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Scattering of Glueballs and the AdS/CFT Correspondence

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Inspired in the AdS/CFT correspondence one can look for dualities between string theory and non conformal field theories. Exact dualities in the non conformal case are intricate but approximations can be helpful in extracting physical results. A phenomenological approach consists in introducing a scale corresponding to the maximum value of the axial AdS coordinate. Here we show that this approach can reproduce the scaling of high energy glueball scattering amplitudes and also an approximation for the scalar glueball mass ratios.

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1. Strings and strong interactions

String theory was originally proposed to describe strong interactions. The motivation was the experimental observation of hadron scattering. Considering the scattering of two particles into two particles as in Fig. 1, one usually introduces Mandelstam variables:

$$s = -(p_1 + p_2)^2 \quad t = -(p_2 + p_3)^2 \quad u = -(p_1 + p_3)^2\,, \tag{1.1}$$

satisfying $(s + t + u = \sum m_i^2)$, where m_i are the particle masses.

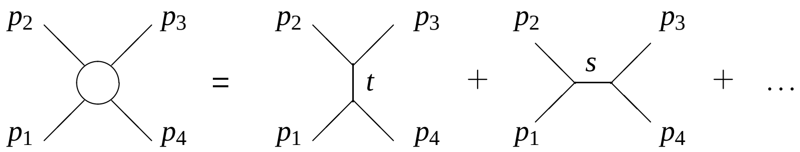

Fig. 1: *Two-two scattering in terms of Mandelstam variables.*

Taking into account the observed properties known at that time, Veneziano proposed an ingenious formula for the hadron scattering amplitude [1]

$$\mathcal{A}(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))}\,, \tag{1.2}$$

where $\alpha(s) = \alpha(0) + \alpha' s$ are the Regge's trajectories. This amplitude can be deduced from string theory which also predicts a relation between mass and angular momentum

$$(m)^2 = \frac{J - \alpha(0)}{\alpha'}\,, \tag{1.3}$$

in agreement with experimental data for hadrons with spin J.

The Veneziano amplitude reproduces the high energy behavior of hadronic amplitudes in the so called Regge regime corresponding to $s \to \infty$ with t fixed (and vice-versa). However, considering hadronic high energy scattering with fixed angles corresponding to $s,\, t \to \infty$ with the ratio s/t fixed, the Veneziano's amplitude presents a soft behavior

$$\mathcal{A}_{Veneziano} \quad \sim \quad e^{-s}\,, \tag{1.4}$$

while experiments show a hard behavior

$$\mathcal{A}_{Experimental} \quad \sim \quad s^{-constant}\,. \tag{1.5}$$

This experimental scaling is reproduced from QCD as shown by Brodsky and Farrar [2], Matveev *et. al.* [3].

Another puzzle for string theory, regarding strong interactions, is the difficulty in reproducing the results of deep inelastic lepton-hadron scattering (Fig. 2) as for instance the Bjorken scaling. These experiments showed also that at high energy protons and neutrons are made of point-like objects later identified with quarks.

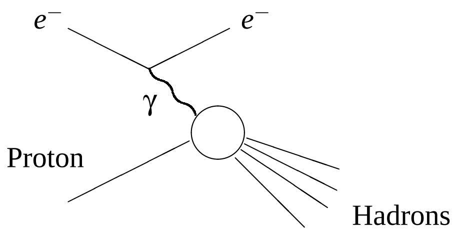

Fig. 2: *Lepton-Hadron scattering.*

Then, string theory *in flat spacetime* does not describe correctly the strong interactions of hadrons.

2. AdS/CFT correspondence and holographic map

In 1974 't Hooft studied in a remarkable work [4] the perturbative expansion of U(N) Gauge theories in the limit $N \to \infty$, with g^2N fixed. He showed that planar diagrams with quarks at the edges dominate the perturbative series with parameter $1/N$. The topological structure of the $1/N$ series is identical to that of the dual model (strings) with $1/N$ as the coupling constant.

Recently, Maldacena [5, 6] proposed that compactifications of M/string theory on various Anti-de Sitter spacetimes are dual to various conformal field theories. The large N limit for $\mathcal{N}=4$ super-Yang-Mills theory in four dimensions is equivalent to type IIB superstrings in five dimensional Anti-de Sitter space times a 5 dimensional sphere ($AdS_5 \times S^5$). This is known as the AdS/CFT correspondence.

It was shown recently by Polchinski and Strassler that the hard scattering behavior of strong interactions can be obtained from string theory based on the AdS/CFT correspondence[7]. They have also studied deep inelastic scattering using this framework [8].

In order to obtain a map between AdS bulk and boundary let us consider type IIB string theory approximated at low energy $(<< 1/\sqrt{\alpha'})$ by supergravity action:

$$S = \frac{1}{2\kappa^2} \int d^{10}x\sqrt{G}\Big[\mathcal{R} + G^{MN}\partial_M\Phi\partial_N\Phi + \ldots.\Big] , \tag{2.1}$$

where G^{MN} is the 10-d metric, $\mathcal{R}$ is the scalar curvature, Φ is the dilaton and $\kappa \sim g(\alpha')^2$. The 10-d spacetime is identified with $AdS_5 \times S^5$

$$ds^2 = \frac{R^2}{z^2}\Big(dz^2 + (d\vec{x})^2 - dt^2\Big) + R^2 d\Omega_5^2 , \tag{2.2}$$

where R is the AdS radius and Ω_5 describe the S^5 sphere.

To set an energy scale we consider just a slice of the AdS space:

$$0 \le z \le z_{max} . \tag{2.3}$$

Assuming the dilaton to be in s-wave state (so that S^5 coordinates are irrelevant), the supergravity action in the slice is:

$$S = \frac{\pi^3 R^8}{4\kappa^2} \int d^4x \int_0^{z_{max}} \frac{dz}{z^3} \Big[\mathcal{R} + (\partial_z\Phi)^2 + \eta^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \ldots.\Big] , \tag{2.4}$$

where $\eta^{\mu\nu}$ is the Minkowiski 4-d metric. The corresponding solution assuming the field to vanish at $z = z_{max}$ is [9, 10]

$$\Phi(z,\vec{x},t) = \sum_{p=1}^{\infty} \int \frac{d^3k}{(2\pi)^3} \frac{z^2 J_2(u_p z)}{z_{max} w_p(\vec{k}) J_3(u_p z_{max})} \{a_p(\vec{k})\, e^{-iw_p(\vec{k})t + i\vec{k}\cdot\vec{x}} + h.c.\}, \tag{2.5}$$

where $w_p(\vec{k}) = \sqrt{u_p^2 + \vec{k}^2}$ and

$$u_p z_{max} = \chi_{2,p}\,, \tag{2.6}$$

such that $J_2(\chi_{2,p}) = 0$. The creation and annihilation operators satisfy the algebra

$$\Big[a_p(\vec{k})\,,\, a^{\dagger}_{p'}(\vec{k}')\Big] = 2\,(2\pi)^3 w_p(\vec{k}) \delta_{p\,p'}\, \delta^3(\vec{k} - \vec{k}')\,. \tag{2.7}$$

On the AdS boundary $z = 0$ we consider massive composite operators $\Theta_i(\vec{x},t)$ associated with glueballs. The corresponding creation-annihilation operators for asymptotic states are assumed to satisfy the algebra

$$\Big[b_i(\vec{K})\,,\, b^{\dagger}_j(\vec{K}')\Big] = 2\delta_{ij}\,(2\pi)^3\, w_i(\vec{K})\, \delta^3(\vec{K} - \vec{K}')\,, \tag{2.8}$$

where $w_i(\vec{K}) = \sqrt{\vec{K}^2 + \mu_i^2}$ and μ_i is the mass of the field Θ_i.

Now let us work out a one to one map between AdS bulk field Φ and boundary fields Θ_i, in particular relating their creation annihilation operators [11]. Note that the dilaton Φ lives in a 5-d space while Θ lives in a 4-d space, but the dilaton has a discrete spectrum associated with the u_p modes.

Let us impose that creation-annihilation operators of both theories be related by

$$\begin{aligned} k\, a_i(\vec{k}) &= K\, b_i(\vec{K})\,, \\ k\, a^{\dagger}_i(\vec{k}) &= K\, b^{\dagger}_i(\vec{K})\,. \end{aligned} \tag{2.9}$$

To preserve the commutation relations of both theories we find an equation which solution reads

$$k = \frac{u_i}{2} \left[\frac{\mathcal{E} + \sqrt{\mathcal{E}^2 + \mu_i^2}}{K + \sqrt{K^2 + \mu_i^2}} - \frac{K + \sqrt{K^2 + \mu_i^2}}{\mathcal{E} + \sqrt{\mathcal{E}^2 + \mu_i^2}} \right], \tag{2.10}$$

where $\mathcal{E}$ is an UV cuttoff for the boundary theory.

3. Glueball scattering amplitudes

The AdS/CFT prescriptions relate the bulk dilaton field to boundary scalar glueball operators. Let us see how the bulk-boundary map of the previous section can be used to reproduce the high energy scaling of scalar glueball amplitudes.

First we relate the mass $\mu_1 \equiv \mu$ of the lightest glueball with the size of the AdS slice:

$$z_{max} \sim \frac{1}{\mu}\,, \tag{3.1}$$

we see that z_{max} corresponds to an IR cutoff of the boundary theory. Since $u_1 z_{max} \sim 1$ this implies that $u_1 \sim \mu$. Also, as $\mathcal{E}_1$ is an UV cutoff then the high energy scattering momenta K of the glueballs satisfy $\mu \ll K \ll \mathcal{E}$. This way, the relation between bulk k and boundary K momenta can be approximated by

$$k \approx \frac{\mathcal{E}\mu}{2K}. \tag{3.2}$$

Note that this relation combined with the condition $\mu \ll K \ll \mathcal{E}$ implies

$$\mu \ll k \ll \mathcal{E} .$$

Setting the string energy scale to the UV boundary cutoff, $\sqrt{\alpha'} \sim 1/\mathcal{E}$, we find that bulk momenta k satisfy $\mu \ll k \ll 1/\sqrt{\alpha'}$ which is consistent with the fact that we are taking the low energy (supergravity) limit of string theory.

Let us consider the scattering of two particles into m particles all in the bulk with axial momentum u_1 (see Fig. 3).

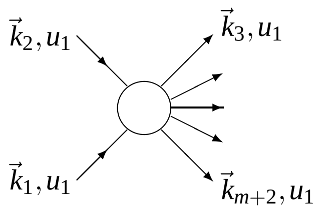

Fig. 3: *The scattering of 2 particles into m particles.*

The S matrix is

$$\begin{aligned} S_{Bulk} &= \langle \vec{k}_3, u_1; ...; \vec{k}_{m+2}, u_1; out | \vec{k}_1, u_1; \vec{k}_2, u_1; in \rangle \\ &= \langle 0 | a_{out}(\vec{k}_3) \ldots a_{out}(\vec{k}_{m+2}) \, a^+_{in}(\vec{k}_1) \, a^+_{in}(\vec{k}_2) \, | 0 \rangle \, , \end{aligned} \tag{3.3}$$

where $a \equiv a_1$ and the *in* and *out* states are defined as $|\vec{k}, u_1\rangle = a^+(\vec{k})|0\rangle$.

Using the map between creation-annihilation operators of bulk and boundary theories and considering fixed angle scattering, $k_i = \gamma_i \, k$ and $K_i = \Gamma_i \, K$, where γ_i and Γ_i are constants $(i = 1, 2, ..., m+2)$ we have

$$\begin{aligned} S_{Bulk} &\sim \langle 0 | \, b_{out}(\vec{K}_3) \ldots b_{out}(\vec{K}_{m+2}) \, b^+_{in}(\vec{K}_1) \, b^+_{in}(\vec{K}_2) | 0 \rangle \left(\frac{K}{k}\right)^{m+2} \\ &\sim \langle \vec{K}_3, \ldots \vec{K}_{m+2}, out \, | \vec{K}_1, \vec{K}_2, in \rangle \left(\frac{K}{k}\right)^{m+2} K^{(m+2)(d-1)} , \end{aligned} \tag{3.4}$$

where d is the dimension of the boundary composite operators and $b \equiv b_1$. The corresponding *in* and *out* states are $|\vec{K}\rangle \cong K^{1-d} b^+(\vec{K})|0\rangle$, since $K \gg \mu$.

Using now the relation (3.2) between bulk and boundary momenta we find

$$S_{Bulk} \sim S_{Bound.} \left(\frac{\sqrt{\alpha'}}{\mu}\right)^{m+2} K^{(m+2)(1+d)} . \tag{3.5}$$

The scattering amplitudes $\mathcal{A}$ are related to the S matrix by

$$\begin{aligned} S_{Bulk} &= \mathcal{A}_{Bulk}\, \delta^4(k_1^\rho + k_2^\rho - k_3^\rho - \ldots - k_{m+2}^\rho)\,, \\ S_{Bound.} &= \mathcal{A}_{Bound.}\, \delta^4(K_1^\rho + K_2^\rho - K_3^\rho - \ldots - K_{m+2}^\rho)\,, \end{aligned} \tag{3.6}$$

so we find

$$\begin{aligned} \mathcal{A}_{Bound.} &\sim \mathcal{A}_{Bulk}\, S_{Bound.}\, (S_{Bulk})^{-1} \left(\frac{K}{k}\right)^4 \\ &\sim \mathcal{A}_{Bulk}\, K^{8-(m+2)(d+1)} \left(\frac{\sqrt{\alpha'}}{\mu}\right)^{2-m} . \end{aligned} \tag{3.7}$$

The bulk scattering amplitude can be estimated from supergravity action in the AdS slice (2.4) using dimensional arguments. Note that R^8/κ^2 is dimensionless. The dimensionfull parameters are $z_{max} \sim 1/\mu$ and the Ricci scalar $\mathcal{R} \sim 1/R^2$. But $\mu \ll k$, so the relevant contribution to the amplitude at high energy will not depend on z_{max}. Further, choosing the AdS radius R such that $1/R \ll k$ (small curvature) we can disregard the contribution from the Ricci scalar to the amplitude. So, the only relevant bulk dimensionfull parameter is the momentum k.

Taking into account the normalization of states $|k, u_1\rangle$ one finds that $\mathcal{A}_{Bulk}$ has dimension of $[\text{Energy}]^{4-n}$, where $n = 2 + m$ is the total number of scattered particles. Then,

$$\mathcal{A}_{Bulk} \sim k^{2-m}\,. \tag{3.8}$$

Using again the relation (3.2) between bulk and boundary momenta and substituting the above equation in (3.7), we find

$$\mathcal{A}_{Bound.} \sim K^{4-\Delta}\,, \tag{3.9}$$

where $\Delta = (m+2)d$ is the total scaling dimension of the scattered particles associated with glueballs on the 4-d boundary.

Considering that $K \sim \sqrt{s}$ we find [12]

$$\mathcal{A}_{Bound.} \sim s^{2-\Delta/2}\,, \tag{3.10}$$

which is the QCD hard scattering result [2, 3] that was reproduced from AdS / CFT correspondence in [7]. Other interesting related discussions can be found in [13, 14, 15, 16, 17].

This phenomenological approach of introducing a mass scale by taking an AdS slice can also de used to calculate glueball mass ratios [18, 19]. This way we get a relation between the dilaton axial modes u_p and the glueball masses μ_p:

$$\frac{u_p}{\mu_p} = constant\,. \tag{3.11}$$

So the glueball masses can be related to the zeros of the Bessel functions by

$$\frac{\mu_p}{\mu_1} = \frac{\chi_{2,p}}{\chi_{2,1}}\,. \tag{3.12}$$

These results are in good agreement with lattice and supergravity calculations as shown in [18, 19].

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