Warm inflation solution to the eta-problem

Arjun Berera[*] [†]

School of Physics, University of Edinburgh, Scotland, EH9 3JZ
E-mail: ab@ph.ed.ac.uk

Abstract: Warm inflationary dynamics is shown to satisfy both the slow-roll and density perturbation constraints for $m_\phi \gg H$ or equivalently $\eta \gg 1$ and for inflaton field amplitudes much below the Planck scale, $\langle \phi \rangle < m_{pl}$. I start by reviewing the two types of inflation dynamics, isentropic or cold inflation and nonisentropic or warm inflation. In the former, inflation occurs without radiation production, whereas in the latter both radiation production and inflation occur concurrently. I then discuss recent, detailed, quantum field theory calculations showing that many generic inflation models, including hybrid inflation, which were believed only to have cold inflation regimes, in fact have regimes of both warm and cold inflation. These results dispel many foregone assumptions generally made up to now about inflation models and bring to the fore various elementary issues that must be addressed to do reliable calculations from inflation models. I also discuss density perturbations and observational consequences of warm inflation, especially related to WMAP. Finally I show that warm inflation has intrinsic features that make the "eta problem" nonexistent, and field amplitudes are below the Planck scale.

1. Introduction

The main requirement of particle physics, arising from the density perturbation and slow-roll conditions of inflation, is a very flat potential. The degree of flatness necessary is typically expressed through the slow-roll parameter

$$\eta \equiv \frac{m_{pl}^2}{8\pi V} \left(\frac{d^2 V}{d\varphi^2} \right). \tag{1.1}$$

In standard inflation models [1, 2, 3, 4], where inflaton evolution is damped by the term $3H\dot{\varphi}$, the slow-roll condition amounts to $\eta \lesssim 1$, which equivalently means the potential can not have mass terms bigger than $\sim H^2\varphi^2$.

[*]Speaker.
[†]PPARC Advanced Fellow

Since Supersymmetry suppresses quantum corrections, thus can preserve the tree level potential, it has been a central idea in realizing such flat inflationary potentials. Of course, since inflation requires a nonzero vacuum energy density, inevitably SUSY must be broken during the inflation period, thus possibly ruining the desired degree of flatness in the potential. In particular, once supergravity effects are included, it becomes very difficult for this symmetry to preserve flatness at the leevl of $\eta < 1$. For F-term inflation, where the nonzero vacuum energy density arises from terms in the superpotential, no symmetry prohibits the appearance of the Planck mass suppressed higher dimensional operators $a_n \varphi^n / m_{pl}^{n-4}$ [5, 6, 7, 8] For the large class of chaotic inflation type models [9], where inflation occurs with the inflaton field amplitude above m_{pl}, to control these higher dimensional operators would require the fine-tuning of a infinite number of parameters. Even for models where inflation occurs for field amplitudes below the Planck scale, dimension six operator terms of the form $V\varphi^2/m_{pl}^2 \sim H^2\varphi^2$ can emerge and ruin the desired flatness. Both minimal and nonminimal Kahler potentials can lead to such terms [8].

One possible solution to the η-problem might be D-term inflation. In such models, the nonzero vacuum energy arises from the supersymmetrization of the gauge kinetic energy. However a closer examination [7, 8] reveals that attaining the required degree of flatness makes such models very restrictive.

Up to now, attempts to solve the eta-problem have sought symmetries that can maintain this desired degree of flatness. One of the few that has proven successful is called the Heisenberg symmetry [6], although it is very restrictive. Another proposal has been a certain shift symmetry [8], which is particularly interesting as it does not require SUSY. In common, all attempts so far have one foregone conclusion, that inflaton dynamics is only viable for $\eta < 1$. However, if the inflaton evolution happened to have a damping term larger than $3H\dot{\varphi}$, then clearly slow-roll can be satisfied for $\eta > 1$. Such a possibility is precisely what occurs in warm inflationary dynamics.

There are two distinct dynamical realizations of inflation. In the original picture, termed cold, supercooled or isentropic inflation [1, 2, 3, 4, 9], the universe rapidly supercools during inflation and subsequently a reheating phase is invoked to end inflation and put the universe back into a radition dominated regime. In the other picture, termed warm or nonisentropic inflation [10], dissipative effects are important during the inflation period, so that radition production occurs concurrently with inflationary expansion. Phenomenologically, the inflaton evolution in simple warm inflation models has the form,

$$\ddot{\varphi} + [3H + \Upsilon(\varphi)]\dot{\varphi} + \xi \mathrm{R}\varphi + \frac{dV_{\mathrm{eff}}(\varphi)}{d\varphi} = 0, \tag{1.2}$$

where

$$R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) \tag{1.3}$$

is the curvature scalar. For $\Upsilon = 0$, this is the familiar inflaton evolution equation for cold inflation, but for a nonzero Υ, it corresponds to the case where the inflaton field is dissipating energy into the universe, thus creating a radiation component.

Despite a historical belief that cold inflation is the most common form of inflationary dynamics, recent work has shown that warm inflationary dynamics also is very generic

[11]. As will be discussed later in the talk, many simple models which up to now have been believed to exclusively yield cold inflation, in fact also have warm inflationary regimes. Very elementary thermodynamic considerations already point to the fact that cold inflation is a very restrictive picture. In particular, even if the inflaton dissipated a minuscule fraction of its energy, say one part in 10^{20}, it still would constitute a significant radiation energy density component in the universe. For example, for inflation with vacuum (i.e. potential) energy at the GUT scale $\sim 10^{15-16}$GeV, leaking one part in 10^{20} of this energy density into radiation corresponds to a temperature of 10^{11}GeV, which is nonnegligible. In fact, the most relevant lower bound that cosmology places on the temperature after inflation comes from the success of hot Big-Bang nucleosynthesis, which requires the universe to be within the radiation dominated regime by $T \stackrel{>}{\sim} 1$GeV. This limit can be met in the above example by dissipating as little as one part in 10^{60} of the vacuum energy density into radiation. Thus, from the perspective of both interacting field theory and basic notions of equipartition, it appears to be a highly tuned requirement of cold inflation to prohibit the inflaton from such tiny amounts of dissipation.

In this talk, I review the progress made in developing warm inflation. In Sec. 2 recent work on quantum field theory first principles calculations which realize the inflaton effective equation of motion (EOM) Eq. (1.2) is given. In particular, a very simple model involving just four fields is shown to be adequate for realizing warm inflation. In Sec. 3, I review the theory of density perturbations in warm inflation. In this picture, density perturbations are thermally induced and formulas for inflaton fluctuations are given. I then discuss recent work which has numerically evolved the cosmological perturbation equations under warm inflationary conditions. The process of dissipation during warm inflation leads to many interesting features in the scalar spectral index. One noteworthy result, which I focus on, is for a simple symmetry breaking potential, which leads to a spectral index that is blue at large scales and red at small scales, similar to the spectrum suggested by recent WMAP results [12, 13]. Finally in Sec. 4, I show how warm inflation dynamics based on Eq. (1.2) and with the thermal inflaton fluctuations of Sec. 3, has no "eta-problem" and has inflaton field amplitudes much below the Planck scale.

2. Quantum field theory dynamics

Our previous considerations of inflaton dynamics were limited to nonexpanding spacetime. In addition the earliest of these works looked for high temperature warm inflation solutions, under rigid adiabatic, equilibrium conditions [14]. Within this limited framework, one type of warm inflation solution was obtained [15]. The high-T regime was examined first, since considerable methodology was already available for treating it. However, intrinsically, the statistical state relevant for warm inflation is not required to be an equilibrium state. The slowly varying nature of the macroscopic variables in warm inflation cosmology suggest that the statistical state may not be far from equilibrium, although this is something that should be proven from the dynamics. Much work remains in order to develop the mathematical formalism necessary to address this problem. As one step in this direction to fill the missing gaps, we studied the zero temperature dissipative dynamics of interacting scalar field systems in Minkowski spacetime [11] (for another interesting direction see [16]). This is useful to understand, since the zero temperature limit constitutes a baseline effect, that will be prevalent in any general statistical state. The key result presented in this talk is that for a broad range of cases, involving interaction with as few as one or two fields, we find dissipative regimes for the scalar field system. This is important for inflationary cosmology, since it suggests that dissipation may be the norm not exception for an interacting scalar field system, thus warm inflation is a natural dynamics once proper treatment of interactions is done.

A key mechanism we identified which leads to robust warm inflation involved the scalar inflaton field ϕ exciting a heavy bosonic field χ which then decays to light fermions ψ_d [11],

$$\phi \to \chi \to \psi_d. \tag{2.1}$$

Recently we studied the simplest model that yields this mechanism [17],

$$\mathcal{L}_I = -\frac{1}{2}g^2\phi^2\chi^2 - g'\phi\bar{\psi}_\chi\psi_\chi - h\chi\bar{\psi}_d\psi_d, \tag{2.2}$$

where ψ_d are the light fermions to which χ-particles can decay, with

$$m_\chi > 2m_{\psi_d}. \tag{2.3}$$

Aside from the last term in Eq. (2.2), this is the typical Lagrangian used in studies of reheating after inflation [18, 19, 20]. Later we will briefly discuss that in minimal SUSY extensions of the typical reheating model, decay channels for the χ or ψ_χ particles are present and the ψ_d field above is simply a representative example. Since in the moderate to strong perturbative regime, reheating models will require SUSY for controlling radiative corrections, Eq. (2.2) with inclusion of the ψ_d field thus is a toy model representative of the typical reheating model.

To study inflation, the effective evolution equation must be derived for the background component of the inflaton, $\varphi \equiv \langle\phi\rangle$. The conventional approach [1, 2, 3, 4, 5, 9, 18, 19, 20, 21], assumes that aside from radiative corrections that modify the φ-effective potential, $V_{\rm eff}(\varphi)$, this equation is the same as its classical counterpart. However, we have shown in earlier works [14, 11], that in addition to radiative corrections, quantum effects also arise in the φ-effective EOM in terms of temporally nonlocal terms, which generically lead to dissipative effects. Moreover, although SUSY can cancel large quantum effects in the local limit, it can not cancel for the dynamical problem the nonlocal quantum effects.

Here results are presented from [17], where we have extended the calculation to the expanding case (for related earlier works see [22, 23, 24]). Also in [17] extensive numerical analysis of dissipative effects was done, which up to now have only been examined in simplified analytic approximations. The φ-effective EOM from [17] for model Eq. (2.2) is,

$$\ddot{\varphi}(t) + 3H(t)\dot{\varphi}(t) + \xi \mathrm{R}(t)\varphi(t) + \frac{dV_{\rm eff}(\varphi(t))}{d\varphi(t)} + g^4\varphi(t)\int_{t_0}^{t} dt'\varphi(t')\dot{\varphi}(t')K(t,t') = 0, \tag{2.4}$$

where

$$K(t,t') = \int_{t_0}^{t'} dt'' \int \frac{d^3q}{(2\pi)^3} \sin\left[2\int_{t''}^{t} d\tau\omega_\chi(\tau)\right] \frac{\exp\left[-2\int_{t''}^{t} d\tau\Gamma_\chi(\tau)\right]}{4\omega_\chi(t)\omega_\chi(t'')}, \tag{2.5}$$

$$\omega_\chi(\tau) = \left[\mathbf{q}^2 \frac{a^2(t)}{a^2(\tau)} + m_\chi^2(\tau) + 2(6\xi - 1)H^2\right]^{1/2}, \tag{2.6}$$

$m_\chi = g\varphi \gg 2m_{\psi_d}$, R the curvature scalar Eq. (1.3), ξ the gravitational coupling, $a(t) = \exp(Ht)$ the scalar factor, $H = \sqrt{8\pi V_{\rm eff}/(3m_p^2)}$ the Hubble parameter, and

$$\Gamma_\chi(t) = \frac{h^2 m_\chi^2}{8\pi}\left(1 - \frac{4m_{\psi_d}^2}{m_\chi^2}\right) \tag{2.7}$$

the decay width. The kernel $K(t,t')$ Eq. (2.5) is obtained by a linear response approximation, equivalent to the closed time path formalism at leading nontrivial order, which treats the effect of the field χ on the evolution of $\varphi(t)$. In the limit $H \to 0$, $a \to constant$, Eq. (2.5) agrees with the corresponding kernel for nonexpanding spacetime in [11]. The physical origin of the nonlocal (dissipative) term in Eq. (2.4) is as follows. The evolving background field φ changes the mass of the χ boson which results in the mixing of its positive and negative frequency modes. This in turn leads to coherent production of χ particles, which then decohere through decay into the lighter ψ_d-fermions. A version of the above calculation in nonexpanding spacetime based on a canonical approach was done in [11, 23]. This approach exhibits much more clearly the above picture relating φ-dissipation and particle creation.

The ψ_χ-fermions in Eq. (2.2) have played no role in the dissipative effects. Within this toy model, these fermions are used to mimic SUSY by canceling the quantum corrections from the χ-boson sector. In particular the one-loop contributions to the effective potential are from the χ-loop

$$V_1(\varphi) = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3} E_{m_\chi}, \tag{2.8}$$

where $E_{m_\chi} = \sqrt{\mathbf{k}^2 + g^2\varphi^2}$. and for the ψ_χ-loop

$$V_1(\varphi) = -2\int \frac{d^3k}{(2\pi)^3} E_{m_{\psi_\chi}}, \tag{2.9}$$

where setting $g' = g$ in Eq. (2.2), $E_{m_{\psi_\chi}} = \sqrt{\mathbf{k}^2 + g^2\varphi^2}$. Here the fields have zero explicit mass terms, so $m_\chi = m_{\psi_\chi} = g\varphi$. Thus if we slightly modify our model Eq. (2.2) so that there are four χ fields for the one ψ_χ-fermion, then summing Eqs. (2.8)+(2.9), these contributions cancel., leaving $V_{eff}(\varphi)$ to be simply whatever is chosen for the φ-potential. Moreover, since the φ-potential must be very flat, thus weakly coupled, quantum corrections arising from the ϕ-field are negligible.

When the motion of φ is slow, which is the regime of interest for inflation, a adiabatic-Markovian approximation can be applied that converts Eq. (2.4) to one that is completely local in time, albeit with time derivative terms. The Markovian approximation amounts to substituting $t' \to t$ in the arguments of the φ-fields in the nonlocal term in Eq. (2.4). The adiabatic approximation then requires that all macroscopic motion is slow on the scale of microscopic motion, thus

$$\begin{aligned} \frac{\dot\varphi}{\varphi} &< H, \Gamma_\chi \\ H &< \Gamma_\chi. \end{aligned} \tag{2.10}$$

Moreover when also $H < m_\chi$, the kernel $K(t, t')$ above is well approximated by the nonexpanding limit $H \to 0$. Combining both these approximations, the effective EOM Eq. (2.4) takes on the form in Eq. (1.2), where we explicitly compute the dissipative coefficient to be,

$$\Upsilon(\varphi) = \frac{g^4 \varphi^2(t) \Gamma_\chi}{32\pi \sqrt{m_\chi^2 + \Gamma_\chi^2} \sqrt{2 m_\chi \sqrt{m_\chi^2 + \Gamma_\chi^2} + 2 m_\chi^2}}. \tag{2.11}$$

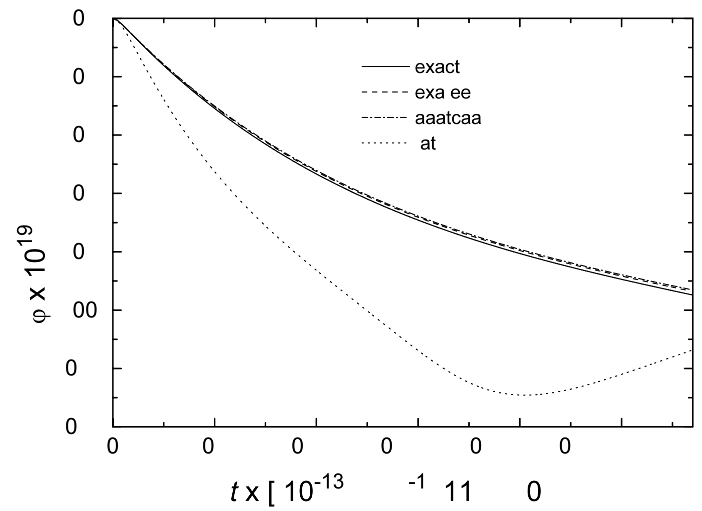

Figure 1: Evolution of $\varphi(t)$ for $\lambda = 10^{-13}$, $g = h = 0.37$, $\xi = 0$, $\varphi(0) = m_p$, $\dot{\varphi}(0) = 0$.

Fig. 1 compares the various approximations for a representative case, where $g = h = 0.37$ and the inflaton potential is that for a chaotic inflation $V_{\rm eff}(\varphi) = \lambda\varphi^4/4$ with $\lambda = 10^{-13}$ [9]. In Fig. 1 evolution has been examined at the final stages of chaotic inflation, where we start with $\varphi(t_0 = 0) = m_p$. The solid line is the exact result based on numerically solving Eq. (2.4). Plotted alongside this, although almost indiscernible, is the same solution expect using the nonexpanding spacetime kernel (dashed line), obtained by setting $H \to 0$, $a \to$ *constant* in Eq. (2.5), and the solution based on the adiabatic-Markovian approximation of Eq. (1.2) (dot-dashed line) for the same parameter set. As seen, the expanding and nonexpanding cases differ by very little and the adiabatic-Markovian approximation is in good agreement with the exact solution. This confirms simplifying approximations claimed in [14, 11, 22, 23] but up to now had not been numerically verified.

More interestingly, and the first major result in [17], the dotted line in Fig. 1 is the solution that would be found by the conventional approach in which the nonlocal term in Eq. (2.4) is ignored. By the conventional approach [3, 4, 9, 18, 19, 20, 5, 21], one expects the inflaton to start oscillating, which is the precursor to entering various stages of pre/re-heating. However with account for dissipative effects, this never happens, since the inflaton remains overdamped till the end when it settles at its minima at $\varphi = 0$. Moreover, throughout inflation, and not just in the final stages, the inflaton is dissipating energy and yielding a radiation component of magnitude

$$\rho_r \approx \frac{\Upsilon \dot{\varphi}^2}{4H}. \tag{2.12}$$

For the case in Fig. 1, the overdamped regime for the inflaton persists until $g \lesssim 0.35$, below which its evolution at the end of inflation has oscillatory features similar to conventional expectations. However the inflaton is still dissipating radiation at the level Eq. (2.12) all throughout the inflation period. If the temperature scale associated with the radiation energy density produced through this dissipative mechanism is greater than the inflaton mass, $T > m_\phi$, then inflaton fluctuations will be significantly altered from their zero temperature behavior. Moreover if $\Gamma_\chi > H$, it is possible for the radiation to thermalize, in which case the inflaton fluctuations will now be thermal. For cold inflation models, it is generally necessary that $m_\phi \lesssim H$. Thus for all cold inflation models, the basic criteria is that if $\rho_r > H^4$, then the radiation has important observational effects in terms of its influence on inflaton fluctuations. For our model Eq. (2.2), $\rho_r > H^4$ and $\Gamma_\chi > H$, if for example $g = h$ and $g > 10^{-2}$ or as another example if $h = 0.1$ and $g > 10^{-3}$. Thus fairly small couplings already lead to observationally significant thermal fluctuations in the inflaton. We thus arrive at the second major result in [17]. In the currently believed treatments of inflaton dynamics, reheating and multifield inflation has been studied for interaction couplings in the range given above. However, in all these studies, the effect of interactions is assumed to be dormant all during inflation. This conclusion once again emerges from neglecting the nonlocal term in Eq. (2.4). In the actually case, where this term is accounted for, it is seen that up to fairly weak coupling, adequate radiation is produced during inflation to alter density perturbations.

Underlying the results here on energy dissipation and inflaton evolution damping is the interaction scheme Eq. (2.1). Such an interaction scheme is very common in particle physics models. For example, the simplest implementation of the Higgs mechanism in the Standard Model has the background Higgs field coupled to W and Z bosons, thereby generating their masses, and these bosons then are coupled to light fermions through the well known charged and neutral current interactions. Also in simple SUSY models, the interaction scheme Eq. (2.1) is very common. For example a minimal SUSY extension that incorporates the $\phi - \chi$ coupling would be

$$W = \sqrt{\lambda}\Phi^3 + g\Phi X^2 + fX^3 + mX^2, \tag{2.13}$$

where $\Phi = \phi + \psi\theta + \theta^2 F$ and $X = \chi + \theta\psi_\chi + \theta^2 F_\chi$ are chiral superfields. In the above model, there would be no additional fermion to associate with ψ_d from our toy model Eq. (2.2). However the χ-field has a decay channel via a ψ_χ triangle-loop into two light inflaton bosons ϕ. For this case, everything in Eqs. (2.4)-(2.6) is unaltered except the decay channel is different with now $\Gamma_\chi \sim (fg^2)^2 m_\chi$. Thus there are additional factors of coupling constants, but at moderate coupling, radiation production would be sufficiently large to affect density perturbations and at the very largest perturbative regime the evolution of φ could be altered into the overdamped behavior similar to the solid line in Fig. 1.

In general when interaction couplings are large, such as for $g \gtrsim 10^{-4}$ for the $\phi - \chi$ interaction in our model Eq. (2.2), to maintain the flat potential $V_{eff}(\varphi)$, SUSY is needed. This situation arises in our warm inflation model as well as in many cold inflation models such as hybrid inflation [5, 21] or any cold inflation model where interactions to fields introduced for reheating are large, such as in [18, 19, 20]. Thus generic SUSY extensions to common cold inflation models can easily have interaction structures of the form Eq. (2.1), and in that case, the resulting dynamics departs radically from the cold inflation picture that is tacitly assumed.

3. Observational tests

As stated in the last section, when a thermalized radiation component is present with $T > m_\phi$, inflaton fluctuations are dominantly thermal rather than quantum. There are two distinct regimes of warm inflation to note. One is the weak dissipative regime [25, 26],

$$\delta\varphi^2 \sim HT \ \text{ warm inflation } (\Upsilon < 3H), \ \ T > m_\phi, \tag{3.1}$$

and the other is the strong dissipative regime [27],

$$\delta\varphi^2 \sim \sqrt{H\Upsilon}T \ \text{ warm inflation } (\Upsilon > 3H), \ \ T > m_\phi. \tag{3.2}$$

For comparison, for the cold inflation case, where the inflaton fluctuations are exclusively quantum [28],

$$\delta\varphi^2 \sim H^2, \ \text{ cold inflation } \ T < m_\phi. \tag{3.3}$$

For both cold and warm inflation, density perturbations are obtained by the same expression, $\delta\rho/\rho \sim H\delta\varphi/\dot{\varphi}$.

In [29] an order of magnitude estimate of density perturbations during warm inflation was computed by matching the thermally produced fluctuations to gauge invariant parameters when the fluctuations cross the horizon (for other phenomenological treatments of warm inflation see [30, 31, 32, 33, 34, 35]). This work provided a clear statement of the consistency condition. Cold inflation has three parameters, related to the potential energy magnitude V_0, slope $\epsilon = m_{pl}^2 V'/(16\pi V)$, and curvature η Eq. (1.1), whereas there are four observable constraints (δ_H, A_g, n_s, n_g). This implies a redundancy in the observations and allows for a consistency relation [36]. This is usually expressed as a relationship between the tensor-to-scalar ratio and the slope of the tensor spectrum. Warm inflation has an extra parameter, the dissipation factor, which implies four constraints for four parameters. Hence we do not expect the consistency relation of standard inflation to hold in warm inflation [29]. Thus, to discriminate between warm and standard inflation, it requires a measure of all four observables. The WMAP and upcoming Planck satellite missions should provide strong constraints on the scalar spectrum and being equipped with polarization detectors, it is hoped the tensor spectrum also will be measured. At the same level of approximation, nongaussian effects from warm inflation models were computed and found to be of the same order of magnitude as in the cold inflation case, and thus too small to be measured [37, 38].

More accurate treatments of density perturbations have followed [30, 32, 33, 34, 35], which use the cosmological perturbation equations. Following our recent results [39], working in the zero-shear gauge with perturbed metric

$$ds^2 = -(1-2\varsigma)dt^2 + a^2(1+2\varsigma)\delta_{ij}dx^i dx^j, \tag{3.4}$$

we numerically evolved the complete set of Einstein equations

$$\begin{aligned}
\dot{\varsigma} + H\varsigma + 4\pi G k^{-1} a(p+\rho)v &= 0, \\
3H\dot{\varsigma} + (3H^2 + k^2 a^{-2})\varsigma - 4\pi G\delta\rho &= 0, \\
\ddot{\varsigma} + 4H\dot{\varsigma} + (2\dot{H} + 3H^2)\varsigma + 4\pi G\delta p &= 0,
\end{aligned} \tag{3.5}$$

and scalar field perturbation equation

$$\delta\ddot{\varphi} + (3H+\Upsilon)\dot{\varphi} + \dot{\varphi}\delta\Upsilon + k^2 a^{-2}\delta\varphi + \delta V_{,\varphi} + 4\dot{\varphi}\dot{\varsigma} - \Upsilon\dot{\varphi}\varsigma - 2V_{,\varphi}\varsigma = 0. \tag{3.6}$$

Here $\delta\varphi$ is the inflaton perturbation, δT is the temperature fluctuation, v is the velocity perturbation, and

$$\begin{aligned}
\delta\rho &= \dot{\varphi}\delta\dot{\varphi} + V_{,\varphi}\delta\varphi + \dot{\varphi}^2\varsigma + T\delta s, \\
\delta p &= \dot{\varphi}\delta\dot{\varphi} - V_{,\varphi}\delta\varphi + \dot{\varphi}^2\varsigma + s\delta T,
\end{aligned} \tag{3.7}$$

where s is the entropy density.

As a example we examined the generic symmetry breaking potential

$$V = \frac{1}{4}\lambda(\phi^2 - \phi_0^2)^2. \tag{3.8}$$

One of the most interesting outcomes of our study was that by accounting for dissipative effects, the spectral index generically runs with wavenumber. As one interesting example, if we consider a model where $\Upsilon \sim c\varphi^2$ in Eq. (1.2), we obtain an index that runs from blue at large scales to red at small scales, such as in Fig. 2. This is an interesting result for the current observational situation. The WMAP CMB first year data suggests a spectral index $n_s < 1$ [12]. However, when this data is taken in combination with large scale structure data [13], the index then has a form similar to Fig. 2.

4. Solution to eta-problem

The warm inflation solution is examined for the simple potential

$$V = \frac{1}{2}m_\phi^2\phi^2. \tag{4.1}$$

In the cold inflation case, such a model requires an initial inflaton amplitude $\langle\phi\rangle = \varphi > m_{pl}$. Moreover, SUSY models that realize a potential like this inevitably lead to an eta-problem based on the reasons discussed in the Introduction.

Let us now treat this model in the warm inflation case. To focus on the essential points, our calculations here will be purely phenomenological, although they can be readily derived

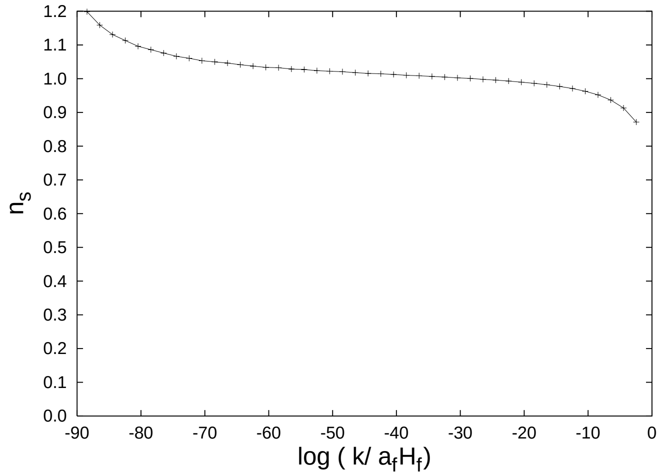

Figure 2: The scalar spectral index for the potential Eq. (3.8) with damping term $\Upsilon \sim \varphi^2$. The wavenumber has been normalized by the horizon size at the end of inflation.

from a first principles quantum field theory calculation. We consider the case where the dissipative coefficient in Eq. (1.2) is independent of both φ and T, $\Upsilon = constant$. The inflaton initially is at a nonzero field amplitude $\varphi \neq 0$, thus supporting a vacuum energy.

The background cosmology for models with constant Υ and monomial potentials has been solved exactly [40]. From this we find $N_e \approx H\Upsilon/m_\phi^2$. The radiation production is determined from the energy conservation equation,

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\varphi}^2. \tag{4.2}$$

During warm inflation $\dot{\rho}_r \approx 0$ [10, 26], so that Eq. (4.2) reduces to Eq. (2.12). Identifying $\rho_r \sim T^4$ permits determination of the temperature during warm inflation. Finally, once T is determined, Eq. (3.2) allows determination of density perturbations.

Combining these expressions, for the model Eq. (4.1) with $\Upsilon = constant$ in Eq. (1.2), we find

$$N_e \approx 2\sqrt{2}\frac{\Upsilon\varphi_0}{m_\phi m_{pl}} \tag{4.3}$$

$$T \approx \frac{m_\phi^{3/4} m_{pl}^{1/4} \varphi_0^{1/4}}{\Upsilon^{1/4}} \tag{4.4}$$

$$\frac{\delta\rho}{\rho} \approx \left(\frac{\varphi_0}{m_\phi}\right)^{3/8} \left(\frac{\Upsilon}{m_{pl}}\right)^{9/8}. \tag{4.5}$$

Imposing observational constraints $N_e = 60$ and $\delta\rho/\rho = 10^{-5}$, leads to the results

$$\frac{m_\phi}{H} \approx 5.5 \times 10^{-9}\frac{m_{pl}}{m_\phi}, \tag{4.6}$$

$$\frac{\varphi_0}{m_{pl}} \approx 5.3 \times 10^{8}\frac{m_\phi}{m_{pl}}, \tag{4.7}$$

$\Upsilon \approx 4\times 10^{-8} m_{pl}$, and $T \approx 10^4 m_\phi$, with the ratio m_ϕ/m_{pl} free to set. For $m_\phi/m_{pl} \lesssim 10^{-9}$, it means $\eta > 1$ and $\varphi < m_{pl}$. Thus we see for sufficiently small inflaton mass, $m_\phi \lesssim 10^{10}$GeV, there is no eta-problem, since $m_\phi \gg H$ and $\varphi < m_{pl}$. Since this warm inflation solution works for $\eta \gg 1$, SUSY models realizing simple monomial potentials like Eq. (4.1) do not require any special symmetries, as is the case discussed in the Introduction for cold inflation models. The "eta" and large φ-amplitude problems simply correct themselves once interactions already present in the models are properly treated.

5. Conclusion

As it emerges, warm inflation is seen to have several remarkable features. In particular, warm inflation

- is generic in quantum field theory
- dissipative effects can produce a running scalar spectral index
- has no eta-problem: $m_\phi \gg H$ ($\eta \gg 1$) is permissible
- has no large φ amplitude problem: $\varphi < m_{pl}$
- has no graceful exit problem: inflation automatically terminates into a radiation dominated regime
- has no quantum-to-classical issues: inflaton fluctuations are classical upon inception.

The dissipative effects discussed in Sec. 2 also can serve to damp kinetic energy contributions before inflation, thus alleviating the initial condition problem of inflation [41]. Warm inflation is also a conducive regime for the creation of large scale cosmic magnetic fields based on the ferromagnetic Savvidy vacuum scenario [42]. Also, spontaneous baryogenesis has been shown to work efficiently in the last stages of warm inflation [43].

Progress toward a theory of inflation requires a complete understanding of the dynamics of inflation models. Our work, more correctly quantum field theory, demonstrates that inflation models generically are dissipative systems. These effects crucially influence inflaton evolution and observational signatures such as density perturbations. Moreover, dissipative effects play a central role in eliminating the eta-problem and the large φ-amplitude problem.

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