

Recent Results on $N=2,4$ -Supersymmetry with Lorentz Symmetry Violating

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In this work, we propose the $N = 2$ and $N = 4$ supersymmetric extensions of the Lorentz-breaking Abelian Chern-Simons term. We formulate the question of the Lorentz violation in 6 and 10 dimensions to obtain the bosonic sectors of $N = 2-$ and $N = 4-$ supersymmetries, respectively. From this, we carry out an analysis in $N = 1 - D = 4$ superspace and, in terms of $N = 1-$ superfields, we are able to write down the $N = 2$ and $N = 4$ supersymmetric extensions of the Lorentz-violating action term.

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1. Introduction

The formulation of physical models for the fundamental interactions in the framework of quantum field theories for point-like objects is based on a number of principles, among which Lorentz covariance and invariance under suitable gauge symmetries. However, mechanisms for the breakdown of these symmetries have been proposed and discussed in view of a number of phenomenological and experimental evidences [1, 2, 3, 4, 5]. Astrophysical observations indicate that Lorentz symmetry may be slightly violated in order to account for anisotropies. Then, one may consider a gauge theory where Lorentz symmetry breaking may be realized by means of a term in the action. A Chern-Simons-type term may be considered that exhibits a constant background four-vector which maintains the gauge invariance but breaks down the Lorentz space-time symmetry [1].

In the context of supersymmetry (SUSY), the issue of Lorentz violation has been considered in the literature in different formulations: in ref. [6], supersymmetry is presented by introducing a suitable modification in its algebra; in ref. [7, 8], one achieves the $N = 1$ -SUSY version of the Chern-Simons term by means of the conventional superspace-superfield formalism; in ref. [9], the authors adopt the idea of Lorentz breaking operators. More particularly, considering the importance of extended supersymmetries in connection with gauge theories, we propose in this work an $N = 2$ and an $N = 4$ extended supersymmetric generalization of the Lorentz-breaking Chern-Simons term in a 4-dimensional Minkowski background. We start off with the Chern-Simons term in $(1+5)$ and $(1+9)$ space-time dimensions and adopt a particular dimensional reduction method, see [10], to obtain the bosonic sector in $D = (1+3)$ of the $N = 2$ and $N = 4$ supersymmetric models, respectively. This is possible because in $N = 1, D = 6$ - and $N = 1, D = 10$ -supersymmetries, the bosonic sector has the same number of degrees of freedom as the bosonic sector of an $N = 2, D = 4$ and $N = 4, D = 4$, respectively [11]. Once the bosonic sectors are identified, we adopt an $N = 1, D = 4$ -superfield formulation to write down the gauge potential and the Lorentz-violating background supermultiplets to finally set up their coupling in terms of $N = 2$ and $N = 4$ actions realized in $N = 1$ -superspace. The result is projected out in component fields and we end up with the complete actions that realize the extended supersymmetric version of the Abelian Chern-Simons Lorentz-violating term.

2. $N = 2$ -Lorentz-violating term

The $N = 2$ supersymmetric generalization of the Abelian Chern-Simons Lorentz breaking term can be built up using superfield formalism in an $N = 1$ superspace background, having coordinates $(x^\mu, \theta^a, \bar{\theta}_{\dot{a}})$ [10]. Using the fact that the bosonic sector for $N = 2$ in $D = 6$ and make the dimensional reduction for $D = 4$. The $D = 4$ Chern-Simons term proposed originally by [1] is

$$\mathcal{L}_{br} = \varepsilon^{\mu\nu\kappa\lambda} A_\mu \partial_\nu A_\kappa T_\lambda. \quad (2.1)$$

We propose for $D = 6$ the Chern-Simons term in the form

$$\mathcal{L}_{br} = \varepsilon^{\hat{\mu}\hat{\nu}\hat{\kappa}\hat{\lambda}\hat{\rho}\hat{\sigma}} A_{\hat{\mu}} \partial_{\hat{\nu}} A_{\hat{\kappa}} T_{\hat{\lambda}\hat{\rho}\hat{\sigma}}, \quad (2.2)$$

where $\hat{\mu} = \mu, 4, 5$. The gauge field has 6 components, then we redefine it as $A_{\hat{\mu}} \equiv (A_\mu; \varphi_1; \varphi_2)$. The background tensor $T_{\hat{\lambda}\hat{\rho}\hat{\sigma}}$ has 20 components, but we can redefine it as $T_{\hat{\lambda}\hat{\rho}\hat{\sigma}} \equiv (R_{\rho\sigma}; S_{\rho\sigma}; \partial_\mu \nu; \partial_\mu u)$.

The fields $R_{\rho\sigma}$ and $S_{\rho\sigma}$ has 6 components each one, and the other 8 components are redefined as 2 vectors that we write as a gradient of the scalars fields v and u . Then, the number of components is reduced to 14. The dimensional reduction is done considering that there are not dependence of the fields in the x^4, x^5 coordinates. It is clear that $\varepsilon^{\hat{\nu}\hat{\kappa}\hat{\lambda}\hat{\rho}\hat{\sigma}}A_{\hat{\mu}}A_{\hat{\kappa}}\partial_{\hat{\nu}}T_{\hat{\lambda}\hat{\rho}\hat{\sigma}} = 0$, so we obtain, integrating by parts, the reduced Lagrangian as follows:

$$\begin{aligned} \mathcal{L}_{br} = & -\frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}A_{\kappa}\partial_{\lambda}v + \frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}\varphi_1R_{\kappa\lambda} + \frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}\varphi_2S_{\kappa\lambda} \\ & + \frac{1}{2}\varphi_1\partial_{\nu}\varphi_2\partial^{\nu}u - \frac{1}{2}\varphi_2\partial_{\nu}\varphi_1\partial^{\nu}u. \end{aligned} \quad (2.3)$$

In order to make the supersymmetrization of the Lagrangian (2.3) using the superspace formalism, we have to define some complex fields that can be found in superfields. We define these bosonic fields as

$$\begin{aligned} B_{\mu\nu} &= S_{\mu\nu} - i\tilde{S}_{\mu\nu}, \\ H_{\mu\nu} &= R_{\mu\nu} - i\tilde{R}_{\mu\nu} \\ \varphi &= \varphi_1 + i\varphi_2, \\ r &= t + iu, \\ s &= w + iv, \end{aligned} \quad (2.4)$$

Notice that we have introduced the new real scalar fields t and w that are bosonic fields that do not appear in the bosonic Lagrangian (2.3). These fields will be necessary in the supersymmetric version to maintain the same number of degree of freedom between bosonic and fermionic sector due the scalar superfields are defined with complex scalar fields. Each tensor field, $R_{\mu\nu}$ and $S_{\mu\nu}$, appears as the real part of the complex tensor field whose imaginary parts are given in terms of their dual fields, as we see in (2.4) and can be found in [14]. The vector superfield V that accommodates A_{μ} in the WZ-gauge is written as:

$$V = \theta\sigma^{\mu}\bar{\theta}A_{\mu} + \theta^2\bar{\theta}\tilde{\lambda} + \bar{\theta}^2\theta\lambda + \theta^2\bar{\theta}^2D, \quad (2.5)$$

which fulfills the reality constraint, $V = V^{\dagger}$. The scalar superfield that accommodates φ and φ^* is written as

$$\Phi = \varphi + i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\varphi - \frac{1}{4}\theta^2\bar{\theta}^2\Box\varphi + \sqrt{2}\theta\psi + \frac{i}{\sqrt{2}}\theta^2\partial_{\mu}\psi\sigma^{\mu}\bar{\theta} + \theta^2f, \quad (2.6)$$

and this complex conjugate $\bar{\Phi}$. These superfields obey the chiral condition: $\bar{D}\Phi = D\bar{\Phi} = 0$. The scalar superfields that accommodate s , r and r and their respective complex conjugate fields are:

$$S = s + i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}s - \frac{1}{4}\theta^2\bar{\theta}^2\Box s + \sqrt{2}\theta\xi + \frac{i}{\sqrt{2}}\theta^2\partial_{\mu}\xi\sigma^{\mu}\bar{\theta} + \theta^2h, \quad (2.7)$$

$$R = r + i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}r - \frac{1}{4}\theta^2\bar{\theta}^2\Box r + \sqrt{2}\theta\zeta + \frac{i}{\sqrt{2}}\theta^2\partial_{\mu}\zeta\sigma^{\mu}\bar{\theta} + \theta^2g, \quad (2.8)$$

and their complex conjugate superfields \bar{S} and \bar{R} which satisfy the chiral condition: $\bar{D}S = D\bar{S} = \bar{D}R = D\bar{R} = 0$. The spinor superfields that contain $R_{\mu\nu}, S_{\mu\nu}$ and their corresponding dual fields are

written as

$$\begin{aligned} \Sigma_a = & \tau_a + \theta^b (\varepsilon_{ba} \rho + \sigma_{ba}^{\mu\nu} B_{\mu\nu}) + \theta^2 F_a + i\theta \sigma^\mu \bar{\theta} \partial_\mu \tau_a \\ & + i\theta \sigma^\mu \bar{\theta} \theta^b \partial_\mu (\varepsilon_{ba} \rho + \sigma_{ba}^{\mu\nu} B_{\mu\nu}) - \frac{1}{4} \theta^2 \bar{\theta}^2 \square \tau_a, \end{aligned} \quad (2.9)$$

$$\begin{aligned} \Omega_a = & \chi_a + \theta^b (\varepsilon_{ba} \phi + \sigma_{ba}^{\mu\nu} H_{\mu\nu}) + \theta^2 G_a + i\theta \sigma^\mu \bar{\theta} \partial_\mu \chi_a \\ & + i\theta \sigma^\mu \bar{\theta} \theta^b \partial_\mu (\varepsilon_{ba} \phi + \sigma_{ba}^{\mu\nu} H_{\mu\nu}) - \frac{1}{4} \theta^2 \bar{\theta}^2 \square \chi_a, \end{aligned} \quad (2.10)$$

and their complex conjugate superfields $\bar{\Sigma}$ and $\bar{\Omega}$ that are also chiral: $\bar{D}_b \Sigma_a = D_b \bar{\Sigma}_a = \bar{D}_b \Omega_a = D_b \bar{\Omega}_a = 0$. We can notice that we have to introduce two extra background complex scalar fields, ρ and ϕ , to match the bosonic and fermionic degrees of freedom.

Now, we are interested in building up the supersymmetric action. For that, we take into consideration the canonical (mass) dimensions of the superfields; based on these dimensionalities, and by analyzing the bosonic Lagrangian (2.3), we propose the following supersymmetric action, S_{br} :

$$\begin{aligned} S_{br} = & \int d^4x d^2\theta d^2\bar{\theta} \left[\frac{1}{4} W^a (D_a V) S + \frac{1}{4} \bar{W}_a (\bar{D}^a V) \bar{S} + \frac{i}{4} \delta(\bar{\theta}) W^a (\Phi + \bar{\Phi}) \Sigma_a \right. \\ & - \frac{i}{4} \delta(\theta) \bar{W}_a (\Phi + \bar{\Phi}) \bar{\Sigma}^a + \frac{1}{4} \delta(\bar{\theta}) W^a (\Phi - \bar{\Phi}) \Omega_a \\ & \left. - \frac{1}{4} \delta(\theta) \bar{W}_a (\Phi - \bar{\Phi}) \bar{\Omega}^a + \frac{1}{4} \Phi \bar{\Phi} (\bar{R} + R) \right], \end{aligned} \quad (2.11)$$

We therefore observe that the action (2.11) is manifestly invariant under $N = 1$ -supersymmetry. The component-field content of the $N = 2$ -supersymmetry is accommodated in the $N = 1$ -superfields). Indeed, the action (2.11) displays a larger supersymmetry, $N = 2$, realized in terms of an $N = 1$ -superspace formulation.

This Lagrangian in its component-field version reads as below:

$$\begin{aligned} \mathcal{L}_{br} = & + \frac{i}{8} \partial_\mu (s - s^*) \varepsilon^{\mu\kappa\lambda\nu} F_{\kappa\lambda} A_\nu - \frac{1}{8} (s + s^*) F_{\mu\nu} F^{\mu\nu} + D^2 (s + s^*) \\ & - \frac{1}{2} i s \lambda \sigma^\mu \partial_\mu \bar{\lambda} - \frac{1}{2} i s^* \bar{\lambda} \bar{\sigma}^\mu \partial_\mu \lambda - \frac{1}{2\sqrt{2}} \lambda \sigma^{\mu\nu} F_{\mu\nu} \xi + \frac{1}{2\sqrt{2}} \bar{\lambda} \bar{\sigma}^{\mu\nu} F_{\mu\nu} \bar{\xi} \\ & + \frac{1}{4} \lambda \lambda h + \frac{1}{4} \bar{\lambda} \bar{\lambda} h^* - \frac{1}{\sqrt{2}} \lambda \xi D - \frac{1}{\sqrt{2}} \bar{\lambda} \bar{\xi} D \\ & + \frac{1}{16} \varepsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} (\varphi + \varphi^*) (B_{\kappa\lambda} + B_{\kappa\lambda}^*) + \frac{i}{8} F^{\mu\nu} (B_{\mu\nu} - B_{\mu\nu}^*) (\varphi + \varphi^*) \\ & - \frac{i\sqrt{2}}{8} \tau \sigma^{\mu\nu} \psi F_{\mu\nu} - \frac{i\sqrt{2}}{8} \bar{\tau} \bar{\sigma}^{\mu\nu} \bar{\psi} F_{\mu\nu} + \frac{1}{4} \tau \sigma^\mu \partial_\mu \bar{\lambda} (\varphi + \varphi^*) \\ & - \frac{1}{4} \bar{\tau} \bar{\sigma}^\mu \partial_\mu \lambda (\varphi + \varphi^*) + \frac{i\sqrt{2}}{4} \psi \sigma^{\mu\nu} B_{\mu\nu} \lambda + \frac{i\sqrt{2}}{4} \bar{\psi} \bar{\sigma}^{\mu\nu} B_{\mu\nu}^* \bar{\lambda} \\ & - \frac{i}{2} D (\varphi + \varphi^*) \rho + \frac{i}{2} D^* (\varphi + \varphi^*) \rho^* \\ & + \frac{i\sqrt{2}}{8} \lambda \psi \rho - \frac{i\sqrt{2}}{8} \bar{\lambda} \bar{\psi} \rho^* - \frac{i\sqrt{2}}{4} D \psi \tau + \frac{i\sqrt{2}}{4} D^* \bar{\psi} \bar{\tau} \end{aligned} \quad (2.12)$$

$$\begin{aligned}
& +\frac{i}{4}f\lambda\tau - \frac{i}{4}f^*\bar{\lambda}\bar{\tau} + \frac{i}{4}(\varphi + \varphi^*)\lambda F - \frac{i}{4}(\varphi + \varphi^*)\bar{\lambda}\bar{F} \\
& - \frac{i}{16}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}(\varphi - \varphi^*)(H_{\kappa\lambda} + H_{\kappa\lambda}^*) + \frac{1}{8}F^{\mu\nu}(H_{\kappa\lambda} - H_{\kappa\lambda}^*)(\varphi - \varphi^*) \\
& - \frac{\sqrt{2}}{8}\chi\sigma^{\mu\nu}\psi F_{\mu\nu} + \frac{\sqrt{2}}{8}\bar{\chi}\bar{\sigma}^{\mu\nu}\bar{\psi}F_{\mu\nu} - \frac{i}{4}\chi\sigma^\mu\partial_\mu\bar{\lambda}(\varphi - \varphi^*) \\
& + \frac{i}{4}\bar{\chi}\bar{\sigma}^\mu\partial_\mu\lambda(\varphi - \varphi^*) + \frac{\sqrt{2}}{4}\psi\sigma^{\mu\nu}H_{\mu\nu}\lambda - \frac{\sqrt{2}}{4}\bar{\psi}\bar{\sigma}^{\mu\nu}H_{\mu\nu}^*\bar{\lambda} \\
& - \frac{1}{2}D(\varphi - \varphi^*)\phi + \frac{1}{2}D^*(\varphi - \varphi^*)\phi^* \\
& + \frac{\sqrt{2}}{8}\lambda\psi\phi + \frac{\sqrt{2}}{8}\bar{\lambda}\bar{\psi}\phi^* - \frac{\sqrt{2}}{4}D\psi\chi - \frac{\sqrt{2}}{4}D^*\bar{\psi}\bar{\chi} \\
& + \frac{1}{4}f\lambda\chi + \frac{1}{4}f^*\bar{\lambda}\bar{\chi} + \frac{1}{4}(\varphi - \varphi^*)\lambda G - \frac{1}{4}(\varphi - \varphi^*)\bar{\lambda}\bar{G} \\
& + \frac{1}{8}\varphi\partial_\mu\varphi^*\partial^\mu(r - r^*) - \frac{1}{8}\varphi^*\partial_\mu\varphi\partial^\mu(r - r^*) \\
& + \frac{1}{4}\partial^\mu\varphi\partial_\mu\varphi^*(r + r^*) - \frac{1}{8}\varphi\varphi^*\square(r + r^*) - \frac{i}{4}\psi\sigma^\mu\partial_\mu\bar{\psi}(r + r^*) \\
& + \frac{1}{4}ff^*(r + r^*) - \frac{i}{4}\psi\sigma^\mu\bar{\psi}\partial_\mu r^* \\
& - \frac{i}{4}\varphi\zeta\sigma^\mu\partial_\mu\bar{\psi} - \frac{i}{4}\varphi^*\psi\sigma^\mu\partial_\mu\bar{\zeta} - \frac{i}{4}\psi\sigma^\mu\bar{\zeta}\partial_\mu\varphi^* \\
& + \frac{1}{4}\varphi f^*g + \frac{1}{4}f\varphi^*g^* - \frac{1}{4}f^*\psi\zeta - \frac{1}{4}f\bar{\psi}\bar{\zeta}.
\end{aligned}$$

We point out the pieces corresponding to the bosonic action (2.3) in the complete component-field action above:

$$\begin{aligned}
& \frac{i}{8}\partial_\mu(s - s^*)\varepsilon^{\mu\kappa\lambda\nu}F_{\kappa\lambda}A_\nu = -\frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}A_\kappa\partial_\lambda\nu, \\
& \frac{1}{16}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}(\varphi + \varphi^*)(B_{\kappa\lambda} + B_{\kappa\lambda}^*) = \frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}\varphi_1R_{\kappa\lambda}, \\
& -\frac{i}{16}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}(\varphi - \varphi^*)(H_{\kappa\lambda} + H_{\kappa\lambda}^*) = \frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}\varphi_2S_{\kappa\lambda}, \\
& \frac{1}{8}\varphi\partial_\mu\varphi^*\partial^\mu(r - r^*) - \frac{1}{8}\varphi^*\partial_\mu\varphi\partial^\mu(r - r^*) = \frac{1}{2}\varphi_1\partial_\nu\varphi_2\partial^\nu u - \frac{1}{2}\varphi_2\partial_\nu\varphi_1\partial^\nu u.
\end{aligned}$$

We can notice that this Lagrangian describes the bosonic sector (2.3) and its superpartners. We find here the $N = 1$ supersymmetrization of the Chern-Simons term presented in [7], where the first term is the same as proposed by [1], considering the constant vector as the gradient of a scalar. Since the gradient vector is a constant, we have that $s = \alpha + \beta^\mu x_\mu$. We see in the Lagrangian the presence of the bosonic real scalar fields, $t = s + s^*$ and $u = r + r^*$, and the complex scalar fields, ρ and ϕ , that do not appear in the bosonic Lagrangian (2.3). These scalar fields appear in the supersymmetric generalization in order to keep the bosonic and fermionic degrees of freedom in equal number. We point out that the bosonic fields $D, D^*, f, f^*, h, h^*, g$ and g^* play all the role of auxiliary fields. The bosonic fields $s, s^*, R_{\mu\nu}, S_{\mu\nu}, \rho, \rho^*, \phi, \phi^*, r, r^*$ and the fermionic fields $\xi, \bar{\xi}, \tau, \bar{\tau}, F, \bar{F}, \chi, \bar{\chi}, G, \bar{G}, \zeta, \bar{\zeta}$ work as background fields also responsible for the breaking the Lorentz invariance.

3. $N = 4$ -Lorentz-violating term

In a very close analogy to the procedure adopted in the previous section, we succeed in writing down the $N = 4$ model by means of a reduction from 10 to 4 dimensions. We propose for $D = 10$ the Chern-Simons term in the form

$$\mathcal{L}_{br} = \varepsilon^{\hat{\mu}\hat{\nu}\hat{\kappa}\hat{\lambda}\hat{\rho}\hat{\sigma}\hat{\delta}\hat{\tau}\hat{\beta}\hat{\gamma}} A_{\hat{\mu}} \partial_{\hat{\nu}} A_{\hat{\kappa}} T_{\hat{\lambda}\hat{\rho}\hat{\sigma}\hat{\delta}\hat{\tau}\hat{\beta}\hat{\gamma}}. \quad (3.1)$$

The background tensor $T_{\hat{\lambda}\hat{\rho}\hat{\sigma}}$ has 120 components, and we can redefine it as

$$T_{\hat{\lambda}\hat{\rho}\hat{\sigma}} \equiv (R_{\rho\sigma}^I; \partial_{\mu} v; \partial_{\mu} u^{IJ}),$$

where $\hat{\mu} = \mu, 4, 5, 6, 7, 8, 9$ is the space-time index and $I, J = 1, 2, 3, 4, 5, 6$ is an internal index. We consider that there is no dependence of the fields on the $x^4, x^5, x^6, x^7, x^8, x^9$ coordinates. Then, we have 6 anti-symmetric tensor fields $R_{\rho\sigma}^I$ with 6 components each one and 15 vectors written as gradients of 15 scalars represented by the anti-symmetric index I, J . Therefore, the number of independent components is reduced to 52.

Next, we need to redefine the gauge field as $A_{\hat{\mu}} \equiv (A_{\mu}; \varphi^I, I = 1, 2, 3, 4, 5, 6)$ where φ^I is real scalar fields. Observing that $\varepsilon^{\hat{\mu}\hat{\nu}\hat{\kappa}\hat{\lambda}\hat{\rho}\hat{\sigma}\hat{\delta}\hat{\tau}\hat{\beta}\hat{\gamma}} A_{\hat{\mu}} A_{\hat{\kappa}} \partial_{\hat{\nu}} T_{\hat{\lambda}\hat{\rho}\hat{\sigma}\hat{\delta}\hat{\tau}\hat{\beta}\hat{\gamma}} = 0$, we obtain, integrating by parts, the Lagrangian as follows:

$$\mathcal{L}_{br} = -\frac{1}{4} \varepsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} A_{\kappa} \partial_{\lambda} v + \frac{1}{4} \varepsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} \varphi^I R_{\kappa\lambda}^I + \frac{1}{2} \varphi^I \partial_{\nu} \varphi^J \partial^{\nu} u^{IJ}. \quad (3.2)$$

This is the bosonic sector of the action term to be supersymmetrized. In this way, is necessary to define new fields to be partners inside the superfields. They are similar to the procedure of the previous section, but now there are internal index. In terms of superfields, we have two sectors:

$$\begin{aligned} \text{Gauge Sector} &: \{V, \Phi^I\} \\ \text{Background Sector} &: \{\Sigma_a^I, \bar{\Sigma}^{\dot{a}I}, S, \bar{S}, R^{IJ}, \bar{R}^{IJ}\}, \end{aligned}$$

and, in components these two sectors encompass the fields cast below:

$$\begin{aligned} \text{Bosonic gauge Sector} &: \{A_{\mu}, \varphi^I, \varphi^{*I}\} \\ \text{Fermionic gauge Sector} &: \{\lambda, \bar{\lambda}, \psi^I, \bar{\psi}^I\} \\ \text{Bosonic background Sector} &: \{s, s^*, R_{\mu\nu}^I, \rho^I, \rho^{*I}, r^{IJ}, r^{*IJ}\} \\ \text{Fermionic background Sector} &: \{\xi, \bar{\xi}, \tau^I, \bar{\tau}^I, F^I, \bar{F}^I, \zeta^{IJ}, \bar{\zeta}^{IJ}\}. \end{aligned}$$

Based on dimensional analysis arguments for the bosonic sector, as it has been done for the $N = 2$ case, and noticing that some superfields now have internal symmetry index, we propose the following $N = 4$ supersymmetric action:

$$\begin{aligned} S_{br} = \int d^4 x d^2 \theta d^2 \bar{\theta} & \left[\frac{1}{4} W^a (D_a V) S + \frac{1}{4} \bar{W}_{\dot{a}} (\bar{D}^{\dot{a}} V) \bar{S} + \frac{i}{4} \delta(\bar{\theta}) W^a (\Phi^I + \bar{\Phi}^I) \Sigma_a^I \right. \\ & \left. - \frac{i}{4} \delta(\theta) \bar{W}_{\dot{a}} (\Phi^I + \bar{\Phi}^I) \bar{\Sigma}^{\dot{a}I} + \frac{1}{4} \Phi^I \bar{\Phi}^J (R^{IJ} - \bar{R}^{IJ}) \right], \end{aligned} \quad (3.3)$$

We can observe that the action (3.3) is invariant under $N = 1$ -supersymmetry and there is a larger symmetry, the $N = 4$ -supersymmetry as well.

This $N = 4$ Lagrangian in its component-field version reads as follows:

$$\begin{aligned}
\mathcal{L}_{br} = & +\frac{i}{8}\partial_\mu(s-s^*)\varepsilon^{\mu\kappa\lambda\nu}F_{\kappa\lambda}A_\nu - \frac{1}{8}(s+s^*)F_{\mu\nu}F^{\mu\nu} + D^2(s+s^*) \\
& -\frac{1}{2}is\lambda\sigma^\mu\partial_\mu\bar{\lambda} - \frac{1}{2}is^*\bar{\lambda}\bar{\sigma}^\mu\partial_\mu\lambda - \frac{1}{2\sqrt{2}}\lambda\sigma^{\mu\nu}F_{\mu\nu}\xi + \frac{1}{2\sqrt{2}}\bar{\lambda}\bar{\sigma}^{\mu\nu}F_{\mu\nu}\bar{\xi} \\
& +\frac{1}{4}\lambda\lambda h + \frac{1}{4}\bar{\lambda}\bar{\lambda}h^* - \frac{1}{\sqrt{2}}\lambda\xi D - \frac{1}{\sqrt{2}}\bar{\lambda}\bar{\xi}D \\
& \frac{1}{16}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}(\varphi^I + \varphi^{*I})(B_{\kappa\lambda}^I + B_{\kappa\lambda}^{*I}) + \frac{i}{8}F^{\mu\nu}(B_{\mu\nu}^I - B_{\mu\nu}^{*I})(\varphi^I + \varphi^{*I}) \\
& -\frac{i\sqrt{2}}{8}\tau^I\sigma^{\mu\nu}\psi^I F_{\mu\nu} - \frac{i\sqrt{2}}{8}\bar{\tau}^I\bar{\sigma}^{\mu\nu}\bar{\psi}^I F_{\mu\nu} + \frac{1}{4}\tau^I\sigma^\mu\partial_\mu\bar{\lambda}(\varphi^I + \varphi^{*I}) \\
& -\frac{1}{4}\bar{\tau}^I\bar{\sigma}^\mu\partial_\mu\lambda(\varphi^I + \varphi^{*I}) + \frac{i\sqrt{2}}{4}\psi^I\sigma^{\mu\nu}B_{\mu\nu}^I\lambda + \frac{i\sqrt{2}}{4}\bar{\psi}^I\bar{\sigma}^{\mu\nu}B_{\mu\nu}^{*I}\bar{\lambda} \\
& -\frac{i}{2}D(\varphi^I + \varphi^{*I})\rho^I + \frac{i}{2}D^*(\varphi^I + \varphi^{*I})\rho^{*I} \\
& +\frac{i\sqrt{2}}{8}\lambda\psi^I\rho^I - \frac{i\sqrt{2}}{8}\bar{\lambda}\bar{\psi}^I\rho^{*I} - \frac{i\sqrt{2}}{4}D\psi^I\tau^I + \frac{i\sqrt{2}}{4}D^*\bar{\psi}^I\bar{\tau}^I \\
& +\frac{i}{4}f^I\lambda\tau^I - \frac{i}{4}f^{*I}\bar{\lambda}\bar{\tau}^I + \frac{i}{4}(\varphi^I + \varphi^{*I})\lambda F^I - \frac{i}{4}(\varphi^I + \varphi^{*I})\bar{\lambda}\bar{F}^I \\
& +\frac{1}{8}\varphi^I\partial_\mu\varphi^{*J}\partial^\mu(r^{IJ} + r^{*IJ}) - \frac{1}{8}\varphi^{*J}\partial_\mu\varphi^I\partial^\mu(r^{IJ} + r^{*IJ}) \\
& +\frac{1}{4}\partial^\mu\varphi^I\partial_\mu\varphi^{*J}(r^{IJ} - r^{*IJ}) - \frac{1}{8}\varphi^I\varphi^{*J}\square(r^{IJ} - r^{*IJ}) - \frac{i}{4}\psi^I\sigma^\mu\partial_\mu\bar{\psi}^J(r^{IJ} - r^{*IJ}) \\
& +\frac{1}{4}f^I f^{*J}(r^{IJ} - r^{*IJ}) + \frac{i}{4}\psi^I\sigma^\mu\bar{\psi}^J\partial_\mu r^{*IJ} \\
& -\frac{i}{4}\varphi^I\zeta^{IJ}\sigma^\mu\partial_\mu\bar{\psi}^J + \frac{i}{4}\varphi^{*J}\psi^I\sigma^\mu\partial_\mu\bar{\zeta}^{IJ} + \frac{i}{4}\psi^I\sigma^\mu\bar{\zeta}^{IJ}\partial_\mu\varphi^{*J} \\
& +\frac{1}{4}\varphi^I f^{*J}g^{IJ} - \frac{1}{4}f^I\varphi^{*J}g^{*IJ} - \frac{1}{4}f^{*J}\psi^I\zeta^{IJ} + \frac{1}{4}f^I\bar{\psi}^J\bar{\zeta}^{IJ}.
\end{aligned} \tag{3.4}$$

We can ascertain the presence of the bosonic sector (3.2) by means of the terms below:

$$\begin{aligned}
\frac{i}{8}\partial_\mu(s-s^*)\varepsilon^{\mu\kappa\lambda\nu}F_{\kappa\lambda}A_\nu &= -\frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}A_\kappa\partial_\lambda\nu, \\
\frac{1}{16}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}(\varphi^I + \varphi^{*I})(B_{\kappa\lambda}^I + B_{\kappa\lambda}^{*I}) &= \frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}\varphi^I R_{\kappa\lambda}^I, \\
\frac{1}{8}\varphi^I\partial_\mu\varphi^{*J}\partial^\mu(r^{IJ} + r^{*IJ}) - \frac{1}{8}\varphi^{*J}\partial_\mu\varphi^I\partial^\mu(r^{IJ} + r^{*IJ}) &= \frac{1}{2}(\varphi^I\partial_\nu\varphi^J + \beta^I\partial_\nu\beta^J)\partial^\nu u^{IJ}.
\end{aligned}$$

We can notice that this Lagrangian fairly accommodates the $N = 4$ bosonic sector (3.2). We re-obtain here the $N = 1$ and $N = 2$ supersymmetrization of the Chern-Simons term presented in ref.[7] and in (2.12), respectively. We notice that $N = 4$ Lagrangian is similar to $N = 2$ but now existing an internal index in same fields. The fields β^I , t , u^{IJ} and ρ^{IJ} , that do not appear in the bosonic Lagrangian (3.2), were introduced in order to keep the bosonic and fermionic degrees of freedom in equal number. We can see that the bosonic fields D, D^* , f^I , f^{*I} , h, h^* , g^{IJ} and g^{*IJ} works as auxiliary

fields. The bosonic fields $s, s^*, R_{\mu\nu}^I, \rho^I, \rho^{*I}, r^{IJ}, r^{*IJ}$ and the fermionic fields $\xi, \bar{\xi}, \tau^I, \bar{\tau}^I, F^I, \bar{F}^I, \zeta^{IJ}, \bar{\zeta}^{IJ}$ work as background fields breaking the Lorentz invariance.

4. Concluding remarks and comments

In the important context of studying the gauge invariant Lorentz-violating term formulated as a Chern-Simons action term, we propose here its $N = 2$ and $N = 4$ supersymmetric versions. This program can be carry out in a simple way with the help of a dimensional reduction method; here, we have chosen the method à la Scherk, but it would also be interesting to contemplate other possibilities, such as the procedures à la Legendre or à la Kaluza-Klein. With our reduction scheme, we could treat the extended supersymmetric version in terms of simple $N = 1$ superspace to supersymmetrize the Chern-Simons like term, as proposed by Jackiw, written in terms of a constant background vector here parametrized as the gradient of the scalar function $\alpha + \beta_\mu x^\mu$, where α and β^μ are constants.

Another interesting point we should consider is the possibility, once we have now the full set of SUSY partners of the Lorentz-breaking vector, to express the central charges of the extended models whenever topologically non-trivial configurations are taken into account. This would allow us to impose bounds on the central charges in terms of the phenomenological constraints already imposed on the vector responsible for the Lorentz covariance breakdown.

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