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Present and future applications of galaxy clusters in cosmology

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A brief summary measurements using X-ray clusters of galaxies of the density of dark matter, the normalization of the matter power spectrum, neutrino masses, and especially the equation of state of the dark energy, the interaction between dark energy and ordinary matter, gravitational holography, and the effects extra-dimensions of brane-world gravity is given.

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1. Ordinary matter

The observed cosmic density fluctuations are very well summarized by a low matter density ΛCDM model. Therefore, many cosmological tests refer to this structure formation scenario. In general, baryonic matter, Cold Dark Matter (CDM), primeval thermal remnants (electromagnetic radiation, neutrinos), and an energy corresponding to the cosmological constant give the total (normalized) density of the present Universe, $\Omega_{\rm tot} = \Omega_{\rm b} + \Omega_{\rm CDM} + \Omega_{\rm r} + \Omega_{\Lambda}$. The normalized density of ordinary matter, $\Omega_{\rm m}$, comprises the first three components. Recent CMB data suggest $\Omega_{\rm tot} = 1.02 \pm 0.02$ (Spergel et al. 2003), i.e., an effectively flat universe with a negligible spatial curvature. The same data suggest a baryon density of $\Omega_{\rm b}h^2 = 0.024 \pm 0.001$ and $h = 0.72 \pm 0.05$. For our purposes, the energy density of thermal remnants can be neglected, so that $\Omega_{\rm m} = \Omega_{\rm b} + \Omega_{\rm CDM}$.

Cluster abundance measurements are a classical application of galaxy clusters in cosmology to determine the present density of ordinary matter and the variance of the matter fluctuations in spherical cells with radius R and Fourier transform $W(kR)$: $\sigma^2(R) = \frac{1}{2\pi^2}\int_0^\infty dk\,k^2\,P(k)\,|W(kR)|^2$. The specific value σ_8 at $R = 8\,h^{-1}$ Mpc characterizes the normalization of the matter power spectrum $P(k)$. Consider the expected number of clusters observed at a certain redshift and flux limit, $dN(z, f_{\rm lim}) = dV(z)\int_{M_{\rm lim}(z,f_{\rm lim})}^{\infty} dM\,\frac{dn(M,z,\sigma^2(M))}{dM}$. The cosmology-dependency of dN stems from the comoving volume element dV, the mass limit $M_{\rm lim}$ at a certain redshift, and the shape of the cosmic mass function dn/dM. The summation in the theoretical mass abundances is over cluster mass whereas observations yield quantities like X-ray luminosity, gas temperature, richness etc. The conversion of such observables into mass is the most crucial step where most of the systematic errors can occure. The overall statistical effect is difficult to quantify, but systematic errors in the cosmological parameters on the 20 percent level can be reached (Randall et al. 2002).

With the REFLEX sample (Böhringer et al. 2004), the classical $\Omega_{\rm m}$-σ_8 test was performed with the Karhunen-Loewe eigenvector base (Schuecker et al. 2002, 2003a). The observed Gaussianity of the matter field directly translates into a multi-variant Gaussian likelihood function, and includes in a natural manner a weighting of the squared differences between KL-transformed observed and modeled cluster counts with the variances of the transformed counts. For the test, further cosmological parameters like the Hubble constant, the primordial slope of the power spectrum, the baryon density, the biasing model, and the empirical mass/X-ray luminosity relation had fixed prior values. The final result is obtained by marginalizing over these parameters and yields the 1σ corridors $0.28 \le \Omega_m \le 0.37$ and $0.56 \le \sigma_8 \le 0.80$. The largest uncertainty in these estimates comes from the empirical mass/X-ray luminosity relation obtained for REFLEX from ROSAT pointed observations. Tests are in preparation with a four-times larger X-ray cluster sample (1 500 clusters), and a more precise M/L-relation obtained over a larger mass range with the XMM-Newton satellite. Errors below the 10-percent level are expected.

White et al. (1993) pointed out that the matter content in rich clusters provides a fair sample of the matter content of the Universe. The ratio of the baryonic to total mass in clusters should thus give a good estimate of $\Omega_{\rm b}/\Omega_{\rm m}$. The combination with determinations of $\Omega_{\rm b}$ from cosmic nucleosynthesis (constrained by the observed abundances of light elements at high z) can thus be used to determine $\Omega_{\rm m}$. At a certain distance from the center of the quite relaxed cooling core clusters, it was found that the observed X-ray gas mass fraction tends to converge to a universal value. After further corrections, the results obtained by Allen et al. (2003) yields

$$\Omega_{\rm m} = 0.29 \pm 0.04 . \tag{1.1}$$

and $\sigma_8 = 0.70 \pm 0.04$. Other measurements show the $\Omega_{\rm m}$-σ_8 degeneracy more pronounced over a larger range. When all measurements are evaluated at $\Omega_{\rm m} = 0.3$, the values of σ_8 appear quite consistent at a comparatively low normalization of

$$\sigma_8 = 0.76 \pm 0.10 , \qquad (\text{formal } 1\sigma \text{ range}) \tag{1.2}$$

within the total range $0.5 < \sigma_8 < 1.0$ (data compiled in Henry 2004).

Recent neutrino experiments are based on atmospheric, solar, reactor, and accelerator neutrinos. All experiments suggest that neutrinos change flavour as they travel from the source to the detector. These experiments give strong arguments for neutrino oscillations and thus nonzero neutrino rest masses m_ν . Further information can be obtained from astronomical data on cosmological scales. The basic idea is measure the normalization of the matter CDM spectrum with CMB anisotropies on several hundred Mpc scales. This normalization is transformed with structure growth functions to $8\,h^{-1}$ Mpc at $z = 0$ assuming various neutrino contributions. This normalization should match the σ_8 normalization from cluster counts. Recent estimates are obtained by combining CMB-WMAP data with the 2dFGRS galaxy power spectrum, X-ray cluster gas mass fractions, and X-ray cluster luminosity functions (Allen, Schmidt & Bridle 2003). For a flat universe and three degenerate neutrino species, they measured the contribution of neutrinos to the energy density of the Universe, and a species-summed neutrino mass, and their respective 1σ errors,

$$\Omega_\nu = 0.0059^{+0.0033}_{-0.0027} , \quad \sum_i m_i = 0.56^{+0.30}_{-0.26}\,{\rm eV} , \tag{1.3}$$

which formally corresponds to $m_\nu \approx 0.2$ eV per neutrino. Estimates from neutrino oscillations suggest $m_\nu \approx 0.05$ eV for at least one of two neutrino species.

2. Dark energy

The combination of recent measurements obtained with three different observational approaches (galaxy clusters: Schuecker et al. 2003b; type-Ia SNe: Riess et al. 2004; CMB-WMAP: Spergel et al. 2003) shows that the cosmic matter density is close to $\Omega_{\rm m} = 0.3$, and that the normalized cosmological constant is around $\Omega_\Lambda = 0.7$. This sums up to unit total cosmic energy density and suggests a spatially flat universe. However, the density of ordinary matter growths with redshift like $(1+z)^3$ whereas the density related to the cosmological constant is independent of z. The ratio $\Omega_\Lambda/\Omega_{\rm m}$ today is close to unity and must thus be a finely-tuned infinitesimal constant set in the very early Universe (cosmic coincidence problem). An alternative hypothesis is to consider a time-evolving ‘dark energy’, where in Einstein’s field equations the time-independent energy density ρ_Λ of the cosmological constant is replaced by a time-dependent dark energy density $\rho_{\rm x}(t)$. For a time-evolving inhomogeneous field (see recent review in Peebles & Ratra 2004) the aim is to understand the coincidence in terms of dynamics. A central role in these studies is assumed by the phenomenological ratio $w_{\rm x} = \frac{p_{\rm x}}{\rho_{\rm x} c^2}$ (equation of state) between the pressure $p_{\rm x}$ of the unknown energy component and its rest energy density ρ_x. Note that $w_x = -1$ for Einstein's cosmological constant.

The resulting phase space diagram of dark energy distinguishes different physical states of the two-component cosmic substratum – separated by two energy conditions of general relativity (Schuecker et al. 2003b). The strong energy condition (SEC): $\rho + 3p/c^2 \geq 0$ *and* $\rho + p/c^2 \geq 0$, derived from the more general condition $R_{\mu\nu}v^\mu v^\nu \geq 0$, where $R_{\mu\nu}$ is the Ricci tensor for the geometry and v^μ a timelike vector. The SEC ensures that gravity is always attractive. Phenomenologically, violation of SEC means $w_x < -1/3$ for a single energy component with density $\rho_x > 0$. For $w_x \geq -1/3$, SEC is not violated and we have a decelerated cosmic expansion. The null energy condition (NEC): $\rho + p/c^2 \geq 0$, derived from the more general condition $G_{\mu\nu}k^\mu k^\nu \geq 0$, where $G_{\mu\nu}$ is the geometry-dependent Einstein tensor and k^μ a null vector (energy-momentum tensors as for SEC). Violations of this condition are recently studied theoretically in the context of macroscopic traversable wormholes and the holographic principle. The breaking of this criterion in a finite local region would have subtle consequences like the possibility for the creation of "time machines". Violating the energy condition in the cosmological case is not as dangerous (no threat to causality, no need to involve chronology protection, etc.), since one cannot isolate a chunk of the energy to power such exotic objects. Nevertheless, violation of NEC on cosmological scales could excite phenomena like super-acceleration of the cosmic scale factor (Caldwell 2002). Phenomenologically, violation of NEC means $w_x < -1$ for a single energy component with $\rho_x > 0$. The sort of energy related to this state of a Friedmann-Robertson-Walker (FRW) spacetime is dubbed phantom energy and is described by super-quintessence models. For $w_x \geq -1$ NEC is not violated, and is described by quintessence or super-quintessence models. Assuming a spatially flat FRW geometry, $\Omega_m + \Omega_x = 1$, and $\Omega_m \geq 0$, the formal conditions for this two-component cosmic fluid translates into $w_x \geq -1/3(1-\Omega_m)$ for SEC, and $w_x \geq -1/(1-\Omega_m)$ for NEC (Schuecker et al. 2003b). These energy conditions, characterizing the possible phases of the dark energy, thus rely on the precise knowledge of Ω_m and w_x. Unfortunately, the effects of w_x are not very large. However, a variety of complementary observational approaches and their combination helps to reduce the measurement errors significantly.

The most direct (geometric) effect of w_x is to change cosmological distances. For example, for a spatially flat universe, comoving distances in dimensionless form where a less negative w_x increases the Hubble parameter and thus reduces all cosmic distances. Structure growth via gravitational instability provides a further probe of w_x. Dark energy, not in form of a cosmological constant or vacuum energy density, is inhomogenously distributed - a smoothly distributed, time-varying component is unphysical because it would not react to local inhomogeneities of the other cosmic fluid and would thus violate the equivalence principle. An evolving scalar field with $w_x < 0$ (e.g. quintessence) automatically satisfies these conditions. The field is so light that it behaves relativistically on small scales and non-relativistically on large scales. The field may develop density perturbations on Gpc scales where sound speeds $c_s^2 < 0$, but does not clump on scales smaller than galaxy clusters. In the linear regime, and when dark energy is modeled as a dynamical scalar field, the rate of growth of linear density perturbations in the CDM is damped by the Hubble parameter. The evolution equation can be solved approximately (Caldwell, Dave & Steinhardt 1999). It is seen that dark energy delays structure growth. To reach the same fluctuations in the CDM field, structures must have formed at higher z compared to the standard CDM model. The sensitivity of

CMB anisotropies to w_x is limited to the integrated Sachs-Wolfe effect because Ω_x dominates only at late z. In the nonlinear regime, the effects of dark energy are not very large (Lahav et al. 1991 for a discussion of the effects of the cosmological constant). However, dark energy delays structure growth so that dark matter haloes are formed at higher z with higher core densities so that they appear more concentrated in $w_x \neq -1$ models compared to the cosmological constant. The first semi-analytic computations of a spherical collapse in a fluid with dark energy with $-1 \leq w_x < 0$ were performed by Wang & Steinhardt (1998). Schuecker et al. (2003b) enlarged the range to $-5 < w_x < 0$, whereas Mota & van de Bruck (2004) discussed the spherical collapse for specific potentials of scalar fields including also a redshift-dependent $w_x(z)$.

The abundance of clusters at redshifts $z > 0.5$ is very important for future planned cluster surveys (e.g. DUO Griffiths et al. 2004) where in the wide (northern) survey about 8 000 clusters will be detected over 10 000 square degrees on top of the SDSS cap up to $z = 1$, and where in the deep (southern) survey about 1 800 clusters will be detected over 176 square degrees up to $z = 2$ (if they exists at such high redshifts). REFLEX has most clusters below $z = 0.3$ and are thus not very sensitive to w_x. However, the resulting likelihood contours of SNe and galaxy clusters appear almost orthogonal to each other in the high-w_x range. Their combination thus gives a quite strong constraint on both w_x and Ω_m. This is a typical example of cosmic complementarity which stems from the fact that SNe probe the homogeneous Universe whereas galaxy clusters test the inhomogeneous Universe as well. The final result of the combination of different SNe samples and REFLEX clusters yields the 1σ constraints $w_X = -0.95 \pm 0.32$ and $\Omega_m = 0.29 \pm 0.10$ (Schuecker et al. 2003b). Averaging all results obtained with REFLEX and various SN-sample yields $w_x = -1.00 \pm 0.20$. The measurements suggest a cosmic fluid that violates SEC and fulfills NEC. In fact, the measurements are quite consistent with the cosmological constant and leave not much room for any exotic types of dark energy. The violation of the SEC gives a further argument that we live in a Universe in a phase of accelerated cosmic expansion. A formal average of the most accurate w_x measurements (Schuecker et al. 2003b, Spergel et al. 2003, Rapetti et al. 2004, and Riess et al. 2004), and their 1σ standard deviation is

$$w_x = -1.00 \pm 0.04 . \tag{2.1}$$

It is save to conclude that all recent measurements are consistent with a cosmological constant, and that the most precise estimates suggest that w_x is very close to -1. This points toward a model where dark energy behaves very similar to a cosmological constant, i.e., that the time-dependency of the dark energy cannot be very large.

The last effect of w_x discussed here is related to a possible non-gravitational interaction between dark energy and ordinary matter (e.g. Amendola 2000). The most obvious candidate for dark energy is presently the cosmological constant with all its catastrophic problems. However, a very small redshift-dependency of the dark energy density can presently not be completely ruled out. The next simplest possibility is a light scalar (quintessential) field ϕ where the self-interaction potential $V(\phi)$ can drive the observed accelerated expansion similar as in the de-Sitter phase of inflationary scenarios. In general, ϕ should interact beyond the gravitational coupling to baryons and CDM with a strength comparable to gravity unless some special symmetry prevents or suppresses the interaction. The following discussion is restricted to a possible interaction between dark energy and dark matter. The general covariance of the energy momentum tensor requires the sum of dark matter (m) and dark energy (ϕ) to be locally conserved so that we can allow for a coupling of the two substrata, e.g., in a simple linear form but more complicated choices are, however, possible. For a given potential $V(\phi)$, the corresponding equation of motion of ϕ can be solved. Amendola (2000) discussed exponential potentials which yield a present accelerating phase. Their model leads to a further suppression of structure growth and thus to smaller σ_8 compared to noninteracting quintessence models. The present observations appear quite stringent. The stronger constraint on $\sigma_8 = 0.76 \pm 0.10$ obtained in Sect. 1 suggests a clear detection of a nonminimal coupling between dark energy and dark matter:

$$\beta = 0.10 \pm 0.01 . \tag{2.2}$$

This would provide an argument that dark energy cannot be the cosmological constant because Λ cannot couple non-gravitationally to any type of matter. In this case, the quite narrow experimental corridor found for $w_{\rm x}$ would be responsible for the nonminimal coupling.

3. The Cosmological Constant Problem

To illustrate the cosmological constant problem (Weinberg 1989), separate the effectively observed dark energy density into a gravitational and non-gravitational part, $\rho_\Lambda^{\rm eff} = \rho_\Lambda^{\rm GRT} + \rho_\Lambda^{\rm VAC} = 6 \cdot 10^{-27}\,{\rm kg\,m^{-3}}$, for $\Omega_\Lambda = 0.7$. The non-gravitational part represents the physical vacuum. A free scalar field offers a convinient way to get an estimate of a plausible vacuum energy density and yields $\rho_\Lambda^{\rm VAC} = \frac{1}{c^5\hbar^3} \int_0^{2\pi E_p/\hbar c} \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + (mc/\hbar)^2} \approx 6 \cdot 10^{+97}\,{\rm kg\,m^{-3}}$ (for $m = 0$). The cosmological constant problem is the extra-ordinary fine-tuning which is necessary to combine the effectively measured dark energy density with the physical vacuum. The answer is probably related to the fact that for the estimation of the physical vacuum, gravitational effects are completely ignored.

A hint how inclusion of gravity could effectively work, comes from black hole thermodynamics. Analyzing quantized particle fields in curved but not quantized spacetimes, it became clear that the information necessary to fully describe the physics inside a certain region and characterized by its entropy, increases with the surface of the region. This is in clear conflict to standard non-gravitational theories where entropy as an extensive variable always increases with volume. Non-gravitational theories would thus vastly overcount the amount of entropy and thus the number of modes and degrees of freedom when quantum effects of gravity become important. ‘t Hooft (1993) and Susskind (1995) elevated the entropy bound as the Holographic Principle to a new fundamental hypothesis of physics. Later studies made the exclusion of states inside their Schwarzschild radii more explict which further strengthen the entropy bound so that a new estimate of the physical is $\rho_\Lambda^{\rm HOL} = \frac{c^2}{8\pi G} \frac{1}{R_{\rm EH}^2} \approx 3 \cdot 10^{-27}\,{\rm kg\,m^{-3}}$, where $R_{\rm EH}$ is the present event horizon of the Universe. This is, however, not a solution of the cosmological constant problem because gravity and the exclusion of microscopic black hole states were put in by hand and not in a self-consistent manner by a theory of quantum gravity. Nevertheless, the similarity of the results might be taken as a hint that gravitational hologpraphy could be relevant to find a more complete theory of physics.

Gravitational holography is based on the validity of the Null Energy Condition (NEC) which offers a way to check for consistency of the principle. However, in contrast to the NEC as discussed in Sec. 2 for the total cosmic fluid, Kaloper & Linde (1999) could show that for the covariance entropy bound each individual component of the cosmic substratum must obey $-1 \le w_i \le +1$.

The most problematic component is the equation of state of the dark energy. The observed values suggest $w_{\rm x} = -1.00 \pm 0.04$. CDM and baryons have $w = 0$, and the primeval thermal remnants have $w = 1/3$. The observed bounds are thus consistent with holography.

t'Hooft (1993) and Susskind (1995) have argued very convincingly that M-theory should satisfy the holographic principle. There is a large class of general models based on higher dimensions which follow the holographic principle. Brane-worlds emerging from the model of Horava & Witten are phenomenological realizations of M-theory ideas. Recent theoretical investigations concentrate on the Randall & Sundrum models where gravity is used in an elegant manner to compactify the extra dimensions. These models also follow the holographic principle. The visible Universe is located on a $(1+3)$-dimensional brane. Non-gravitational forces, described by open strings, are confined to the brane. Gravity, described by closed strings, can propagate also into the $(1+4)$-dimensional bulk and thus 'dilutes' differently than Newton or Einstein gravity. Table-top experiments of classical gravity and BBN already confines the size of the extra dimension to values smaller than about 0.2 mm. Rhodes et al. (2003) discussed the effects of extra dimensions on CMB anisotropies and large scale structure formation showing that $P(k)$ gets flatter on scales around $300\,h^{-1}$ Mpc with increasing size of the extra dimension. A careful statistical analysis shows that more than 30 000 galaxy clusters are needed to clearly detect the presence of an extra dimension on scales below 0.2 mm.

4. Summary and conclusions

X-ray clusters of galaxies are used – partially in combination with other measurements – to get the observational constraints on the matter density $\Omega_{\rm m} = 0.29 \pm 0.04$, the normalization of the matter power spectrum $\sigma_8 = 0.76 \pm 0.10$, the neutrino energy density $\Omega_\nu = 0.006 \pm 0.003$, the equation of state of the dark energy $w_{\rm x} = -1.00 \pm 0.04$, and the interaction $\beta = 0.10 \pm 0.01$ between dark energy and dark matter. Future observations will include precise determinations of the normalized cosmological constant Ω_Λ and the redshift-dependency of $w_{\rm x}$. Unfortunately, this set of observations does not provide a consistent picture. The problem is related to the comparatively low σ_8 value. This normalization leads to an overestimate of the neutrino mass compared to laboratory experiments. The low normalization also suggests a significant interaction between dark energy and dark matter. Such a high interaction is not consistent with a dark energy with $w_{\rm x} = -1.00 \pm 0.04$ because the latter suggests that dark energy behaves quite similar to a cosmological constant which cannot exchange energy beyond gravity. A more convincing explanation is that $\sigma_8 = 0.76$ should be regarded as a lower limit. Systematic underestimates of σ_8 by 20-30 percent are not unexpected in the light of recent hydrodynamical simulations. A higher σ_8 would immediately lead to a consistent neutrino mass, and dark energy in form of a cosmological constant without nonminimal couplings. Present data do not allow such definite conclusions. It is, however, seen that the inclusion of neutrino mass and interaction parameters significantly improves our abilities for internal tests. A further problem is related to the measurement $w_{\rm x} = -1.00 \pm 0.04$. Many published proposals for $w_{\rm x}$ measurements expect errors on the 5-percent level within the next five years. Is this expected accuracy really enough to detect deviations from the cosmological constant?

Another aim of the present paper was to point out that a precise measurement of $w_{\rm x}$ corresponds to tests of general relativitistic energy conditions. They form the bases of many phenomena

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related to gravitation and are also important for tests of gravitational holography as a new emerging principle of physics which is expected to provide a new guideline towards a more complete theory of physics. M-theory should also come holographic as well as brane-world gravity as a phenomenological realization of M-theory ideas. Tests of the resulting cosmologies will in the end confront alternative theories of gravity. Observational tests of gravity on cosmological scales as illustrated by the effects of an extra-dimension on the cluster power spectrum probably need the ultimate cluster survey, i.e. a census of possibly all 10^6 galaxy clusters which might exist down to redshifts of $z = 2$ in the visible Universe.

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