_{\nu_a}-\mathbf{V}\times\nabla\times\delta\mathbf{j}_{\nu_a})+$$
$$+\frac{G_F\sqrt{2}}{2\mid e\mid}\sum_{\nu_a}c_A^{(a)}\left[\nabla\times\left(\frac{n_0^{(-)}+n_0^{(+)}}{n_e}\right)\left(\hat{\mathbf{b}}\frac{\partial\delta n_{\nu_a}}{\partial t}\right)+\right.$$
$$\left.+\nabla\times\left(\frac{N_0^{(-)}+N_0^{(+)}}{n_e}\right)\nabla(\hat{\mathbf{b}}\cdot\delta\mathbf{j}_{\nu_a})\right]\ . \qquad (3.2)$$

Here the equalities $\delta\mathbf{V}_++\delta\mathbf{V}_-=0$, or $\mathbf{V}_++\mathbf{V}_-=2\mathbf{V}$, $\mathbf{V}_+-\mathbf{V}_-=2\delta\mathbf{V}_+\equiv2\delta\mathbf{V}$ followed from the $e\gamma$-equilibrium are taken into account; the magnetic diffusion coefficient $\eta=(4\pi\sigma_{cond})^{-1}$ stems from the third term in the electric field (3.1) given by the electromagnetic collision frequency ν_{em}, which enters the plasma conductivity $\sigma_{cond}=\omega_p^2(\varepsilon_e/w_e)/4\pi\nu_{em}$ with ε_e, $\omega_p=\sqrt{4\pi\alpha n_e/\varepsilon_e}$ being the internal energy and the plasma frequency correspondingly. In the non-relativistic plasma the enthalpy w_e coincides with the internal energy, $w_e\approx\varepsilon_e\approx m_e$, while in the hot relativistic plasma $w_e=4T$, $\varepsilon_e=3T$. For the uniform conductivity the second term takes the standard form $+\eta\nabla^2\mathbf{B}$.

The first term in the r.h.s. (3.2) represents the nonlinear dynamo effect, the third one is the Biermann battery effect. The fourth term can be neglected for small fluctuations $\delta\mathbf{P}\ll\mathbf{P}$, $\delta\mathbf{V}\ll\mathbf{V}$.

In an unpolarized medium we can omit all terms in last lines which are proportional to the axial vector coupling $c_A^{(a)}$. The remaining standard terms and weak interaction vector terms ($\sim c_V^{(a)}$) reproduce the Faradey equation (5.7) in [2] for the *ideal* pair plasma ($\eta = 0$) interacting with neutrinos (antineutrinos).

The neutrino (antineutrino) currents $\mathbf{j}_{\nu_a,\bar{\nu}_a}$ entering (3.2) are given in Eq. (2.22) by their generalized momenta $\mathbf{K}_{\nu_a,\bar{\nu}_a}$ which in turn obey the motion equations (2.10), (2.19).

In the next section we consider an application [1] of the generalized Faradey equation (3.2) which includes weak interaction terms violating parity to the actual problem of magnetic field generation in the early universe plasma.

4. Large-scale magnetic field generation in early universe

The main problem of primordial magnetic field generation that leads to a seed of observable galactic magnetic fields is an inconsistency of their values B and correlation lengths L_0 obtained in the different scenarios.

There are many ways how to generate small-scale random magnetic fields with large values of $B_{rms} = \sqrt{< B^2 >}$, e.g. using some causal mechanisms like bubble collisions at phase transitions, while the correlation length of such magnetic fields evolved (via inverse cascade) during expansion of universe into large-scale magnetic fields turns out to be too small at present time, $L_0 \sim$ *tens parsecs*, to reach the size $L_0 \sim 100$ *kps* for galactic magnetic field, or even more ($\gg$ Mps) for extragalactic magnetic fields. The other way using inflation scenario allows, vice versa, to get large-scale (a few Mps) magnetic fields while their strength occurs too small for observable magnetic fields.

Let us simplify the Faradey equation (3.2) rewriting it as a simple governing equation for mean magnetic field evolution

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \alpha \mathbf{B} + \eta \nabla^2 \mathbf{B} \ , \tag{4.1}$$

where we omitted: dynamo term neglecting any macroscopic rotation in plasma of early universe, Biermann battery effect and weak interaction terms given by the vector coupling $c_V^{(a)}$.

Here we approximate the tensor α_{ij} coming in $\mathbf{E}$ from the axial vector force in (2.17) by the first diagonal ($\sim \alpha\delta_{ij}$) term:

$$\alpha = \frac{G_F}{2\sqrt{2} \mid e \mid B} \sum_a c_{e\nu_a}^{(A)} \left[\left(\frac{n_0^{(-)} + n_0^{(+)}}{n_e} \right) \frac{\partial \delta n_{\nu_a}}{\partial t} \right] \simeq$$
$$\simeq \frac{\ln 2}{4\sqrt{2}\pi^2} \left(\frac{10^{-5} T}{m_p^2 \lambda_{\rm fluid}^{(\nu)}} \right) \left(\frac{\delta n_\nu}{n_\nu} \right) \ , \tag{4.2}$$

where densities $n_0^{(\pm)}$ are given by Eq. (2.16), $n_\nu / n_e = 0.5$, and we assume a scale of neutrino fluid inhomogeneity $t \sim \lambda_{\rm fluid}^{(\nu)}$, that is small comparing with a large Λ-scale of the mean magnetic field $\mathbf{B}$, $\lambda_{\rm fluid}^{(\nu)} \ll \Lambda$.

The diffusion coefficient $\eta \approx 4\pi/137\ T$ is given by the relativistic plasma conductivity.

[1]Results in section below were obtained together with D.D. Sokoloff in [6].

For a small neutrino chemical potential μ_ν, $\xi_{\nu_a}(T) = \mu_{\nu_a}(T)/T \ll 1$, the neutrino asymmetry in the r.h.s. of Eq. (4.2) is the algebraic sum following the sign of the axial coupling, $c^{(A)}_{e\nu_a} = \pm 0.5$,

$$\frac{\delta n_\nu}{n_\nu} \equiv \sum_a c^{(A)}_{e\nu_a}\frac{\delta n_{\nu_a}}{n_{\nu_a}} = \frac{2\pi^2}{9\zeta(3)}[\xi_{\nu_\mu}(T) + \xi_{\nu_\tau}(T) - \xi_{\nu_e}(T)]\ . \tag{4.3}$$

We stress that the Eq. (4.1) is the usual equation for mean magnetic field evolution (see e.g. [8]) with α-effect based on particle effects rather on the averaging of turbulent pulsations. It is well-known (see e.g. [9]) that Eq. (4.1) describes a self-excitation of a magnetic field with the spatial scale $\Lambda \approx \eta/\alpha$ and the growth rate $\alpha^2/4\eta$.

Substituting α into $\Lambda = \eta/\alpha$ we arrive now to the estimate

$$\frac{\Lambda}{l_H} = 1.6\times 10^9\left(\frac{T}{\mathrm{MeV}}\right)^{-5}\left(\frac{\lambda^{(\nu)}_{fluid}}{l_\nu(T)}\right)(\mid \xi_{\nu_e}(T)\mid)^{-1}\ , \tag{4.4}$$

where the neutrino mean free path $l_\nu(T) = \Gamma_W^{-1}$ is given by the weak rate $\Gamma_W = 5.54\times 10^{-22}(T/\mathrm{MeV})^5$ MeV, $l_H(T) = (2\mathcal{H})^{-1}$ and $\mathcal{H}$=$4.46\times 10^{-22}(T/\mathrm{MeV})^2$ MeV is the Hubble parameter.

If the neutrino fluid inhomogeneity scale $\lambda^{(\nu)}_{\mathrm{fluid}}$ is of the order $l_\nu(T_0) \sim 4$ cm $\ll l_H(T_0) \sim 10^6$ cm, we have $\Lambda/l_H \geq 1$ at the beginning of the lepton era ($T = T_0 \sim 10^2$ MeV). The magnetic field time evolution is given by

$$B(t) = B_{EW}\exp\left(\int_{t_{EW}}^t \frac{\alpha^2(t')}{4\eta(t')}dt'\right)\ , \tag{4.5}$$

where B_{EW} is some seed value at the electroweak instant T_{EW} (here we imbed the standard estimates of α^2-dynamo into the context of expanding Universe).

For $\lambda^{(\nu)}_{fluid}(T) \sim l_\nu(T)$ we can estimate the index in the exponent (4.5) substituting in the integrand the expansion time
$t(T) = 3.84\times 10^{21}(T/\mathrm{MeV})^{-2}\mathrm{MeV}^{-1}/\sqrt{\mathrm{g}^*}$ with the effective number of degrees of freedom $g^* \sim 100$ at the temperatures $T > 1$ GeV. Then from our estimates of $\alpha(T)$ and $\eta(T)$ with the change of the variable $(T/10^5\mathrm{MeV}) \to x$ one finds the fast growth of the mean field (4.5) in hot plasma, $x \leq 1$,

$$B(x) = B_{EW}\exp\left(3.2\times 10^8\int_x^1\left(\frac{\xi_{\nu_e}(x')}{0.07}\right)^2 x'^{10}dx'\right) \tag{4.6}$$

given by the upper limit $x_{EW} = 1$. The behaviour of $\xi_{\nu_e}(T)$ at high temperatures is unknown as well as a value of the neutrino density asymmetry. We can state only that this value changes due to neutrino oscillations somewhere below $T < 10$ MeV not overcoming the primordial nucleosynthesis limit $\mid \xi_{\nu_e} \mid < 0.07$ at the BBN time ($T \sim 0.1$ MeV, $x = 10^{-6}$) [10]. Nevertheless, even for $\xi_{\nu_e}(x) \sim 10^{-4}$ there remains an enhancement of a weak large-scale magnetic field $B_{EW} \ll T^2_{EW}/\mid e\mid$ by collective neutrino-plasma interactions considered here, or this mechanism can be efficient and important in cosmology. For a decrease of the neutrino fluid inhomogeneity scale $\lambda^{(\nu)}_{\rm fluid} \ll l_\nu$ entering the α-parameter this conclusion remains valid even for very small neutrino chemical potentials, $\mid \xi_{\nu_a} \mid \ll 1$.

Note that the inflation mechanism (with a charged scalar field fluctuations at super-horizon scales) explains the origin of mean field at cosmological scales. However, the value of this field is too small for seeding the galactic magnetic fields. The amplification mechanism suggested in our paper [6] can improve this very low estimate by a substantial factor from Eq. (4.6).

Thus, while in the temperature region $T_{EW} \gg T \gg T_0 = 10^2$ MeV there are many small random magnetic field domains, a weak mean magnetic field turns out to be developed into the uniform *global* magnetic field. The global magnetic field can be weak enough to preserve the observed isotropy of cosmological model [11] while strong enough to be interesting as a seed for galactic magnetic fields. This scenario was intensively discussed by experts in galactic magnetism [12], however until now no viable origin for the global magnetic field has been suggested. We believe that the α^2-dynamo based on the α-effect induced by particle physics [6] solves this fundamental problem and opens a new and important option in galactic magnetism.

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