PROCEEDINGS OF SCIENCE

# Introduction to Cosmology

**V. A. Rubakov***
*Institute for Nuclear research of the Russian Academy of Sciences, Moscow*
*E-mail:* rubakov AT ms2.inr.ac.ru

An introduction to cosmology is given.

*RTN Winter School on Strings, Supergravity and Gauge Theories*
*31/1-4/2 2005*
*SISSA, Trieste, Italy*

*Speaker.

## Contents

## 1. INTRODUCTION

The evolution of the Universe is determined, to large extent, by microscopic laws of physics — the same laws that govern particle interactions at high energies. Hence, discoveries in particle physics are of direct relevance to the theory of the Universe. Conversely, cosmology provides important insights for high energy physics. Amazingly, many fundamental aspects of cosmology require dramatic extensions beyond known physics; it may even happen that some aspects of cosmology will be possible to understand only by invoking hints from string theory.

How well do we understand the present and early Universe? Why cosmologists are so confident when inventing new physics, even though it has not been discovered yet by high energy physics community? What lessons should high energy physicists learn from advances in cosmology? These lectures are an attempt to address these issues, which are at the core of high energy physicists' interest in cosmology.

There are many excellent books and reviews on cosmology and astrophysics. Here we mention a few [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. When discussing specific subjects in these lectures, we will mostly refer to review papers; an interested reader may find references to original literature there.

## 2. EXPANDING UNIVERSE

### 2.1 Friedmann–Robertson–Walker metric

Two basic facts about the Universe are that it is **homogeneous and isotropic** at large spatial scales, and that it **expands**.

There are three types of homogeneous and isotropic three-dimensional spaces, which are conventionally labeled by a parameter $\kappa = 0, +1, -1$. These are[1] three-sphere ($\kappa = 1$), flat space ($\kappa = 0$) and three-hyperboloid ($\kappa = -1$). Accordingly, one speaks about closed, flat and open Universe ($\kappa = +1, 0$ and $-1$, respectively); in the latter two cases the spatial size of the Universe is infinite, whereas in the former the Universe is compact.

The homogeneity and isotropy of the Universe mean that its hypersurfaces of constant time are either three-spheres or three-planes or three-hyperboloids. The distances between points may (and in fact, do) depend on time, i.e., the interval has the form

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 \tag{2.1}$$

where $d\mathbf{x}^2$ is the distance on unit three-sphere/plane/hyperboloid. Metric (2.1) is usually called Friedmann–Robertson–Walker (FRW) metric, and $a(t)$ is called scale factor. In our Universe

$$\dot{a} \equiv \frac{da}{dt} > 0$$

which means that the distance between points of fixed spatial coordinates $\mathbf{x}$ grows

$$dl^2 = a^2(t)d\mathbf{x}^2$$

[1] Strictly speaking, this statement is valid only locally: in principle, homogeneous and isotropic Universe may have complex global properties. As an example, spatially flat Universe may have topology of three-torus. There is some discussion of such a possibility in literature, and fairly strong limits have been obtained by the analyses of cosmic microwave background [11].

The Universe expands.

The coordinates $\mathbf{x}$ are often called comoving coordinates. It is straightforward to check that $\mathbf{x} = \text{const}$ is a time-like geodesic, so a galaxy put at a certain $\mathbf{x}$ at zero velocity will stay at the same $\mathbf{x}$. Furthermore, as the Universe expands, non-relativistic objects loose their velocities $\dot{\mathbf{x}}$, i.e., they get frozen in the comoving coordinate frame.

### 2.2 Hubble law

Let us discuss the propagation of photons in expanding Universe. The action of free electromagnetic field in curved space-time is

$$S = -\frac{1}{4}\int \sqrt{-g} d^4x \, g^{\mu\nu} g^{\lambda\rho} F_{\mu\lambda} F_{\nu\rho} \tag{2.2}$$

While the time coordinate $t$ is convenient because it coincides with proper time for a particle at rest (i.e., a particle whose spatial coordinates $\mathbf{x}$ do not change in time), one can introduce another convenient time coordinate $\eta$ instead of $t$, such that

$$a(\eta) d\eta = dt$$

that is

$$\eta = \int \frac{dt}{a(t)} \tag{2.3}$$

In terms of this time coordinate, the FRW metric takes the form

$$ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2) \tag{2.4}$$

If the three-dimensional space is flat, one has

$$ds^2 = a^2(\eta) \eta_{\mu\nu} dx^\mu dx^\nu \tag{2.5}$$

The time coordinate $\eta$ is called conformal time, because metric (2.5) differs from Minkowski metric $\eta_{\mu\nu}$ only by the conformal factor $a^2(\eta)$. The form (2.5) may be used for closed and open Universe as well, as long as distances much shorter than the radius of spatial curvature are considered.

In terms of conformal time (and neglecting spatial curvature), one has $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$. Plugging this into the action of electromagnetism, eq. (2.2), one finds that the action reduces to Minkowski form,

$$S = -\frac{1}{4}\int d^3x d\eta \, \eta^{\mu\nu} \eta^{\lambda\rho} F_{\mu\lambda} F_{\nu\rho} \tag{2.6}$$

This means that in coordinates $(\eta, \mathbf{x})$, photon propagates in exactly the same way as in Minkowski space-time. However, the coordinates $\eta$ and $\mathbf{x}$ do not correspond to physical time intervals and physical distances, and photons get redshifted as they propagate through the Universe. Indeed, it is clear from the form of the action (2.6) that solutions to the corresponding field equations are superpositions of plane waves,

$$A_\mu \propto \mathrm{e}^{i\mathbf{kx} - i|\mathbf{k}|\eta} \tag{2.7}$$

where $\mathbf{k}$ is a constant *coordinate* momentum. The *coordinate* wavelength $\lambda_x = 2\pi/|\mathbf{k}|$ stays constant, but the physical wavelength

$$\lambda = a(\eta)\lambda_x$$

increases. The physical momentum and frequency

$$\mathbf{p}(t) = \frac{\mathbf{k}}{a(t)} , \quad \omega(t) = \frac{|\mathbf{k}|}{a(t)}$$

decrease in time, i.e., they get redshifted[2].

We will always label the present values of time-dependent quantities by subscript 0: the present physical wavelength of a photon is thus denoted by $\lambda_0$, the present time is $t_0$, the present value of the scale factor is $a_0 \equiv a(t_0)$, etc. If a photon was emitted at some moment of time $t_e$ in the past, and its physical wavelength at the moment of emission was $\lambda_e$ ($\lambda_e$ is fixed by physics of the source, say, it is the wavelength of a photon emitted by an excited hydrogen atom), then we receive today a photon whose physical wavelength is longer,

$$\frac{\lambda_0}{\lambda_e} \equiv 1 + z = \frac{a_0}{a(t_e)}$$

Here we introduced the redshift $z$ which, on the one hand, is directly measurable[3], and, on the other hand, is related to the time of emission, and hence to the distance to the source.

Let us consider a "nearby" source, for which

$$z \ll 1$$

This corresponds to relatively small $(t_0 - t_e)$. Expanding $a(t_e)$, one writes

$$a(t_e) = a_0 - \dot{a}(t_0)(t_0 - t_e) \tag{2.8}$$

To the leading order in $z$, the difference between the present time and the emission time is equal to the distance to the source $r$ (the speed of light is set equal to 1). Let us define the Hubble parameter

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

and denote its present value by $H_0$. Then eq. (2.8) takes the form

$$a(t_e) = a_0(1 - H_0 r)$$

and we get for the redshift, again to the leading non-trivial order in $z$,

$$1 + z = \frac{1}{1 - H_0 r} = 1 + H_0 r$$

In this way we obtain the Hubble law,

$$z = H_0 r , \quad z \ll 1 \tag{2.9}$$

[2]Although in this derivation we made use of conformal invariance of electromagnetism, the results are valid for any massless field, for waves with frequencies much greater than $H$. Indeed, WKB solutions of the wave equations always have the form (2.7), possibly with slowly varying pre-exponential factors, see section 2.7.

[3]One identifies a series of emission or absorption lines, thus obtaining $\lambda_e$, and measures their actual wavelength $\lambda_0$. These spectroscopic measurements give very accurate values of $z$ even for distant sources.

Traditionally, one tends to interpret the expansion of the Universe as runaway of galaxies from each other, and red shift as the Doppler effect. Then at small $z$ one writes $z = v$, where $v$ is the radial velocity of the source with respect to the Earth, so $H_0$ is traditionally measured in units "velocity per distance". Observational data, which we will discuss briefly in section 3, give

$$H_0 \approx 70 \frac{\text{km}}{\text{s} \cdot \text{Mpc}} \tag{2.10}$$

Traditionally, the present value of the Hubble parameter is written as

$$H_0 = h \cdot 100 \ \frac{\text{km}}{\text{s} \cdot \text{Mpc}} \tag{2.11}$$

(1 Mpc$\approx$ 3 mln. light yrs.$\approx 3 \cdot 10^{24}$ cm). Thus

$$h \approx 0.7$$

We will use this value in further estimates.

Let us point out that the interpretation of redshift in terms of the Doppler shift is actually not adequate, at least for large enough $z$. In fact, there is no need in this interpretation at all: the "radial velocity" enters neither theory nor observations, so this notion may be safely dropped. Physically meaningful quantity is redshift $z$ itself.

A final comment is that $H_0^{-1}$ has dimension of time, or length. Clearly, this quantity sets the cosmological scales of time and distance at the present epoch. We will discuss this point in section 2.5.

### 2.3 Hot Universe

Our Universe is filled with cosmic microwave background. Cosmic microwave background as observed today consists of photons with excellent black-body spectrum of temperature

$$T_0 = 2.725 \pm 0.001 \text{ K} \tag{2.12}$$

The spectrum has been precisely measured by various instruments and does not show any deviation from the Planck spectrum, as shown in fig. 1.

Thus, the present Universe is "warm". Earlier Universe was warmer; it cooled down because of the expansion. While the CMB photons freely propagate today, it was not so at early stage. When the Universe was hot, the usual matter (electrons and protons with rather small admixture of light nuclei) was in the plasma phase. At that time photons strongly interacted with electrons and protons in the plasma, so all these particles were in thermal equilibrium. As the Universe cooled down, electrons "recombined" with protons into neutral hydrogen atoms, and the Universe became transparent to photons. The temperature scale of recombination is, very crudely speaking, determined by the ionisation energy of hydrogen, which is of order 10 eV. In fact, recombination occured at lower temperature[4],

$$T_{rec} \approx 3000 \text{ K}$$

[4]The reason is that the number density of electrons and protons was small compared to the number density of photons, i.e., there was large entropy per electron/proton; thus, recombination at higher temperatures was not thermodynamically favourable because of entropy considerations.

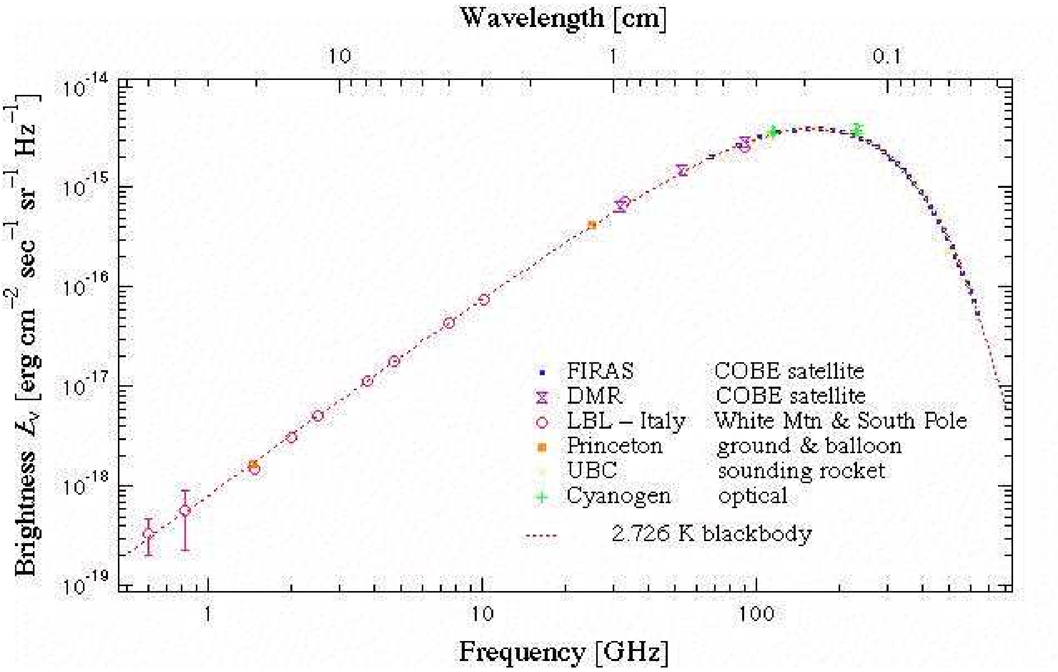


**Figure 1:** Measured CMB spectrum, compilation from Ref. [19]. Dashed line shows the black body (Planck) spectrum. Recent analysis has lead to the value of the temperature given in the text, rather than $T = 2.726$ , as indicated in this figure.

An important point is that recombination process lasted quite a bit less than the Hubble time at that epoch; to a reasonable approximation, recombination occured instantaneously.

Another point is that even though after recombination photons no longer were in thermal equilibrium with anything, the shape of the photon distribution function has not changed, except for overall redshift. Indeed, the thermal distribution function for *ultra-relativistic* particles, the Planck distribution, depends only on the ratio of frequency to temperature,

$$f_{Planck}(p,T) = f\left(\frac{\omega_p}{T}\right) , \quad \omega_p = |p|$$

As the Universe expands, the frequency gets redshifted, $\omega_p \to \omega_p/(1+z)$, but the shape of the spectrum remains Planckian, with temperature $T/(1+z)$. Hence, the Planckian form of the observed spectrum is no surprise. Generaly speaking, this property does not hold for massive particles[5].

At even earlier times, the temperature of the universe was even higher. The earliest time which has been oservationally probed to date is the Big Bang Nucleosynthesis epoch (see below), and corresponds to temperature of order 1 MeV.

To summarize, the effective temperature of photons scales as

$$T(t) \propto \frac{1}{a(t)} \tag{2.13}$$

[5]Similar property holds, however, for particles that decouple being non-relativistic (hydrogen, cold dark matter). At the decoupling, they have Maxwell–Boltzmann distribution function, which is a function of the ratio $p^2/(2mT)$. As the momentum gets redchifted, $p \to p/(1+z)$, the shape of this distribution function remains Maxwell–Boltzmann, with effective temperature $T/(1+z)^2$.

This behavious is characteristic to *ultra-relativistic* free spieces (at zero chemical potential) only. The same formula is valid for ultra-relativistic particles (at zero chemical potential) which are in thermal equilibrium. Thermal equilibrium means adiabatic expansion; during adiabatic expansion, the temperature of ultra-relativistic gas scales as the inverse size of the system[6], hence eq. (2.13).

Both for free photons, and for photons in thermal equilibrium, the number density behaves as follows,

$$n_\gamma = \text{const} \cdot T^3 \propto \frac{1}{a^3}$$

and the energy density is

$$\rho_\gamma = \frac{\pi^2}{30} \cdot 2 \cdot T^4 \propto \frac{1}{a^4} \tag{2.14}$$

where the factor 2 accounts for two photon polarizations. Present number density of relic photons is about

$$n_{\gamma,0} \approx 410 \ \text{cm}^{-3} \tag{2.15}$$

and their energy density is

$$\rho_{\gamma,0} = 2.7 \cdot 10^{-10} \ \frac{\text{GeV}}{\text{cm}^3} \tag{2.16}$$

Let us now turn to non-relativistic particles: baryons, massive neutrinos, possible exotic "dark matter" spieces, etc. If they are not destroyed during the evolution of the Universe (that is, they are stable and do not co-annihilate), their number density merely gets diluted,

$$n \propto \frac{1}{a^3} \tag{2.17}$$

This means, in particular, that the baryon-to-photon ratio stays constant,

$$\eta \equiv \frac{n_B}{n_\gamma} = \text{const} \tag{2.18}$$

The energy density of non-relativistic particles scales as

$$\rho(t) = m \cdot n(t) \propto \frac{1}{a^3(t)} \tag{2.19}$$

in contrast to more rapid fall off (2.14) characteristic to ultra-relativistic spieces.

As we will discuss later, there exists strong evidence for *dark energy* in the Universe, whose density does not decrease in time as fast as in eqs. (2.14) or (2.19). For the moment it suffices to mention that this property holds for vacuum, whose energy density stays constant (in locally Lorentz frame),

$$\rho_{vac} = \text{const} \tag{2.20}$$

while the vacuum energy-momentum tensor in arbitrary frame is, by general covariance,

$$T_{\mu\nu}^{vac} = \rho_{vac} g_{\mu\nu}$$

[6]This follows from usual thermodynamics. The energy density of ultra-relativistic gas scales as $\rho \propto T^4$, and pressure is $p = \rho/3$. During the adiabatic expansion, energy decreases as follows, $dE \equiv d(\rho V) = -pdV$, where $V$ is the volume of the system. The latter relation immediately gives $dT/T = (1/3)(dV/V)$.

i.e., vacuum has negative pressure (see eq. (2.23)),

$$p_{vac} = -\rho_{vac} \tag{2.21}$$

In this context, the vacuum energy density is the same thing as the cosmological constant, or $\Lambda$-term.

### 2.4 Friedmann equation

The basic equation governing the expansion rate of the Universe is the Friedmann equation,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{\kappa}{a^2} \tag{2.22}$$

where dot denotes derivative with respect to time $t$, $\rho$ is the total energy density in the Universe, the parameter $\kappa$ has been introduced in section 2.1 and distinguishes closed ($\kappa = +1$), flat ($\kappa = 0$) and open ($\kappa = -1$) Universes, and $G$ is Newton's gravity constant. In natural units $G = M_{Pl}^{-2}$ where $M_{Pl} = 1.2 \cdot 10^{19}$ GeV is the Planck mass. The first and second terms on the right hand side of eq. (2.22) may be viewed as the contributions of matter and spatial curvature, respectively, to the expansion rate.

The Friedmann equation (2.22) is nothing but one of the Einstein equations of General Relativity specialized to homogeneous and isotropic space. Other Einstein equations are satisfied automatically for this simple geometry. The relations (2.14), (2.19) and (2.20) may in fact be viewed as consequences of the covariant conservation of energy-momentum,

$$\nabla_\mu T^{\mu\nu} = 0$$

For energy-momentum tensor of a fluid,

$$T^\mu_\nu = \text{diag}\,(\rho, -p, -p, -p) \tag{2.23}$$

and FRW metric, the covariant conservation equation reduces to

$$\frac{d\rho}{\rho + p} = -3\frac{da}{a} \tag{2.24}$$

whose solutions are (2.14), (2.19) and (2.20) for $p = \rho/3$ (ultra-relativistic matter), $p = 0$ (non-relativistic matter) and $p = -\rho$ (vacuum), respectively.

As we will discuss later, the Universe is spatially flat today to a good approximation. The curvature term $\kappa/a^2$ in eq. (2.22) today is less than about 2 per cent of the matter term. This can be also phrased in the following way. One defines the critical density $\rho_c$ according to

$$\frac{8\pi}{3}G\rho_c = H_0^2 \tag{2.25}$$

The meaning of this quantity is as follows. If the actual energy density $\rho$ of all forms of matter in the Universe (including vacuum, quintessence, etc.) is larger than $\rho_c$, then $\kappa > 0$, and the Universe is closed; if $\rho < \rho_c$, the Universe is open, and it is flat for $\rho = \rho_c$. Observationally,

$$\rho = (1 \pm 0.02)\rho_c \tag{2.26}$$

At earlier times, the curvature term was even less significant, so we will neglect it in the study of the evolution of the Universe, and write the Friedmann equation as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho \tag{2.27}$$

To get an idea of numerics, one plugs the present value of the Hubble parameter (2.11) into the definition (2.25) and converts $\rho_c$ into

$$\rho_c = h^2 \cdot 1 \cdot 10^{-5}\ \frac{\text{GeV}}{\text{cm}^3} \approx 5 \cdot 10^{-6}\ \frac{\text{GeV}}{\text{cm}^3} \tag{2.28}$$

where we use our fiducial value $h = 0.7$.

### 2.5 Sample solutions

Solutions to the Friedmann equation (2.27) are most easily obtained in cases when matter of definite type gives dominant contribution into the energy density $\rho$. Let us present some solutions of his sort.

**Matter dominated Universe**

If the dominant contribution to the energy density comes from non-relativistic particles, then $\rho = \text{const} \cdot a^{-3}$, and the Friedmann equation reads

$$\frac{\dot{a}}{a} = \frac{\text{const}}{a^{3/2}}$$

The solution is

$$a = \text{const} \cdot t^{2/3} \tag{2.29}$$

Note that this solution describes decelerating Universe,

$$\ddot{a} < 0$$

As $t \to 0$, the scale factor tends to zero, and the energy density tends to infinity. This is a cosmological singularity, "beginning of the Universe" ("Big Bang"), and $t$ is the lifetime of the Universe. Note that the lifetime is related to the Hubble parameter, since

$$H \equiv \frac{\dot{a}}{a} = \frac{2}{3t}$$

Until fairly recently, our Universe was indeed matter dominated, so this relation may be used to obtain a crude estimate of its present age,

$$t_0 \sim \frac{2}{3}H_0^{-1} \approx 1 \cdot 10^{10}\ \text{yrs}$$

(again with $h = 0.7$). In fact, the lifetime of 10 billion years was a bit of a problem at some point, as independent estimates of lifetimes of old objects in our Universe suggested that their lifetimes were close, and sometimes even larger than 10 billion years. This was one of the reasons for suggesting, even before the observational evidence for accelerating Universe, that our Universe is not in the matter dominated stage today — rather, it is in dark matter dominated stage.

Let us make use of this solution to introduce another notion, the cosmological horizon. Suppose that at $t = 0$, signals were emitted everywhere in space, and then propagate in the Universe with the speed of light. We ask at what distance today are the sources of signals we receive now. This sphere is precisely the cosmological horizon for the solution (2.29): interior of this sphere is causally connected to us, while the part of the Universe outside this sphere is causally disconnected from us. The world line of a signal propagating with the speed of light obeys $ds = 0$, which, in view of eq. (2.4), implies that the coordinate distance to the horizon is

$$r_H = \eta = \int_0^t \frac{dt'}{a(t')}$$

The physical distance to the horizon at the time $t$ is thus

$$l_H(t) = a(t) r_H = a(t) \int_0^t \frac{dt'}{a(t')} \tag{2.30}$$

For the solution (2.29) this distance is finite,

$$l_H(t) = 3t = 2H^{-1}$$

Hence, the present size of the visible part of the Universe is estimated as

$$l_{H,0} \sim 2H_0^{-1} \approx 3 \cdot 10^{28} \text{ cm} \approx 10^4 \text{ Mpc}$$

Note that in matter dominated Universe, the integral in eq. (2.30) is saturated at large $t'$, so the relic photons, which were actually emitted somewhat after the Big Bang, traveled almost the same distance as $l_{H,0}$.

Let us stress that the above estimates for the present age and horizon size are not quite correct, since during good part of the evolution, the expansion of the Universe was *not* dominated by non-relativistic matter; rather, it is dark matter dominated.

**Radiation dominated Universe**

If dominant contribution to the energy density comes from ultra-relativistic particles, then $\rho = \text{const} \cdot a^{-4}$, and the solution to the Friedmann equation (2.27) is

$$a(t) = \text{const} \cdot t^{1/2} \tag{2.31}$$

Qualitative features of this solution are similar to matter dominated case: the Universe starts from the singularity, its expansion decelerates, age and horizon size are finite at given $t$.

**Vacuum dominated Universe**

Qualitatively different solution occurs if the Universe is vacuum dominated. The vacuum energy density $\rho_{vac}$ is time-independent, so the solution to the Friedmann equation (2.27) is

$$a(t) = \text{const} \cdot e^{H_{vac} t} \tag{2.32}$$

where the time-independent Hubble parameter is determined by the vacuum energy density,

$$H_{vac} = \sqrt{\frac{8\pi}{3} G \rho_{vac}}$$



PoS(RTN2005)003

One notices that the Universe accelerates, rather than decelerates,

$$\ddot{a} > 0$$

We note in passing that spatially flat FRW metric with scale factor (2.32) describes (part of) the de Sitter space-time. Unlike other cosmological solutions, de Sitter geometry does not have past singularity.

**Equation of state** $p = w\rho$

Let us now consider general case of a fluid with equation of state

$$p = w\rho$$

where $w$ is a constant. For definiteness, let us restrict to the case

$$w > -1$$

Then the solution to eq. (2.24) is

$$\rho = \frac{\text{const}}{a^{3(1+w)}}$$

With $\kappa = 0$ (spatially flat Universe) one finds from eq. (2.22)

$$a = \text{const} \cdot t^{\alpha} \tag{2.33}$$

where

$$\alpha = \frac{2}{3}\frac{1}{1+w}$$

The behaviour of solutions is qualitatively different for $w > -1/3$ and $w < -1/3$, i.e., for $\alpha < 1$ and $\alpha > 1$:

$$\begin{aligned} w > -\frac{1}{3} \quad &: \quad \ddot{a} < 0\,, \quad \text{decelerated expansion} \\ w < -\frac{1}{3} \quad &: \quad \ddot{a} > 0\,, \quad \text{accelerated expansion} \end{aligned} \tag{2.34}$$

Thus, accelerated expansion of the Universe requires *negative pressure*.

It is worth noting that the two cases differ in another respect: in the former case there exists cosmological horizon, while in the latter the entire Universe is causally connected. Indeed, we have seen that the cosmological horizon exists, if the following integral converges (see eq. (2.30)),

$$\int_0^t \frac{dt'}{a(t')}$$

This integral is convergent for $\alpha < 1$, i.e. $w > -1/3$. Otherwise this integral diverges, so the cosmological horizon is absent. Note that this observation has to do with *early times*, $t \to 0$; it is of relevance for inflation rather than for the present epoch.

### 2.6 Changing regimes

As we will discuss in sections 4 and 5, the present expansion of the Universe is dark energy dominated; to a reasonable approximation, the expansion of the Universe today follows the exponential law (2.32). This was not so at earlier times. Indeed, the dark energy density is constant in time (or almost constant), while the energy density of non-relativistic particles scales like $a^{-3}(t)$. Hence, even though the present energy density of non-relativistic matter is smaller than dark energy, non-relativistic matter dominated at earlier epoch, when the scale factor was smaller than today. Even earlier, the expansion of the Universe was dominated by ultra-relativistic matter. Indeed, the energy density of the latter scales as $a^{-4}(t)$, so at small enough $a(t)$ it was higher than the energy density of non-relativistic particles. The curvature term $\varkappa/a^2$ never dominated the expansion of the Universe: at present it contributes much less than not only the dark energy density, but also the matter energy density. Its contribution was even less significant at earlier times, as it scales as $a^{-2}$, whereas the matter energy density scales as $a^{-3}$. This is why it is legitimate to neglect the curvature term for describing the entire evolution of the Universe.

We will quantify this discussion later, when we have better idea of the composition of the Universe.

### 2.7 Linear perturbations in the expanding Universe

The Universe is of course not exactly homogeneous and isotropic. Thus, it is important to understand the basic properties of perturbations about homogeneous and isotropic background. At early times, density perturbations were small (we will discuss their magnitude in section 3), so a linearized theory is a good approximation. We will discuss qualitative features of perturbations in the expanding Universe by making use of an example of a massless scalar field $\varphi$ minimally coupled to gravity; the analysis of density perturbations is more technically involved, so we are not going to present it here, and only make comments in appropriate places. An interested reader may consult, e.g., refs. [12, 13].

The action of the scalar field is

$$S_\varphi = \int d^4x\,\sqrt{-g}\,\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \tag{2.35}$$

In spatially flat FRW background it has the form

$$S_\varphi = \int d^3x dt \frac{1}{2}a^3\left[(\dot\varphi)^2 - \frac{1}{a^2}(\partial_i\varphi)^2\right]$$

Hence, the field equation reads

$$\ddot\varphi + 3H(t)\dot\varphi - \frac{1}{a^2}\partial_i\partial_i\varphi = 0 \tag{2.36}$$

where $H(t) = \dot a/a$ is still the Hubble parameter. Due to homogeneity and isotropy of space, it is convenient to use the momentum representation, i.e., search for solutions in the form

$$\mathrm{e}^{i\mathbf{kx}}\varphi_k(t)$$

where $\varphi_k$ obeys

$$\ddot\varphi_k + 3H(t)\dot\varphi_k + \frac{k^2}{a^2}\varphi = 0 \tag{2.37}$$

Note that $k$ is a *coordinate* momentum; the physical momentum at time $t$ is

$$p = \frac{k}{a(t)}$$

It depends on time and gets redshifted.

Now, the second term in eq. (2.37) acts as friction. Thus, there are two regimes with qualitively different properties:

— Subhorizon modes:

$$p \equiv \frac{k}{a} \gg H$$

Modes obeying this property are called subhorizon modes, since their physical wavelength $\lambda \sim p^{-1}$ is much shorter than the Hubble distance $H^{-1}$ (which is horizon size in matter dominated or radiation dominated Universe). Subhorizon modes oscillate in time,

$$\varphi_k = \frac{1}{a} \mathrm{e}^{\pm \int dt \; \omega_k(t)} \; , \quad \frac{k}{a} \gg H \tag{2.38}$$

where

$$\omega_k = p \equiv \frac{k}{a}$$

This behaviour is nothing peculiar: modulo slowly varying prefactor, the solutions describe oscillations with the frequency experiencing redshift.

— Superhorizon modes:

$$p \equiv \frac{k}{a} \ll H$$

In this case, the last term in eq. (2.37) is negligible, and the solutions are

– constant mode

$$\varphi_k = \text{const} \; , \qquad \frac{k}{a} \ll H \tag{2.39}$$

– "growning" mode

$$\varphi_k = \text{const} \cdot \int \frac{dt}{a^3(t)} \; , \qquad \frac{k}{a} \ll H \tag{2.40}$$

The traditional name for the "growing" mode is somewhat misleading: the mode actually decreases with time. The point, though, is that the latter mode grows as $t$ decreases; in the radiation dominated (and matter dominated) Universe it blows up at small $t$.

The gravitational waves obey precisely the same equation (2.37), so they have exactly the same behavior. The (adiabatic) density perturbations at the radiation dominated epoch also have similar behavior; in particular, for given $k$, one of the superhorizon modes blows up at small $t$. The whole picture of the FRW Universe with small pertubations is thus self-consistent only if this mode *vanishes* (or almost vanishes, if the hot stage begins at large but finite temperature) at finite times. Any mechanism producing small density perturbations is likely to have this property.

Now, we recall that for radiation dominated and matter dominated Universe $H \sim t^{-1}$, while the scale factor behaves as given in eqs. (2.31) and (2.29), respectively. Thus, the ratio of the physical momentum to $H$ behaves as

$$\begin{aligned} \frac{p(t)}{H(t)} &\propto t^{1/2} \; , \quad \text{radiation dominated} \\ &\propto t^{1/3} \; , \quad \text{matter dominated} \end{aligned} \tag{2.41}$$

This means that all modes start off as superhorizon, and then enter the horizon. In the scalar field example, the requirement that the "growing" mode vanishes determines the initial data for each **k** up to overall amplitude. Thus the solution is

$$\begin{aligned}\varphi_k &= c_k\,, \quad \frac{k}{a} \ll H \\ &= c_k \cos\left(\int_0^t dt\ \omega(t)\right)\,, \quad \frac{k}{a} \gg H \end{aligned} \tag{2.42}$$

For density perturbations, the oscillating behavior means that at late enough times, there are sound waves in the primordial plasma, whose wavelengths are shorter than the horizon size at each moment of time (in fact, since the speed of sound in the cosmic plasma is different from the speed of light, it is sound horizon that actualy matters here). The point of the whole analysis is that the *phase* of these waves is fixed. This property holds for all types of density perturbations (and gravity waves) and is very important for interpreting data on CMB.

Briefly speaking, the fate of the primordial density perturbations is as follows. They stay constant until they enter the horizon at radiation dominated or matter dominated stage. After that they make sound waves. The amplitudes of these waves grow during the matter dominated stage due to the gravitational instability: overdense regions tend to gravitationally attract matter and become even more overdense. This growth becomes non-linear when $(\delta\rho/\rho)$ becomes roughly of order 1; the dense regions collapse and form gravitationally bound structures.

## 3. OVERVIEW OF COSMOLOGICAL DATA

In the last 10 to 15 years, cosmology has become qualitative science. Detailed data on the present and earlier Universe are now available, and even more precise data are due to come. Before going into furher theoretical discussion, let us briefly consider what kinds of data are there, and what gross features of the Universe they show.

**Distribution of luminous matter** (galaxies, quasars) is obtained by deep surveys. The largest surveys (2dF, SDSS) measure angular positions of, and distances (redshifts) to hundreds of thousand galaxies, with depth of the order of 2000 Mpc (about 6 billion light years). This is a fairly large portion of the present Universe. Even larger part of the Universe is sampled by quasars. Thus, by now we have a "map" of our "neighbourhood", and can discuss matter distribution on various length scales.

At large scales, the Universe is **homogeneous and isotropic**, as illustrated in fig. 2.

At shorter scales, the Universe is of course inhomogeneous. The largest structures visible (superclusters of galaxies, giant voids) extend to several dozens of Mpc. This is seen from fig. 3 taken from older Las Campanas survey.

The comparison of the observed structure to simulations, at scales ranging from a few kpc (size of a galaxy) and smaller, to thousands Mpc, tells a lot about the primordial density perturbations in the early Universe, the composition of the Universe and the rate of its expansion at relatively late epoch.

**Observation of "standard candles"**, the objects whose absolute luminocity is believed to be known. What is measured is the visible luminocity $F$ and redshift $z$. At relatively short distances,

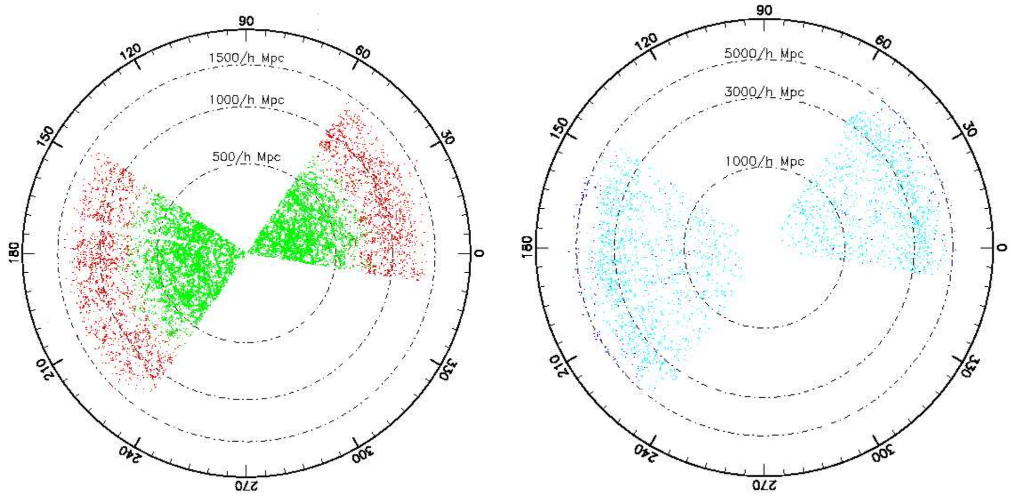


**Figure 2:** Spatial distribution of galaxies (left panel) and quasars (right panel), according to SDSS survey [14]. Shown are samples of usual and brighter galaxies and quasars. The parameter $h$ is defined in the text.

the visible luminocity is related to the distance,

$$F = \frac{L}{4\pi r^2} \tag{3.1}$$

where $L$ is the absolute luminocity. This relation becomes confusing for $z \sim 1$, yet one often *defines* the luminocity distance by eq. (3.1), and talks about redshift–distance relation (the notion of velocity is usually not used for large $z$).

At $z \ll 1$, the relation between $z$ and $r$ follows linear Hubble law (2.9), as shown in fig. 4. The present value of the Hubble parameter consistent with virtually all measurements is

$$H_0 = (71 \pm 3) \frac{\text{km}}{\text{s} \cdot \text{Mpc}} \tag{3.2}$$

In terms of the parameter $h$ in eq. (2.11) this means

$$h = 0.71 \pm 0.03$$

At large $z$, the linear relation (2.9) no longer holds. The relation between $z$ and $r$ tells about the expansion rate of the Universe at relatively late epoch. Presently, the data for large $z$ come from the observations of type 1a supernovae (SNe 1a), figs. 5 and 6. Surprisingly, they show that the Universe undergoes **accelerated** expansion today (and at relatively small $z$, i.e., at late times), while at higher redshift the expansion was decelerating. We will discuss the significance of this result shortly.

**Cosmic microwave background radiation** (CMB) is an extremely important source of information about the properties of the earlier Universe.

The CMB photons were last scattered/emitted at the recombination epoch, when the Universe was only about $3 \cdot 10^5$ years old; for comparison, the present age of the Universe is about $1.4 \cdot 10^{10}$

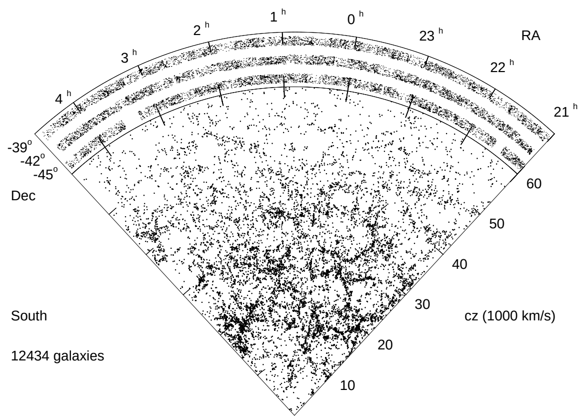


**Figure 3:** Distribution of galaxies in the south galactic cap [15]. Each point represents a galaxy. Radial distances are measured in redshift *cz*; redshifts $20 \cdot 10^3$km/s, $40 \cdot 10^3$km/s and $60 \cdot 10^3$km/s correspond approximately to radial distances 300, 600 and 900 Mpc, respectively.

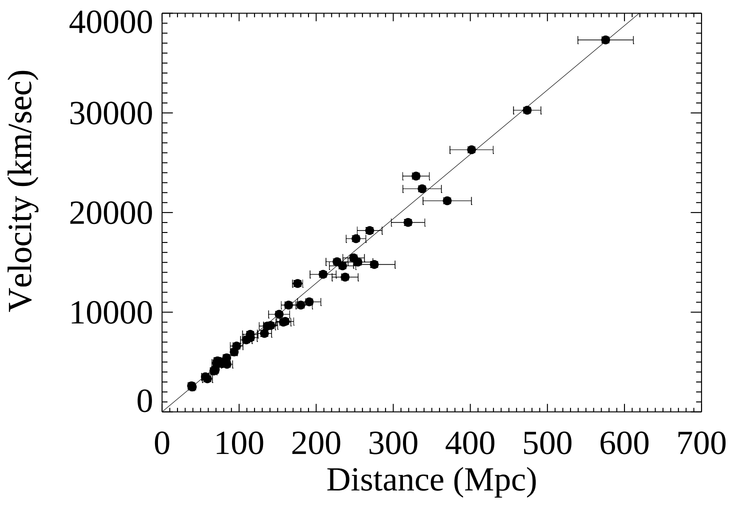


**Figure 4:** Hubble diagram for Supernovae 1a as standard candles, see Refs. [5, 16]. Straight line corresponds to the Hubble law.

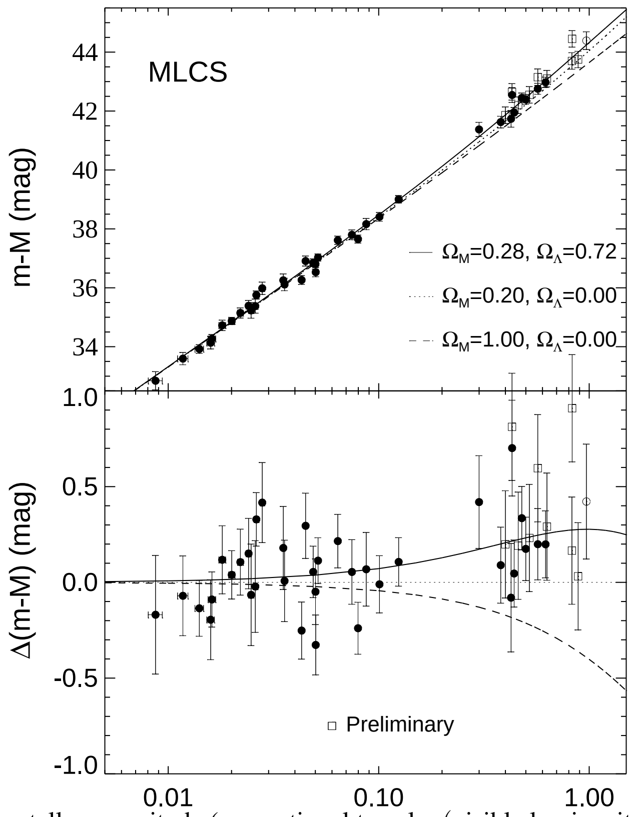


**Figure 5:** Upper panel: stellar magnitude (proportional to $-\log(\text{visible luminocity})$) of SNe 1a as function of redshift, according to relatively old data [17]. The larger the magnitude, the dimmer the object. For definition of Ω's see the text. Lower panel: the same figure, with expectation from the model $\Omega_M = 0.2$, $\Omega_\Lambda = 0$ subtracted.

years. The observations of CMB give the photographic picture (literally) of the "young" Universe, which had quite different properties than it has today. One of these properties is much higher level of homogeneity and isotropy: photons coming from different directions in the sky have almost (but not exactly!) the same temperature. Crudely speaking, relative angular anisotropy of CMB temperature, $\delta T/T_0$, is of order $10^{-4}$ to $10^{-5}$. This means that the Universe was homogeneous and isotropic at the level better than $10^{-4}$, when it was 300 thousand years old.

Yet the angular anisotropy of the CMB temperature exists, and has been measured at various angular scales. In a wide range of angular scales this anisotropy was accurately measured by WMAP satellite, see fig. 7, while at smaller angular scales some data are available from measurements made by ground-based interferometers.

It is convenient to decompose the temperature, as function of the direction $\vec{n}$, in spherical harmonics $Y_{lm}(\vec{n})$, which make a complete set of functions on a sphere. One writes[7]

$$\delta T(\vec{n}) \equiv T(\vec{n}) - T_0 - \delta T_{\text{dipole}} = \sum_{l,m} C_{lm} Y_{lm}(\vec{n}) \ ,$$

The angular momentum $l$ corresponds to fluctuations with typical angluar scale $\pi/l$. Figure 8 shows

[7]The dipole component is due to the motion of the Earth with respect to the rest frame of the CMB photon gas.

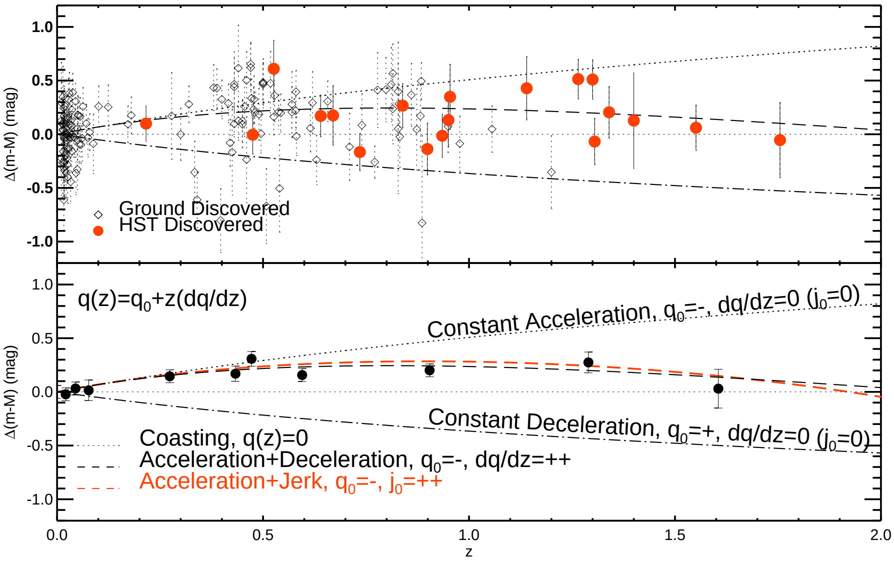


**Figure 6:** Recent data [18] on SNe 1a; subtracted is the expectation from a model in which the Universe expands at constant velocity, $\dot{a} = const$.

the measured anisotropy as function of $l$.

To understand what is shown, one notices that the data are mostly consistent[8] with Gaussian fluctuations, for which $C_{lm}$ are statistically independent. For isotropic Universe this means

$$\langle C_{lm} C^*_{l'm'} \rangle = C_l^2 \delta_{ll'} \delta_{mm'} \ ,$$

The coefficients $C_l$ determine the correlation function

$$\langle \delta T(\vec{n}_1) \delta T(\vec{n}_2) \rangle = \sum_l \frac{2l+1}{4\pi} C_l^2 P_l(\cos\theta) \ ,$$

where $P_l$ are the Legendre polynomials, and $\theta$ is the angle between $\vec{n}_1$ and $\vec{n}_2$. In particular, for mean fluctuation of the temperature one has

$$\langle \delta T^2 \rangle = \sum_l \frac{2l+1}{4\pi} C_l^2 \approx \int \frac{l(l+1)}{2\pi} C_l^2 \, d\ln l \ .$$

Hence, the quantity

$$\delta T(l) \equiv \sqrt{\frac{l(l+1)}{2\pi}} C_l$$

is a measure of the contribution coming from angular momenta in a decimal interval of $l$. It is this quantity that is shown in fig. 8.

[8] Recently, it has been argued that there is non-Gaussianity in WMAP data at large angular scales [23]. This feature is still under debate.

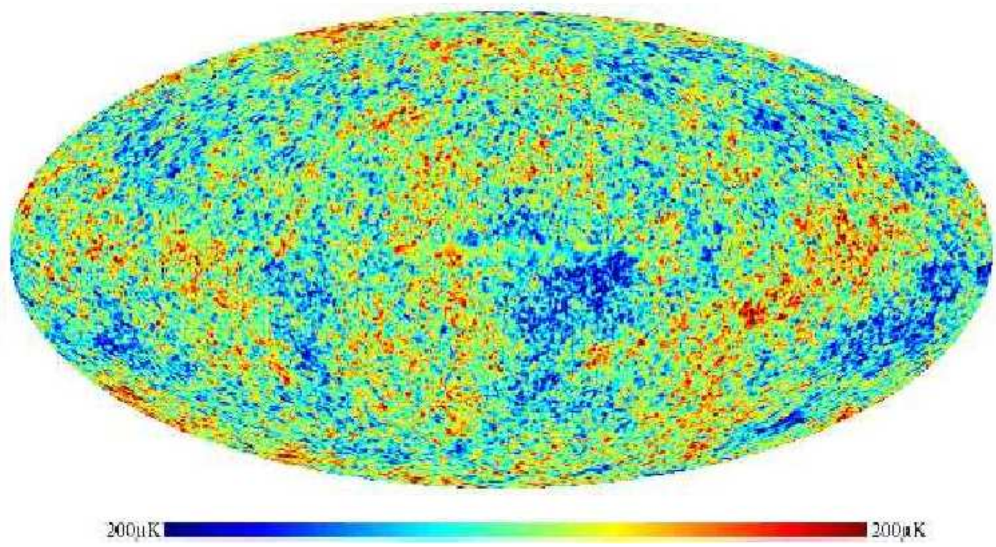


**Figure 7:** WMAP data [20]: temperature of photons coming from different directions in the sky. Darker regions correspond to lower temperatures. Average temperature $T_0$ and dipole component are subtracted. The angular variation of the temperature is at the level of $T \sim 100\ \mu$K, i.e., $\delta T/T_0 \sim 10^{-4} - 10^{-5}$.

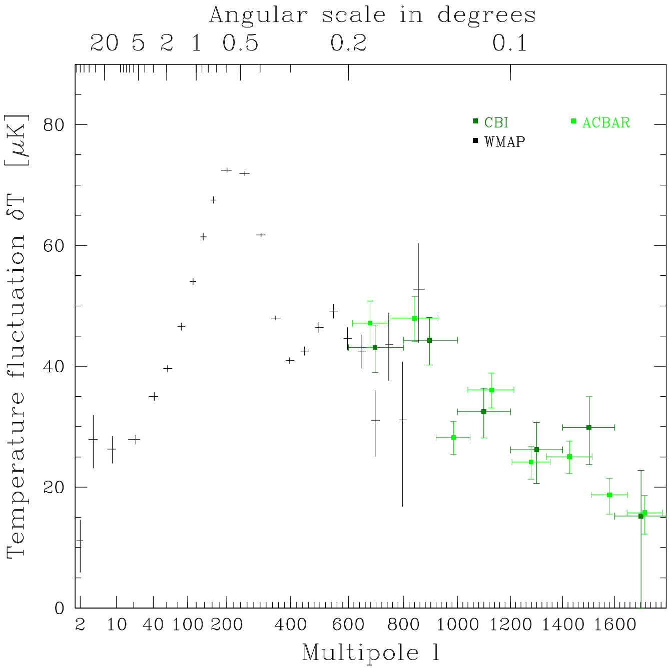


**Figure 8:** Measured angular anisotropy of cosmic microwave background [20, 21, 22].

It is clear from fig. 8 that the anisotropy as function of $l$ has fairly complex behaviour. There are peaks (at least two of them are clearly visible) and dips. The physics beyond these features is roughly as follows. The CMB anisotropy has its origin in the density perturbations[9], i.e., sound

[9]Another possible source is gravity waves; there effect is small, see section 8.3.

waves of all possible wavelengths. Waves of longer wavelength are seen in the photographic picture at larger angles; this is small-$l$ region in fig. 8. Conversely, the region of large $l$ corresponds to shorter wavelengths of the sound waves. The properties of these waves are different for modes which are superhorizon and subhorizon at recombination. We saw in section 2.7 that the density perturbations in plasma, whose wavelength exceeds (sound) horizon at recombination, did not oscillate in time by recombination. Their effect on temperature is mostly due to the gravitational potential they produce (Sachs–Wolfe effect): roughly speaking, light from denser reginos with stronger (negative) gravitational potentials has to climb the gravitational well, so it gets more redshifted as compared to light from less dense regions. This is the flat region of small $l$ in fig. 8. For shorter waves there is another effect: the waves oscillate, i.e. the particles in plasma move; this causes the Doppler effect leading to the CMB anisotropy. The waves of "just right" frequency (and hence wavelength) are in the phase of maximum motion at recombination (it is important here that the phase of oscillations is fixed, see section 2.7). These are the waves seen at an angle of 0.7 degrees ($l \sim 200$), where $C_l$ has the first peak. Further peaks correspond to higher harmonics.

All physical processes involved — the expansion of the Universe during its first 300 thousand years, the evolution of density perturbations and the recombination itself — are very well understood, so the calculations of the CMB anisotropy are very reliable. The predictions depend, of course, on a number of parameters characterising the early and present Universe, so the CMB data are used for extracting these parameters.

Let us point out two results coming from the analysis of the CMB anisotropy. The first is that our Universe is **spatially flat** to rather high precision. The positions of the peaks in $C_l$ (in particular, of the first peak measured with good accuracy) are sensitive to the spatial curvature: the absolute wavelength of the corresponding sound waves is reliably calculable, while its angular size strongly depends on whether space is a 3-sphere, 3-hyperboloid or 3-plane. Quantitatively, the result is usually expressed in terms of the contribution of the curvature term to the right hand side of the Friedmann equation (2.22): this contribution is less than about 2 per cent (we mentioned this already, see (2.26)). In more physical terms, this means that the radius of the spatial curvature of our Universe, $a$, is quite a bit greater than the length of the visible part (size of the cosmological horizon) $l_{H,0}$,

$$a > 4l_{H,0} \tag{3.3}$$

Another way to phrase this is to say that if the Universe were 3-sphere, its volume would still be a lot larger than the volume we can observe,

$$\frac{V_{tot}}{V_{obs}} = \frac{2\pi^2 a^3}{\frac{4}{3}\pi l_{H,0}^3} > 100$$

Hence, even if the Universe has finite volume, we know from observations that we are able to observe not more than 1 per cent of it. It is worth stressing that eq. (3.3) is an observational *bound*; it is likely that the actual radius of the spatial curvature of the Universe is much, much greater than the horizon size.

The second result concerns the baryonic content of the Universe. The height of the second peak in $C_l$ is sensitive to the dissipation rate of the sound waves in primordial cosmic plasma. The latter in turn depends on the number density of electrons, which is equal to the number density of protons by electric neutrality. The analysis gives for the present number density of baryons

$$n_{B,0} = 2.5 \cdot 10^{-7} \text{ cm}^{-3} \tag{3.4}$$

with precision better than 10 per cent. In terms of time-independent parameter $\eta$ defined in (2.18) this corresponds to

$$\eta = 6 \cdot 10^{-10} \tag{3.5}$$

It is remarkable that the same value of this parameter comes from an entirely different set of observations, which we are about to discuss in brief.

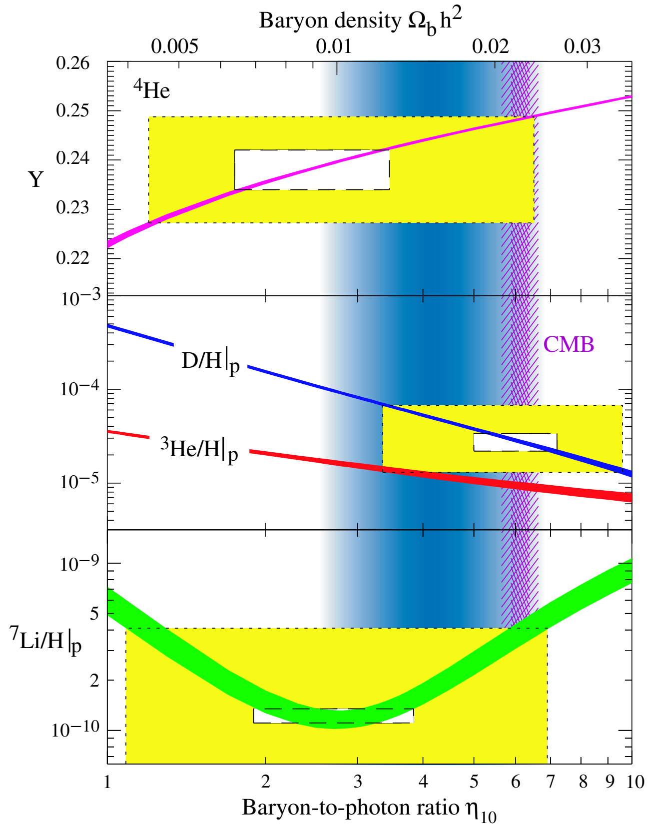


**Figure 9:** Big Bang Nucleosynthesis [24]: theoretical predictions (lines) versus observations (smaller boxes: $2\sigma$ errors, statistical only; larger boxes: $2\sigma$ with systematic uncertainties included). Vertical lines are the results from CMB anisotropy (the widths of bands depend on priors on other cosmological parameters).

**Theory of Big Bang Nucleosynthesis and observations of primordial abundances of light elements** probe the earliest epoch of the evolution of the Universe, accessible to observations today. This epoch corresponds to temperatures ranging from 1 MeV to a few $\cdot$ 10 keV, and age of the Universe from 1 to 200 s. At temperatures above 1 MeV, there is thermal equilibrium with respect to reactions

$$p + e^- \leftrightarrow n + \nu_e \tag{3.6}$$

As the Universe cools down below $T \approx 1 MeV$, neutrons are no longer produced or destroyed; their concentration (relative to protons) "freezes out". At temperatures of 100 keV and somewhat lower, these neutrons combine with protons into light nuclei, mostly $^4$He, but also deuterium $^2$H, lithium $^7$Li and others. These elements remain in the Universe, so their primordial abundance is measurable today. The calculations of the thermonuclear reactions are again based on well known physics, and the results are sensitive to the only unknown parameter[10], $\eta$. The results of the calculations and data are shown in fig. 9.

It is worth pointing out that Big Bang Nucleosynthesis serves also as a source of constraints on particle physics exotica. The very fact that the temperature of the Universe reached at least 1 MeV or so, and that the expansion was described by known physics at that stage, constrain significantly some extensions of the Standard Model, like models with large extra dimensions [25] (for a review see Ref. [26]). More generally, constraints from BBN are important in models with stable or long-living new particles: these are produced at earlier stages and may contribute too much to the energy density at the nucleosynthesis epoch, modifying in this way the expansion rate, and hence predictions of the BBN theory. Another example are particles that decay at the BBN epoch: these may destroy thermal equilibrium and therefore affect BBN; an example of this sort is provided by some theories with light gravitino. Any extension of the Standard Model has to be checked against cosmology, in particular, Big Bang Nucleosynthesis.

There are other data of cosmological significance, notably, measurements of mass distributions in galaxies and galactic clusters. We will briefly present them in appropriate places, and now proceed to immediate consequences.

## 4. COMPOSITION OF THE PRESENT UNIVERSE

As we will see in this section, the cosmological data correspond to a very surprising composition of the Universe.

Before proceeding, let us introduce a notion traditional in the analysis of the composition of the present Universe. For every type of matter $i$ with the present energy density $\rho_{i,0}$, one defines a parameter

$$\Omega_i = \frac{\rho_{i,0}}{\rho_c} \tag{4.1}$$

Then eq. (2.26) tells that

$$\sum_i \Omega_i = 1 \pm 0.02 \tag{4.2}$$

where the sum runs over all forms of energy. Let us now discuss contributions of different spieces to this sum.

We begin with **baryons**. The result (3.4), together gives

$$\rho_{B,0} = m_B \cdot n_{B,0} \approx 2.5 \cdot 10^{-7} \ \frac{\text{GeV}}{\text{cm}^3} \tag{4.3}$$

Comparing this result with the value of $\rho_c$ given in (2.28), one finds

$$\Omega_B = 0.05 \tag{4.4}$$

[10] Assuming the Standard Model particle content, see below.

Thus, baryons constiute rather small fraction of the present energy density in the Universe. One point to note is that most of the baryons in our Universe are dark: direct measurements of the mass density of stars give an estimate

$$\Omega_{stars} \sim 0.005$$

which is about an order of magnitude smaller than $\Omega_B$. There is nothing particularly dramatic about this observation: baryons may hide in dust and neutral gas clouds, brown dwarfs, Jupiters, etc.

**Photons** contribute even smaller fraction, as is clear from (2.16):

$$\Omega_\gamma \approx 6 \cdot 10^{-4} \tag{4.5}$$

From electric neutrality, the number density of **electrons** is about[11] the same as that of baryons, so electrons contribute negligible fraction to the total mass density. The remaining known stable particles are **neutrinos**. Their number density is calculable in Hot Big Bang theory and these calculations are nicely confirmed by Big Bang Nucleosynthesis. The number density of each type of neutrinos is

$$n_{\nu_\alpha} = 115 \, \frac{1}{\text{cm}^3}$$

where $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$. Direct limit on the mass of electron neutrino, $m_{\nu_e} < 2.6$ eV, together with the observations of neutrino oscillations suggest that every type of neutrino has mass smaller than 2.6 eV (neutrinos with masses above 0.03 eV must be degenerate, according to neutrino oscillation data). The energy density of all types of neutrinos is thus smaller than $\rho_c$:

$$\rho_{\nu,total} = \sum_\alpha m_{\nu_\alpha} n_{\nu_\alpha} < 3 \cdot 2.6 \text{ eV} \cdot 115 \, \frac{1}{\text{cm}^3} \sim 8 \cdot 10^{-7} \, \frac{\text{GeV}}{\text{cm}^3}$$

which means

$$\Omega_{\nu,total} < 0.16$$

This estimate does not make use of any cosmological data. In fact, cosmological observations give stronger bound

$$\Omega_{\nu,total} < 0.01 \tag{4.6}$$

This bound is mostly due to the analysis of the structures at small length scales, and has to do with streaming of neutrinos from the gravitational potential wells at early times when neutrinos were ultra-relativistic. In terms of the neutrino masses the bound (4.6) reads [27]

$$\sum m_{\nu_\alpha} < 0.42 \text{ eV}$$

so every neutrino must be lighter than 0.14 eV. On the other hand, atmospheric neutrino data and K2K experiment tell that the mass of at least one neutrino must be larger than 0.02 eV. Comparing these numbers, one sees that it may be feasible to measure neutrino masses by cosmological observations (!) in the future.

Coming back to our main topic here, we conclude that most of the energy density in the present Universe is not in the form of known particles; most energy in the present Universe must

[11] There are neutrons, whose number is somewhat smaller than he number of protons.

be in “something unknown”. Furthermore, there is strong evidence that this “something unknown” has two components: clustered (dark matter) and unclustered (dark energy).

**Clustered dark matter** consists presumably of new stable massive particles. These make clumps of energy density which encounter for much of the mass of galaxies and most of the mass of galactic clusters. There are a number of ways of estimating the contribution of non-baryonic dark matter into the total energy density of the Universe (see Ref. [28] for details):

– Composition of the Universe affects the angular anisotropy of cosmic microwave background. Quite accurate measurements of the CMB anisotropy, available today, enable one to estimate the total mass density of dark matter.

– Composition of the Universe, and especially the density of non-baryonic dark matter, is crucial for structure formation of the Universe. Comparison of the results of numerical simulations of structure formation with observational data gives reliable estimate of the mass density of non-baryonic clustered dark matter.

The bottom line is that the non-relativistic component constitutes about 30 per cent of the total present energy density, which means that non-baryonic “cold dark matter” has

$$\Omega_{CDM} \approx 0.25 \tag{4.7}$$

There is direct evidence that dark matter exist in the largest gravitationally bound objects – clusters of galaxies. There are various methods to determine the gravitating mass of a cluster, and even mass distribution in a cluster, which give consistent results. To name a few:

– One measures velocities of galaxies in galactic clusters, and makes use of the gravitational virial theorem,

$$\text{Kinetic energy of a galaxy} = \frac{1}{2}\,\text{Potential energy}$$

In this way one obtains the gravitational potential, and thus the distribution of the total mass in a cluster.

– Another measurement of masses of clusters makes use of intracluster gas. Its temperature obtained from X-ray measurements is also related to the gravitational potential through the virial theorem.

– Fairly accurate reconstruction of mass distributions in clusters is obtained from the observations of gravitational lensing of background galaxies by clusters.

These methods enable one to measure mass-to-light ratio in clusters of galaxies. Assuming that this ratio applies to all matter in the Universe[12], one arrives at the estimate for the mass density of clumped matter in the present Universe. Remarkably, this estimate coincides with (4.7).

Finally, dark matter exists also in galaxies. Its distribution is measured by the observations of rotation velocities of distant stars and gas clouds around a galaxy. An example is shown in fig. 10.

Thus, cosmologists are confident that much of the energy density in our Universe consists of new stable particles. We will see that natural candidates are particles which participate in weak interactions (or, more generally, particles whose annihilation cross section is determined by a scale of the order of electroweak scale, $M_{EW} \sim 100$ GeV). Of course, this is only a hypothesis for the time being, and there are many other candidates for dark matter spieces.

[12]This is a strong assumption, since only about 10 per cent of galaxies are in clusters.

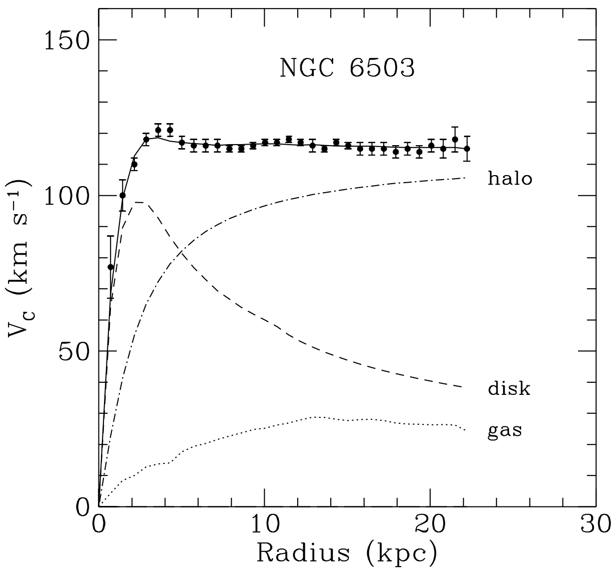


**Figure 10:** Rotation velocities of gas clouds for galaxy NGC 6503 [29]. Curves show contributions of different components of the galaxy; "halo" is dark.

**Unclustered dark energy**

Non-baryonic clustered dark matter is not the whole story. Making use of the above estimates, one obtains an estimate for the energy density of all particles,

$$\Omega_\gamma + \Omega_B + \Omega_{\nu,total} + \Omega_{CDM} \approx 0.3$$

We note in passing that the contribution of photons and possible massless neutrinos is very small here, so the left hand side is the contribution of all non-relativistic matter, and it is often denoted by $\Omega_M$. Thus, $\Omega_M \approx 0.3$. Equation (4.2) implies then that 70 per cent of the energy density is unclustered.

All this fits nicely to the observations of SNe 1a. Indeed, we have seen that neither relativistic, nor non-relativistic matter can lead to the accelerated expansion of the Universe. So, the accelerated expansion requires energy stored in something dramatically different from conventional particles. Furthermore, we have seen that this "something" — dark energy — must have negative pressure. In fact the analysis of the entire set of cosmological data [27] in terms of dark energy with the phenomenological equation of state[13]

$$p = w\rho \; , \quad w = \text{const}$$

gives

$$\Omega_\Lambda = 0.72 \pm 0.02$$

[13]The data are consistent with time-independent $w$, although allow for slight time variation.

(here subscript $\Lambda$ traditionally refers to dark energy) and

$$-1.2 < w < -0.8$$

It is worth noting that the vacuum value, $w = -1$ is right in the middle of the allowed region.

To conclude, the composition of the present Universe is fairly complex. Most of the energy density comes from spieces which particle physicists are unfamiliar with: vacuum or vacuum-like dark energy and non-baryonic clumped dark matter (presumably, non-relativistic, weakly interacting particles). This poses serious problems for both fundamental physics and cosmology:

**What are the particles of non-baryonic dark matter?** This appears to be a less difficult problem, as compared to some others listed below. Currently, a popular option is the lightest supersymmetric particle, which is stable in many supersymmetric extensions of the Standard Model. Indeed, we will estimate in what follows that the present mass density of such particles is naturally predicted to be in the right ballpark. Of course there are many other options, such as axions, gravitinos, Q-balls, to name a few. In any case, experimental discovery of the dark matter particle would be a great achievement of both particle physics and cosmology. This discovery may come either from experiments attempting to detect dark matter or from collider searches, or both.

**Why there are baryons, and no anti-baryons in our Universe?** In other words, what is the origin of matter-antimatter asymmetry of the Universe? This also appears to be a less difficult problem; we will discuss this issue later in these lectures. Here we notice only, that any solution of this problem requires an extension of the Standard Model.

**Why the mass density of the non-baryonic dark matter is so similar (within less than an order of magnitude) to the mass density of baryons?** Both these densities scale as $a^{-3}(t)$, so their ratio stays constant during most of the evolution of the Universe. It is not inconceivable that mechanisms which create baryons and dark matter particles in the early Universe are related to each other, so that the approximate equality of the mass densities is not a mere coincidence. It is, however, difficult to construct a corresponding particle physics model, and it is fair to say that existing attempts are far from being compelling.

**What is the origin of dark energy? If this is vacuum, why vacuum has non-zero energy density, which, however, is very small by particle physics standards?** This is a very fundamental problem of microscopic physics. In natural units, the vacuum energy density is about $\rho_c \sim 10^{-46}$ GeV$^4$ while on dimensional grounds one would expect values like 1 GeV$^4$ (QCD scale) or $10^8$ GeV$^4$ (electroweak scale). This enormous discrepancy cries for explanation, but despite numerous attempts it remains essentially an open problem. It may very well be that the solution of this "cosmological constant problem" will lead to entirely new concept of physics at ultra-large distances.

**Why now?** The energy densities of non-relativistic dark matter and dark energy scale differently: the former scales like $a^{-3}(t)$ while the latter stays approximately constant. Hence, at small $a(t)$ (early Universe) the energy density of non-relativistic matter exceeded by far the dark energy density. Conversely, future expansion of the Universe will be dominated by dark energy. Yet these energy densities are of the same order of magnitude today. Why is this the case? What is special about the present epoch of the evolution of the Universe?



PoS(RTN2005)003

## 5. DARK ENERGY

It appears that by far the most difficult problem is the origin of dark energy. The most disappointing possibility would be that the carrier of dark energy is **vacuum**; in that case we will hardly ever be confident of a mechanism responsible for tiny, but non-zero vacuum energy density. As the last resort, we will possibly have to rely upon anthropic considerations [30, 31], which are based on the observation that if the vacuum energy density were substantially larger (in absolute value) than observed, the Universe would not be suitable for the existence of observers like us. Indeed, large negative cosmological constant would give rise to early recollapse of the Universe, while with large positive value, the accelerated expansion would start much earlier. In either case there would not be enough time for stars and galaxies to form, hence life would not develop. These considerations in fact provide rather strong bounds on the vacuum energy density, within two orders of magnitude of the observed value. The idea is then that there may be infinitely many regions in the Universe (or even infinitely many universes) where fundamental parameters like vacuum energy density are different, and span entire range $(-\infty,+\infty)$. The observers like us can only find themselves in a region where the values of these parameters are suitable for their existence. There have been various suggestions for how such a picture can occur, ranging from wormholes/branching universes to "ethernal inflation" and, most recently, to string theory landscape [32]. The problem is that it hardly will be possible to check this picture experimentally even in distant future.

Another option, more promising from observational viewpoint, is that dark energy is due to some light field [33, 34, 35], dubbed **quintessence**. As an example, consider a homogeneous scalar field $\phi(t)$ in an expanding Universe. The action of the scalar field is

$$S = \int d^4x \, \sqrt{-g} \, \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right)$$

where $V(\phi)$ is a scalar potential. For homogeneous scalar field in the FRW metric this action reduces to

$$S = \int dt \, a^3 \left( \frac{1}{2} \dot{\phi}^2 - V(\phi) \right)$$

so the scalar field equation is

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -\frac{\partial V}{\partial \phi}$$

Formally, this equation may be viewed as Newton's equation of classical mechanics with potential $V$ and friction, where $\phi$ plays the role of particle coordinate and $\dot{a}/a \equiv H$ is the time-dependent friction coefficient. If the scalar potential is a slowly varying function of $\phi$, the Hubble friction makes the field slowly rolling in the potential, cf. section 7.3. For homogeneous field, the energy-momentum tensor

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - L$$

has the form of the energy-momentum tensor of a fluid, with the energy density and pressure equal to

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

For slowly varying field, both are dominated by the scalar potential, which is assumed to be small. In this way one has approximately vacuum equation of state,

$$p_\phi \approx -\rho_\phi \tag{5.1}$$

but the energy density $\rho_\phi$ slowly decreases in time (hence the equation of state (5.1) is only approximate). Of course, this proposal raises several questions: why the genuine vacuum energy density is zero, so that it *does not* contribute to dark energy density? what is the physics behind the field $\phi$? where does the small energy scale, giving $V(\phi) \sim 10^{-46}$ GeV$^4$ today, come from? why the *present* value of $\phi$ is in the right place? While the first three questions remain unanswered, for certain scalar potentials the fourth one has an elegant answer: irrespectively of the initial conditions, the field rolls down to correct place just in time (tracking solutions [36]). An exapmple is the theory with the scalar potential

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^\alpha}$$

where $\alpha > 0$, and $M$ is adjusted as follows,

$$M^{4+\alpha} \sim M_{Pl}^\alpha \rho_c \tag{5.2}$$

(where $\rho_c$ is again the present critical density). This is precisely the adjustment needed to ensure that the energy of the scalar field today is close to $\rho_c$, but the point is that the initial conditions for the evolution of the field $\phi$ *need not* be adjusted. Furthermore, once the relation (5.2) is granted, the transition from matter dominated to dark energy dominated regime occurs at $z \sim 1$, which is precisely what is required by observations (see Ref. [36] for details).

Yet another option for explaining the accelerated expansion of our Universe is that **gravity** deviates from General Relativity at cosmological distances and time scales, so that the Friedmann equation (2.22) merely is not valid at the present epoch. This option would probably have to address similar questions as the quintessence proposal, but even before that one meets a serious problem of constructing theoretically consistent and phenomenologically acceptable theory which would reduce to General Relativity at distances from a fraction of a millimeter (down to which gravity is experimentally known to obey Newton's law) to at least tens Megaparsecs, and deviate from General Relativity at cosmological scales. In known Lorentz-invariant examples of such a theory[14], there either exist ghosts (fields with negative energy unbounded from below) or gravity becomes strongly coupled at quantum level (and hence not tractable) at an unacceptably low "ultraviolet" energy scale corresponding to distance of order 1000 km. A consistent theory of this sort would probably require "gravitational Higgs mechanism" and violation of Lorentz-invariance, but even this, rather exotic idea, has not yet lead to a consistent model capable of explaining the accelerated expansion of the Universe.

Quintessence models (and most likely models with infrared-modified gravity, if the latter exist) imply that the effective dark energy density is *not* constant in time. Needless to say, an observational evidence for the time-dependence of $\rho_\Lambda$ would have enormous impact on fundamental physics. On the other hand, it is hard to foresee any method to probe dark energy in a laboratory, so we have to rely on cosmological observations when trying to reveal the origin of dark energy.

[14]See, however, Ref. [37].

## 6. HOT BIG BANG

The usual matter and dark matter have their origin in the early Universe. Before discussing plausible scenarios of their generation, let us give some details of the evolution of the Universe in its hot stage.

One assumes that it makes sense to extrapolate the evolution of the Universe back in time by making use of known microscopic physics (electrodynamics, nuclear physics, QCD and electroweak physics) and General Relativity. This theory is called "Hot Big Bang theory". According to this theory, the Universe was hotter at earlier stages (i.e., at smaller values of the scale factor $a(t)$), as the temperature scales as $a^{-1}(t)$. Also, the Universe was denser: the particle number densities scale as $a^{-3}(t)$ both for relativistic and non-relativistic particles. At high enough temperature the Universe was quite different from what we observe today: instead of almost empty space with galaxies here and there, there was hot, dense and almost homogeneous plasma filling the Universe. This is why the microscopic physics played a role in the early Universe: at temperatures of the order of nuclear physics scale, roughly a few MeV, one has to deal with nuclear reactions; at temperatures of the order of the strong interaction scale, $(\text{a few}) \cdot 100$ MeV, the relevant microscopic theory was largely QCD, etc.

### 6.1 Expansion of the Universe

We begin with the analysis of the evolution of the Universe, i.e., the behavior of the scale factor $a(t)$ as function of time. As we already discussed in section 2.5, at early times the Universe was radiation dominated, then matter dominated, and presently dark energy dominated, while the curvature term $\kappa/a^2$ was never important.

**Deceleration to acceleration**

Due to the dark energy dominance, the Universe accelerates today. When matter was dominating, the Universe was decelerating. To figure out when the change in the regime occurred, we write down the Friedmann equation in the following form (assuming dark matter equation of state $p = -\rho$, neglecting spatial curvature and also neglecting ultra-relativistic matter for the moment; we will see that ultra-relativistic matter dominated the expansion at much earlier stage),

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3\rho_c}\left(\Omega_M \frac{a_0^3}{a^3} + \Omega_\Lambda\right)$$

where $a_0$, $\Omega_M$, $\Omega_\Lambda$ and $\rho_c$ are present values, thus time-independent constants. Therefore,

$$\dot{a}^2 = \frac{8\pi G}{3\rho_c}\left(\Omega_M \frac{a_0^3}{a} + \Omega_\Lambda a^2\right)$$

and $\ddot{a}$ is equal to zero when

$$\frac{a_0^3}{a^3} \equiv (1+z)^3 = \frac{2\Omega_\Lambda}{\Omega_M}$$

With $\Omega_\Lambda = 0.7$ and $\Omega_M = 0.3$, we have

$$\text{deceleration} \rightarrow \text{acceleration} : \quad z \approx 0.7$$

The Universe was decelerating until fairly recently. Before $z \approx 0.7$, the expansion was dominated by the non-relativistic matter.

**Radiation domination to matter domination**.

Since energy densities of non-relativistic and ultra-relativistic matter ("radiation") scale as $a^{-3}$ and $a^{-4}$, respectively, dominant contribution to the energy density of the Universe at very small $a$, i.e., at very early epoch, came from ultra-relativistic matter. To estimate $z_{eq}$ at which the equilibrium between matter and radiation occurred, i.e., at which the expansion regime changed from the dominance of ultra-relativistic particles to the dominance of non-relativistic matter, we write

$$\frac{\rho_M(t)}{\rho_{rad}(t)} = \left(\frac{\rho_M}{\rho_{rad}}\right)_0 \frac{a(t)}{a_0}$$

where the subscript 0 still refers to the present values. Equilibrium occurs at

$$\frac{\rho_M(t_{eq})}{\rho_{rad}(t_{eq})} \approx 1$$

which gives

$$\frac{a_0}{a(t_{eq})} \equiv 1 + z_{eq} \approx \left(\frac{\rho_M}{\rho_{rad}}\right)_0 = \frac{\Omega_M}{\Omega_{rad}}$$

We already know the energy density of relic photons; massless neutrinos[15] of all tree types have $\rho_{\nu,0} \approx 0.7\rho_\gamma$. Thus, $\Omega_{rad} \approx 10^{-4}$, see eq. (4.5). With $\Omega_M = 0.3$ we have

$$\text{radiation domination} \rightarrow \text{matter domination}: \quad z_{eq} \approx 3000$$

The corresponding temperature is

$$T_{eq} = T_0(1 + z_{eq}) \approx 10^4 \text{ K} \approx 1 \text{ eV} \tag{6.1}$$

At higher temperatures, the expansion of the Universe was dominated by ultra-relativistic matter.

It is important for the theory of structure formation that during much of its lifetime, the Universe was dominated by non-relativistic matter. The expansion rate at both radiation dominated and vacuum dominated stages is such that gravitational perturbations grow slowly, and only during matter dominated stage their growth is fast enough to account for the existing structures in the Universe. The bottom line is that the present composition of the Universe plus simple extrapolation back to the past are consistent with the theory of structure formation. Various ingredients of the standard cosmology nicely fit together.

### 6.2 Epochs in the early Universe

We have already mentioned two important epochs in the evolution of the Universe: **recombination epoch** (transition from plasma to neutral gas that occured at $T \sim 3000$ K, $t \sim 3 \cdot 10^5$ yrs and lasted much less than Hubble time) and **nucleosynthesis epoch** ($T = 1$ MeV to a few $\cdot$ 10 keV). Another "event" is **neutrino decoupling.** At high temperatures, weakly interacting particles, including neutrinos, were in thermal equilibrium with the rest of cosmic plasma. The plasma became

[15]Whether or not neutrinos are exactly massless is inessential for the estimate of $z_{eq}$: the estimate remains valid if every neutrino has mass smaller than 1 eV.

effectively transparent for neutrinos at temperature of about 1 MeV. The temperature of decoupling of neutrinos from cosmic plasma is of importance for nucleosynthesis, as it affects the neutron–proton ratio just before nucleosynthesis (and hence the abundances of light elements, which need neutrons for their formation), and also the expansion of the Universe at the nucleosynthesis epoch. The fact that neutrinos decoupled much earlier than photons implies that the present neutrino-to-photon ratio is less than one[16].

As we move further back in time, the cosmic plasma has more and more components. At temperatures above roughly 0.5 MeV (set by the mass of electron), there are lots of electrons and positrons which frequently are pair created and annihilate; at $T > 100$ MeV the plasma contains muons and pions, etc. Simple estimates given in the next subsection show that the plasma remains in thermal equilibrium except possibly for **phase transitions**.

– QCD phase transition.

At temperatures well above 100 MeV (QCD scale), strongly interacting particles are dissolved into quarks and gluons. This quark-gluon plasma converts into hadronic matter (mostly pions) during the quark-hadron phase transition. Theoretical estimates and lattice simulations in QCD suggest that the temperature of this phase transition is about 170 MeV.

– Electroweak transition.

Loosely speaking, at temperatures well above 100 GeV, electroweak symmetry is unbroken, the Higgs expectation value vanishes, and $W$- and $Z$-bosons are massless. At $T \sim 100$ GeV, the phase transition of the electroweak symmetry breaking takes place. In fact, there is no local, gauge invariant order parameter in the standard electroweak theory, and the electroweak transition is similar to vapor-liquid transition: for some values of parameters, there is the first order phase transition, whereas for other values, a smooth cross-over takes place instead. With existing constraints on the Higgs boson mass, the *Standard Model* predicts cross-over; what actually happened in the early Universe depends on what exactly is the extension of the Standard Model. Uncovering physics in 100 GeV – 1 TeV energy range will thus allow cosmologists to study quantitatively quite early epoch of the evolution of the Universe.

– GUT transition.

Extrapolating further back is dangerous, but if we do so, we come to the Grand Unification epoch, whose temperature is set by the GUT scale, $T_{GUT} \sim 10^{16}$ GeV. At this temperature, one expects Grand Unified phase transition to occur. However, many models of inflation suggest that the Universe never had such a high temperature after inflation.

Even more interesting are the **epoch of the generation of dark matter** and **epoch of the generation of baryon asymmetry**. We can only make guesses about these epochs, and some of the guesses is the subject of these lectures. Before turning to them, it is convenient to consider the expansion of the Universe at early times in little more detail.

### 6.3 Expansion rate and lifetime at radiation domination

Before proceeding further, let us consider in little more detail the expansion of the Universe at

[16]This is because photons are additionally heated, after neutrino decoupling, due to annihilations of $e^+$ and $e^-$ which were abundant at $T \sim 1 MeV$.

the radiation dominated stage, assuming thermal equilibrium of all ultra-relativistic spieces[17]. The energy density of all ultra-relativistic spieces, which enters the Friedmann equation, is

$$\rho = \frac{\pi^2}{30} g_* T^4$$

where $g_*$ is the effective number of massless degrees of freedom at temperature $T$. The contribution of bosons into $g_*$ is equal to the number of spin states (e.g., for photons $g_\gamma = 2$, while for $W$-bosons at temperature above 100 GeV, $g_W = 6$ because of two charges and three projections of spin), while fermions contribute $7/8$ of the number of spin states (electrons plus positrons contribute $4 \cdot 7/8$, each type of left-handed neutrino plus its antineutrino gives $2 \cdot 7/8$, etc.). The parameter $g_*$ is the sum of contributions of all ultra-relativistic spieces; it slightly depends on time because at higher temperatures, more spieces are ultra-relativistic (say, electrons contribute at $T > 0.5$ MeV and do not contribute at lower temperatures).

It is convenient to introduce the effective Planck mass

$$M_{Pl}^* = \frac{M_{Pl}}{1.66\sqrt{g_*}}$$

This parameter slightly depends on temperature, and numerically is of order

$$M_{Pl}^* = (\text{a few}) \cdot 10^{18} \text{ GeV}$$

With this notation, the expansion rate is related to temperature in a simple way,

$$H(t) = \frac{T^2(t)}{M_{Pl}^*}$$

One recalls that the expansion law at the radiation dominated stage (neglecting the dependence of $g_*$ on temperature) is

$$a(t) = \text{const} \cdot \sqrt{t}$$

so the Hubble parameter is related to the lifetime as follows,

$$H \equiv \frac{\dot{a}}{a} = \frac{1}{2t}$$

We immediately deduce the relation between lifetime and temperature,

$$t = \frac{M_{Pl}^*}{2T^2}$$

Let us make use of the latter formula to estimate the age of the Universe at different epochs:

– Nucleosynthesis

The temperatures relevant for BBN range from a few MeV to about 70 keV. From these we obtain that the earliest time directly probed by observations is about

$$t \sim \frac{10^{18} \text{ GeV}}{10^{-6} \text{ GeV}^2} \sim 1 \text{ s}$$

[17]The assumption of thermal equilibrium is in fact not valid for neutrinos at temperatures below 1 MeV. The corresponding modification of our discussion is straightforward, however.

whereas BBN ends at $t \sim 200$ s $\sim 3$ min. We do have a handle on the Universe one second old!

– Earlier epochs:

$$\begin{aligned} \text{QCD phase transition}: \quad & T \sim 200 \text{ MeV}, \quad t \sim 3 \cdot 10^{-5} \text{ s} \\ \text{Electroweak epoch}: \quad & T \sim 100 \text{ GeV}, \quad t \sim 10^{-10} \text{ s} \end{aligned} \tag{6.2}$$

One may wonder whether equilibrium thermodynamics, which we use throughout, is applicable at these early times, when the Universe expands so rapidly. To see that this is indeed the case, let us consider, as an example, electromagnetic scattering of light particles at $T > 1$ MeV. It is clear on dimensional grounds that the mean free time of a charged particle at temperature $T$ is

$$\tau \sim (\alpha^2 T)^{-1}$$

(the electromagnetic cross section is proportional to $\alpha^2$). For thermal equilibrium with respect to the electromagnetic interactions be established, the interaction rate $\tau^{-1}$ must be smaller than the expansion rate of the Universe, $H$, which gives

$$\alpha^2 T \gg \frac{T^2}{M_{Pl}^*}$$

This inequality is indeed valid at $T \gg \alpha^2 M_{Pl}^* \sim 10^{14}$ GeV, so electromagnetic (and many other) microscopic processes are in thermal equilibrium at all temperatures of interest to us.

Thermal equilibrium is not a particularly interesting state of the Universe. What we are going to discuss in the rest of this section are in fact *inequilibrium* phenomena. It is these phenomena that may leave relics behind, and hence may have observable consequences.

### 6.4 Heavy relic: Best guess for cold dark matter

As we discussed above, the observational data strongly suggest that a good part of the energy density in the present Universe comes from new stable (or practically stable) particles. At least in some cases, the Hot Big Bang theory is capable of predicting the density of such particles in terms of their interaction cross sections and masses. Here we present the corresponding estimate in the simplest possible scenario; needless to say, this estimate can be (and has been) refined by more careful calculations.

Let us assume that there exists a heavy stable particle $Y$ and its anti-particle $\bar{Y}$. Let us assume for definiteness that the dominant process in which these particles can be destroyed or created is their pair-annihilation or creation, with annihilation products being the particles of the Standard Model (the analysis for a neutral particle $Y$, which coincides with its own anti-particle, is very similar, provided that these particles pair-annihilate). Let us further assume that there is no asymmetry between $Y$ and $\bar{Y}$ in the early Universe, i.e., the densities of $Y$ and $\bar{Y}$ are equal to each other[18]. We will see that the overall cosmological behaviour of these particles is as follows. At high temperatures, $T \gg M_Y$, the $Y$-particles are in thermal equilibrium with the rest of cosmic plasma; there are

[18]This is actually a strong assumption. It is valid in many, but not all, realistic extensions of the Standard Model. In fact, an alternative scenario with the generation of $Y$-asymmetry is appealing too, because it might relate the baryon asymmetry to the density of dark matter [38].

lots of $\bar{Y}$-$Y$ pairs in the plasma, which are continuously created and annihilate. As the temperature drops below $M_Y$, the equilibrium number density decreases. At some "freeze-out" temperature $T_f$ the number density becomes so small, that $Y$ and $\bar{Y}$ can no longer meet each other during the Hubble time, and their annihilation terminates. After that the number density of survived $Y$ and $\bar{Y}$ decreases like $a^{-3}$, and these relic particles contribute to the mass density in the present Universe. Our purpose is to estimate the range of properties of $Y$-particles, in which their present mass density is of the order of the critical density $\rho_c$, so that $Y$-particles may serve as dark matter candidates.

Assuming thermal equilibrium, elementary considerations of mean free path of a particle in gas give for the lifetime of a non-relativistic $Y$-particle in cosmic plasma, $\tau_{ann}$,

$$\sigma_{ann} \cdot v \cdot \tau_{ann} \cdot n_{\bar{Y}} \sim 1$$

where $v$ is the velocity of $Y$-particle, $\sigma_{ann}$ is the annihilation cross section at velocity $v$ and $n_{\bar{Y}} = n_Y$ is the equilibrium number density (Boltzmann law at zero chemical potential, i.e., at $n_{\bar{Y}} = n_Y$)

$$n_Y = g_Y \cdot \left(\frac{m_Y T}{2\pi}\right)^{3/2} \mathrm{e}^{-\frac{m_Y}{T}}$$

Let us assume for definiteness that the annihilation occurs in $s$-wave (other cases give similar results), so at non-relativistic velocities

$$\sigma_{ann} = \frac{\sigma_0}{v}$$

where $\sigma_0$ is a constant about which we will have to say more later. One should compare the lifetime with the Hubble time, or annihilation rate $\Gamma_{ann} \equiv \tau_{ann}^{-1}$ with the expansion rate $H = T^2/M_{Pl}^*$. At $T \sim m_Y$, the equilibrium density is of order $n_Y \sim T^3$, and $\Gamma_{ann} \gg H$ for not too small $\sigma_0$. This means that annihilation (and, by reciprocity, creation) of $\bar{Y}$-$Y$ pairs is indeed rapid, and $Y$-particles are indeed in thermal equilibrium with the plasma. At very low temperature, on the other hand, the number density $n_Y$ is exponentially small, and $\Gamma_{ann} \ll H$. At low temperatures we cannot, of course, make use of equilibrium formulas: $Y$-particles no longer annihilate (and, by reciprocity, are no longer created), there is no thermal equilibrium with respect to creation–annihilation processes, and the number density $n_Y$ gets diluted only because of the cosmological expansion.

The freeze-out temperature $T_f$ is determined by the relation

$$\tau_{ann}^{-1} \equiv \Gamma_{ann} \sim H$$

where we can still use the equilibrium formulas, as $Y$-particles are in thermal equilibrium (with respect to annihilation and creation) just before freeze-out. We find

$$\sigma_0 \cdot n_Y(T_f) \sim \frac{T_f^2}{M_{Pl}^*} \tag{6.3}$$

or

$$\sigma_0 \cdot g_Y \cdot \left(\frac{m_Y T_f}{2\pi}\right)^{3/2} \mathrm{e}^{-\frac{m_Y}{T_f}} \sim \frac{T_f^2}{M_{Pl}^*}$$

The latter equation gives the freeze-out temperature, which, up to loglog terms, is

$$T_f \approx \frac{m_Y}{\ln(M_{Pl}^* m_Y \sigma_0)}$$

Note that this temperature is quite a bit smaller than $m_Y$, if the relevant microscopic mass scale is much below $M_{Pl}$. This means that $Y$-particles freeze out when they are indeed non-relativistic, hence the term "cold dark matter". The fact that the annihilation and creation of $Y$-particles terminates at relatively low temperature has to do with rather slow expansion of the Universe, which should be compensated for by the smallness of the number density $n_Y$.

At the freeze-out temperature, we make use of eq. (6.3) and obtain

$$n_Y(T_f) = \frac{T_f^2}{M_{Pl}^* \sigma_0}$$

Note that this density is inversely proportional to the annihilation cross section (up to logarithms). The reason is that for higher annihilation cross section, the creation–annihilation processes are longer in equilibrium, and less $Y$-particles survive.

To estimate the present density of $Y$-particles, it is convenient to consider ratio $n_Y/s$ where $s$ is the entropy density,

$$s = \frac{2\pi^2}{45} g_* T^3$$

The point is that during the adiabatic expansion after freeze-out, both entropy density and $n_Y$ behave as $a^{-3}$, so this ratio stays constant. Up to a factor of order 1, this ratio at freeze-out is

$$\frac{n_Y}{s} \sim \frac{1}{g_*(T_f) M_{Pl}^* T_f \sigma_0}$$

At late times, the entropy density, again up to a factor of order 1, is equal to the number density of photons, so the present number density of $Y$-particles is of order

$$n_{Y,0} \sim n_{\gamma,0} \cdot \left(\frac{n_Y}{s}\right)_{freeze-out}$$

and the mass density is

$$\begin{aligned} \rho_{Y,0} &= m_Y n_{Y,0} \\ &\sim n_{\gamma,0} \cdot \frac{\ln(M_{Pl}^* m_Y \sigma_0)}{g_*(T_f) M_{Pl}^* \sigma_0} \end{aligned} \tag{6.4}$$

This formula is remarkable. The mass density depends mostly on one parameter, the annihilation cross section $\sigma_0$. The dependence on the mass of $Y$-particle is through the logarithm and $g_*(T_f)$, and is very mild. From this formula we immediately derive the condition ensuring that $Y$-particles are dark matter candidates, i.e., their present mass density is of the order of $\rho_c$,

$$\sigma_0 \sim \frac{n_{\gamma,0}}{g_*(T_f) M_{Pl}^* \rho_c} \ln(M_{Pl}^* m_Y \sigma_0)$$

The value of the logarithm here is between 20 and 40, depending on parameters (this means, in particular, that freeze-out occurs when the temperature drops 20 to 40 times below the mass of $Y$-particle). Plugging in other numerical values ($\rho_c \sim 10^{-5}$ GeV cm$^{-3}$, $n_{\gamma,0} \sim 400$ cm$^{-3}$, $g_*(T_f) \sim 100$, $M_{Pl}^* \sim 10^{18}$ GeV), we obtain an estimate

$$\sigma_0 \sim 10^{-11}\ \text{GeV}^{-2}$$

which already crudely tells us what the relevant range of mass scales is. In fact, the annihilation cross section may be parametrized as

$$\sigma_0 = \frac{\alpha^2}{\mathcal{M}^2}$$

where $\alpha$ is some coupling constant, and $\mathcal{M}$ is the mass scale. This parametrization is suggested by the picture of $\bar{Y}$-$Y$ annihilation via exchange of another particle of mass $\mathcal{M}$, which may be somewhat higher than $m_Y$. With $\alpha \sim 10^{-2}$, the estimate for the mass scale is roughly

$$\mathcal{M} \sim 1 \text{ TeV}$$

Thus, with very mild assumptions, we find that the non-baryonic dark matter may naturally originate from the TeV-scale physics!

In supersymmetric extensions of the Standard Model, the lightest supersymmetric particle (LSP, most likely, neutralino — a mixture of superpartners of photon, $Z$-boson and neutral Higgs bosons) is often stable, and its annihilation cross section is automatically in the right ballpark. This is illustrated in fig. 11.

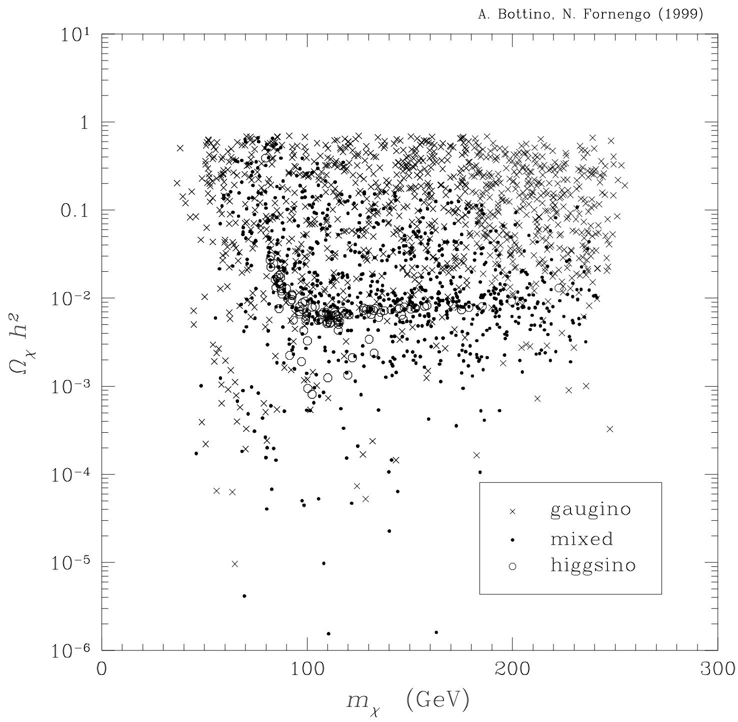


**Figure 11:** Present mass density of relic neutralino LSP's (denoted by $\chi$), as function of the neutralino mass [39] in Minimal Supersymmetric Standard Model (MSSM). Each point corresponds to a randomly chosen set of MSSM parameters. Crosses, dots and circles correspond to different neutralino content. Note that observational data suggest $\Omega_\chi \approx 0.25$, $h \approx 0.7$, so that $\Omega_\chi h^2 \approx 0.12$.

Naturally, search for both direct and indirect signals from neutralino dark matter (and more generally, weakly interacting massive particles, WIMPs) is an active area of experimental research. For discussions of the potential of existing and future experiments, see, e.g., Refs. [39, 40].

If dark matter particles are indeed WIMPs, and the relevant energy scale is of order 1 TeV, then the Hot Big Bang theory will be probed experimentally up to temperatures of (a few) $\cdot$ 10 GeV and down to age $10^{-9}$ s in relatively near future (compare to 1 MeV and 1 s accessible today). With microscopic physics to be known from collider experiments, the WIMP density will be reliably calculated and checked against data from observational cosmology. Thus, WIMP scenario (and also some others) offers a window to a very early stage of the evolution of the Universe.

The mechanism discussed here is by no means the only mechanism capable of producing cold dark matter, and WIMPs by no means are the only candidates for dark matter particles. Other dark matter candidates include very heavy relics produced towards the end of inflation, axions, gravitinos, Q-balls, massive gravitons, etc.

### 6.5 Baryon asymmetry of the Universe

In the present Universe, there are baryons and almost no anti-baryons. The number density of baryons today is characterized by the ratio $\eta$, see eq. (2.18). In the early Universe, the appropriate quantity is

$$\Delta_B = \frac{n_B - n_{\bar{B}}}{s}$$

where $n_{\bar{B}}$ is the number density of anti-baryons, and $s$ is the entropy density. If the baryon number is conserved, and the Universe expands adiabatically, $\Delta_B$ is constant, and its value is, up to a numerical factor, equal to $\eta$, so that

$$\Delta_B \approx 10^{-10}$$

Back at early times, at temperatures well above 100 MeV, cosmic plasma contained many quark-antiquark pairs, whose number density was of the order of the entropy density,

$$n_q + n_{\bar{q}} \sim s$$

while baryon number density was related to densities of quarks and antiquarks as follows (baryon number of a quark equals 1/3),

$$n_B = \frac{1}{3}(n_q - n_{\bar{q}})$$

Hence, in terms of quantities characterizing the very early epoch, the baryon asymmetry may be expressed as

$$\Delta_B \sim \frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}}$$

We see that there was one extra quark per about 10 billion quark-antiquark pairs! It is this tiny excess that is responsible for entire baryonic matter in the present Universe.

There is no logical contradiction to suppose that the tiny excess of quarks over antiquarks was built in as an initial condition. This is not at all satisfactory for a physicist, however. Furthermore, inflationary scenario does not provide such an initial condition for Hot Big Bang; rather, inflation theory predicts that the Universe was baryon-symmetric just after inflation. Hence, one would like to explain the baryon asymmetry dynamically.

The baryon asymmetry may be generated from initially symmetric state only if three necessary conditions, dubbed Sakharov's conditions, are satisfied. These are

(i) baryon number non-conservation;

(ii) C- and CP-violation;

(iii) deviation from thermal equilibrium.

All three conditions are easily understood. (i) If baryon number were conserved, and initial net baryon number in the Universe was zero, the Universe today would be symmetric rather than asymmetric. (ii) If C or CP were conserved, then the rate of reactions with particles would be the same as the rate of reactions with antiparticles. In other words, if the initial state of the Universe was C- and CP-symmetric, then the asymmetry between particles and antiparticles may develop only if C and CP is violated. (iii) Thermal equilibrium means that the system is stationary (no time dependence at all). Hence, if the initial baryon number is zero, it is zero forever, unless there are deviations from thermal equilibrium.

There are two well understood mechanisms of baryon number non-conservation. One of them emerges in Grand Unified Theories and is due to the exchange of super-massive particles. It is very similar, say, to the mechanism of charm non-conservation in weak interactions which occurs via the exchange of heavy $W$-bosons. The scale of these new, baryon number violating interactions is the Grand Unification scale, presumably of order $10^{16}$ GeV.

Another mechanism is non-perturbative and is related to the triangle anomaly in the baryonic current (a keyword here is "sphaleron"). It exists already in the Standard Model, and, possibly with slight modifications, operates in all its extensions. The two main features of this mechanism, as applied to the early Universe, is that it is effective over a wide range of temperatures, $100\ \text{GeV} < T < 10^{11}$ GeV, and that it conserves $(B-L)$.

Realistic mechanisms of baryon number non-conservation are thus not numerous, yet there are several ways the baryon asymmetry could have been generated. They differ by the characteristic temperature at which the asymmetry is produced.

(i) *Grand Unification mechanisms* operate at extremely high temperatures, $T \sim 10^{15} - 10^{16}$ GeV. The most commonly discussed source of the baryon asymmetry in this context are $B$- and CP-violating decays of ultra-heavy particles. At later times, the baryon number is violated by anomalous electroweak processes whose effect is basically to wash out $(B+L)$. They would therefore reprocess part of the baryon asymmetry, but if non-zero $(B-L)$ is generated at GUT temperatures, then this $(B-L)$ would survive until the present epoch (provided there are no strong lepton number violating interactions at intermediate temperatures, $100\ \text{GeV} < T < 10^{11}$ GeV, otherwise all fermion quantum numbers would be violated at those temperatures, and no asymmetry would survive). Part of this $(B-L)$ would be carried by baryons.

(ii) *Electroweak baryogenesis* is a scenario in which the baryon asymmetry is generated entirely due to the anomalous electroweak processes. Its generation would occur at temperature of order 100 GeV, at which these anomalous processes are switching off. Since the Universe expands slowly at the electroweak epoch (as compared to rates of microscopic interactions), considerable departure from thermal equilibrium is possible only due to the first order phase transition. Indeed, the latter transition, which proceeds through the nucleation, expansion and collisions of bubbles of the new phase, is quite a violent phenomenon. With LEP limits on the Higgs boson mass, no first order electroweak phase transition occurs in the Standard Model, so the electroweak baryogenesis requires that SM be extended. It is unlikely also that electroweak baryogenesis operates within the Minmal Supersymmetric Standard Model (MSSM), though there still exists a window in its parameter space where the phase transition is of strong enough first order. On the other hand, introducing extra scalars, one can construct extensions of the Standard Model and/or MSSM where electroweak baryogenesis is successfull. An extension of SM is useful for electroweak baryogenesis also from the point of view of CP-violation, as the CKM mechanism alone is insufficient to generate the realistic baryon asymmetry. Electroweak baryogenesis is a particularly fascinating possibility, as physics involved is being probed at Tevatron and will soon be explored at LHC.

(iii) Currently popular mechanism is *leptogenesis*. It may occur at some intermediate temperature (the estimates range from $10^7$ GeV to $10^{11}$ GeV), say, due to $L$- and CP-violating decays of heavy Majorana neutrinos [41]. Of course, the generation of lepton asymmetry requires lepton number violation, i.e., extension of the Standard Model, but such an extension is favored by neutrino oscillation data anyway. The lepton asymmetry would then be partially reprocessed into baryon asymmetry by anomalous electroweak processes. Interestingly, the range of Majorana neutrino masses compatible with this mechanism is indeed consistent with the range inferred from neutrino oscillations. Let us discuss leptogenesis scenario in little more detail.

Let us assume that ordinary neutrinos get their small masses via see-saw mechanism. This mechanism invokes heavy "sterile" (neutral with respect to the Standard Model gauge interactions) neutrinos $N_i$, where $i = 1,2,3$ is the generation number. They have large Majorana masses; in an appropriate basis the corresponding mass matrix is diagonal and real. They are assumed to have Yukawa interaction with conventional lepton doublets $L$ and the Standard Model higgs field $H$. Thus, besides the kinetic terms, the Lagrangian involving $N_i$ contains two terms

$$L = \sum_i M_i \bar{N}_i^c N_i + (\sum_{ij} h_{ij} \bar{L}_i N_j \tilde{H} + h.c.)$$

where the fermions $N$ are right, the Yukawa couplings $h_{ij}$ are in general complex, and $\tilde{H}_\alpha = \varepsilon_{\alpha\beta} H^*_\beta$ is a weak doublet whose vacuum expectation value is $(v,0)$. Due to this vacuume expecation value, there is mixing between ordinary neutrinos and $N$'s. Upon diagonalising the mass matrix, one finds that there are heavy states, predominantly $N$'s, with masses $M_i$, and light states with mass matrix

$$m_{ij} = \sum_k h^*_{ik} h^*_{jk} \frac{v^2}{M_k} \tag{6.5}$$

These masses are naturally small for large $M_i$, which is the advantage of the see-saw mechanism. Obviously, the original Lagrangian does not conserve any of the lepton numbers.

Now assume that at high temperatures, heavy neutrinos are in thermal equilibrium[19]. As the Universe cools down below $T \sim M_1$ (the smallest of $M_i$), the lightest of $N$'s (call it $N_1$) starts decaying[20]. There is also inverse decay process which tends to keep $N_1$ in thermal equilibrium. The lepton asymmetry may be produced only if $N_1$ is *not* in thermal equilibrium with respect to decay and inverse decay at $T \sim M_1$. This requires that its width is small enough[21]

$$\Gamma_1 < H(T \sim M_1) \tag{6.6}$$

[19]This may be either due to some new interactions in which $N$'s participate, or due to the Yukawa interactions themselves. In the latter case, thermal equilibrium is established with *large enough* Yukawa couplings only, i.e., usual neutrinos must be sufficiently *heavy*, $m_\nu > 3 \cdot 10^{-3}$ eV. This is consistent with neutrino oscillation data.

[20]Generically, the lightest of $N$'s is most relevant, since any asymmetry produced in decays of heavier $N$'s will be washed out by intractions involving the lightest one.

[21]There are cases when this relation is violated, and still required asymmetry is generated. The vaiolation should not be by many orders of magnitude anyway.

Now, since the Yukawa couplings contain CP-phases, there is an asymmetry in decays of $N_1$'s,

$$\delta \equiv \frac{\Gamma(N_1 \to LH) - \Gamma(N_1 \to \bar{L}\bar{H})}{\Gamma(N_1 \to LH) + \Gamma(N_1 \to \bar{L}\bar{H})} \neq 0$$

This asymmetry occurs due to the interference of the tree diagram and one-loop diagram in which all $N_i$ run in intermediate states. This "microscopic" asymmetry is

$$\delta = \frac{1}{8\pi} \frac{1}{(h^\dagger h)_{11}} \sum_{k=2,3} \mathrm{Im}\left[(h^\dagger h)^2_{1k}\right] F\left(\frac{M_1}{M_k}\right)$$

where $F(M_1/M_k) \sim M_1/M_k$ for $M_1 \ll M_k$, up to a factor of order 1. Provided the out-of-equilibrium condition (6.6) is satisfied, the generated lepton asymmetry is of order

$$\frac{n_L}{s} \sim 0.01\delta \tag{6.7}$$

where the factor 0.01 is due to large number of ultra-relativistic spieces at temperature $T \sim M_1$. Good part of this asymmetry gets reprocessed into the baryon asymmetry by electroweak processes, so the generated baryon asymmetry is roughly of the same order of magnitude as (6.7).

Let us first see what the out-of-equilibrium condition (6.6) means in terms of neutrino masses. Let $L_1$ be a linear combination of lepton fields to which $N_1$ is coupled (generally, this combination is neither flavor nor mass eigenstate),

$$L_1 \propto \sum_k h^*_{k1} L_k$$

After rotating into the lepton basis whose first basis vector is $L_1$, the lightest $N$ produces the only term in the neutrino mass matrix,

$$m_{11} \bar{\nu}^c_1 \nu_1$$

where

$$m_{11} = (h^*_{11})^2 \frac{v^2}{M_1}$$

The width of $N_1$ is

$$\Gamma_1 = \frac{1}{16\pi} |h_{11}|^2 M_1 = \frac{|m_{11}| M_1^2}{16\pi v^2}$$

while $H(T \sim M_1) = M_1^2/M^*_{Pl}$, so the relation (6.6) gives

$$m_{11} < \frac{16\pi v^2}{M^*_{Pl}}$$

Inserting numbers, one finds

$$m_{11} < 3 \cdot 10^{-3}\ \mathrm{eV}$$

Miraculously, this number is in the right ballpark of neutrino masses, suggested by neutrino oscillations!

Let us see that the masses of heavy neutrinos must be quite large. For a crude estimate, let us assume that all these masses are of the same order of magnitude; let us also assume that all Yukawa couplings are of the same order. Then, assuming that CP phases are large,

$$\delta \sim \frac{1}{8\pi} h^2$$

We need $\delta > 10^{-8}$, so

$$\frac{1}{8\pi}h^2 \sim \frac{m_\nu M}{8\pi v^2} > 10^{-8}$$

This gives for $m_\nu \sim 0.1$ eV the following estimate,

$$M > 10^8 \text{ GeV}$$

which is not unreasonable for the see-saw mechanism. The conclusion is that the mechanism described can indeed produce right amount of the baryon asymmetry. Unfortunately, the CP phases responsible for the asymmetry are generally *different* from CP phases in the mass matrix of light neutrinos. So, future observation of CP violation in neutrino oscillations, though will add weight to the leptogenesis scenario, will not be a proof of it.

The bottom line is that the observed baryon asymmetry may be explained by a number of mechanisms, all of which, however, exist in *extensions* of the Standard Model only. The problem is that, with a notable exception of the electrowaek baryogenesis, direct proof that any given mechanism is indeed responsible for the baryon asymmetry, does not seem possible. For reviews of baryogenesis see, e.g., Refs. [42, 43, 44, 45].

## 7. INFLATION

The Hot Big Bang theory, being very successful in many aspects, is not free of problems. These have to do with initial conditions for the cosmological evolution: the initial data required are very special, and in several respects very unnatural. This situation is improved dramatically if the Universe underwent inflationary epoch before the Hot Big Bang stage. In this section we first discuss the motivation for inflation, and then briefly study mechanisms of inflation and observational predictions of the inflationary theory. Major success of inflation, from the observational point of view, is that it provides a mechanism of the generation of primordial density perturbations in the early Universe, whose spectrum is almost flat. The approximate flatness of the spectrum has been confirmed by the measurements of the angular anisotropy of cosmic microwave background radiation, while many alternative mechanisms of the generation of density perturbations, like topological defects, are ruled out by the CMB data. Inflation also predicts a certain spectrum of primordial gravitational waves in our Universe, which in principle is observable through CMB. In these lectures we will illustrate the basic mechanism of the generation of density perturbations and gravitational waves at inflation. Of course, the corresponding theory is rather involved, so our discussion here will be fairly qualitative. For reviews on inflationary cosmology, see Refs. [46, 47, 48].

### 7.1 Problems of Hot Big Bang theory

Within Hot Big Bang theory, the Universe started its expansion from the singularity. Of course, the singularity is a property of *classical* General Relativity, and it may be replaced by something else in full quantum theory of gravity and matter. In other words, it is not legitimate to extrapolate the evolution, by making use of classical Einstein equations, to curvatures and energy densities of the order of the Planck values or higher. Still, the classical theory is applicable soon after the Planck epoch, and we may ask what were the properties of the Universe soon after that epoch. This is what we mean by initial conditions in the Hot Big Bang theory.



PoS(RTN2005)003

Off hand, one would think that these initial conditions are more or less random: different parts of the Universe would have different properties after the Planck epoch, both spatial curvature and energy density would be close to the Planck values, etc. It is straightforward to see, however, that with such random initial conditions, the Universe would *not* evolve into the state of high homogeneity and isotropy, as we see today. When quantifying this statement, one arrives at several "problems of Hot Big Bang theory".

**Horizon problem.**

As we know, relic photons were emitted/last scattered when the Universe was rather young, $t_{rec} \approx 3 \cdot 10^5$ yrs. In the Hot Big Bang theory, the horizon size at that time was about $l_{hor,rec} = 3t_{rec} \approx 10^6$ light yrs. In the Hot Big Bang theory, there was no cross talk, by the time of recombination, between regions separated by distance larger than $l_{hor,rec}$, i.e., these regions were not in causal contact with each other. Hence, there is no reason for such regions to have the same properties, e.g., the same temperature; CMB photons coming from different regions should a priori have quite different temperatures.

The present size of the region whose size at recombination was $l_{hor,rec}$ is

$$\begin{aligned} l_{hor,rec}^{today} &= l_{hor,rec} \cdot (1 + z_{rec}) \\ &\approx 10^6 \cdot 10^3 \text{ light yrs} \\ &\approx 300 \text{ Mpc} \end{aligned} \tag{7.1}$$

while the present horizon size is $l_{H,0} \approx 10^4$ Mpc. This means that the present angular size of the horizon at recombination is

$$\theta_{rec} = \frac{l_{hor,rec}^{today}}{l_{H,0}} \approx 0.03 \approx 2^o \tag{7.2}$$

We conclude that there is no reason for angular isotropy of CMB temperature at angular scales greater than $2^o$. Yet CMB *is* isotropic to better than $10^{-4}$ ! Why this is so? Why the initial conditions for Hot Big Bang are so homogeneous and isotropic even over causally disconnected regions of space? This is the horizon problem, which cannot be solved in the context of the Hot Big Bang theory.

**Flatness problem**

The Universe today is almost spatially flat. Quantitatively,

$$|\Omega_{curv}| \equiv \frac{1}{a^2} \frac{1}{\frac{8\pi}{3} G\rho_c} < 0.02$$

The curvature contribution into the Friedmann equation,

$$|\rho_{curv}| \equiv \frac{1}{a^2} \frac{1}{\frac{8\pi}{3} G}$$

scales as $1/a^2$, while matter and radiation contributions scale as $1/a^3$ and $1/a^4$, respectively. Thus, the curvature term in the Friedmann equation was even less important at earlier epochs, for example

$$\text{nucleosynthesis epoch}: \quad |\Omega_{curv}| < 10^{-16}$$

$$\text{electroweak epoch}: \quad |\Omega_{curv}| < 10^{-26}$$

Thus, the spatial curvature of the Universe was tiny at the beginning. Why the initial conditions were so flat? In other words, one can compare the radius of spatial curvature $a$ with the radius of space-time curvature, the latter being of the order of the inverse Hubble parameter. One obtains, e.g., at the electroweak epoch,

$$\text{electroweak epoch}: \quad a > 10^{13} H^{-1}$$

Why the initial conditions are such that the radius of the Universe is so large? This "flatness problem" again cannot be solved within Hot Big Bang theory.

**Entropy problem**

Let us estimate the entropy of the visible part of the Universe, i.e., entropy inside a sphere of size $l_{H,0}$. This entropy is of the order of the number of photons inside this sphere,

$$S \sim N_\gamma \sim n_\gamma l_{H,0}^3$$

which gives

$$S \sim 10^{88}$$

In the Hot Big Bang theory, the expansion of the Universe is almost adiabatic, so this huge entropy should be built in as an initial condition. Certainly, this initial condition is very special: off hand, one would rather expect that all dimensionless quantities are roughly of order 1 in the beginning of the Universe.

Besides these problems which basically mean that the Hot Big Bang theory does not explain why our Universe is so large, hot and homogeneous, there is another problem of a different kind. This is **the problem of primordial perturbations**. At early times (e.g., at recombination epoch), the Universe was not exactly homogeneous: there were density perturbations at the level $\delta\rho/\rho \sim 10^{-5}$. These density perturbations grew up, and finally gave rise to structures in our Universe (galaxies, galactic clusters, etc.). The problem is that in the Hot Big Bang theory, the density perturbations are to be built in as initial conditions, and there is no way to explain their origin.

Inflation is a dynamical mechanism that makes the Universe large, homogeneous, flat and hot. As a bonus, it provides a mechanism for the generation of primordial density perturbations (and also gravitational waves). These perturbations originate from *vacuum fluctuations* of quantum fields, which get enhanced during the inflationary epoch.

### 7.2 Basic picture

The idea of inflation is that before the Hot Big Bang (but after the Planck era), the Universe was in vacuum-like state and underwent the exponential expansion[22],

$$a(t) = \text{const} \cdot e^{\int H_{infl}\, dt}$$

where $H_{infl}$ is almost constant in time. Due to the exponential expansion, a small patch of the Universe expands to great size. Say, if the duration of inflation $t_{infl}$ exceeds 140 Hubble times,

$$t_{infl} > \frac{140}{H_{infl}}$$

[22]It is not absolutely necessary that the expansion is exponential. What is needed is that by the end of inflation, the size of the cosmological horizon is very large. As an example, power-law behaviour of the scale factor (2.33) with $\alpha > 1$ would also correspond to inflation.

then a patch of initial Planck size $l_{Pl} = 1/M_{Pl} \sim 10^{-33}$ cm expands to the size exceeding the present horizon size $l_{H,0} \sim 10^{28}$ cm. Obviously, the Universe flattens out, any initial inhomogeneities get diluted and, by the end of inflation, the Universe becomes spatially flat, homogeneous and isotropic at exponentially large spatial scales. This solves the horizon and flatness problems.

A natural and most popular way to ensure that the Universe expands exponentially is to assume that the matter at inflationary stage is in the vacuum-like state characterized by energy density $\rho_{infl}$ which is almost constant in time. At some point, however, this energy density should transform into conventional energy density of hot plasma. This transformation is called reheating, and after reheating the Hot Big Bang era begins. During reheating, huge entropy is released, and this solves the entropy problem.

This scenario automatically solves three problems mentioned above, which have to do with horizon, flatness and entropy. It is not at all obvious that inflation solves the fourth problem of primordial perturbations, but it does!

### 7.3 Simple model: one field inflation

Presently, most popular models of inflation invoke a new scalar field — inflaton, which drives inflation and then automatically provides exit from inflationary stage. Depending on the scalar potential, several versions of inflationary scenario have been designed, of which the simplest one is one field inflation, or "chaotic inflation".

Suppose that the action of the scalar field is

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right)$$

where $V(\phi)$ is a simple power-law potential at large $\phi$, say $m^2\phi^2$ or $\lambda\phi^4$. Assume further that "at the beginning" there is a sufficiently large patch in the Universe, where the field $\phi(\mathbf{x})$ is reasonably homogeneous. We should stress that these assumptions are rather mild: if "the beginning" is just after the Planck era, then "sufficiently large" means "somewhat larger than the Planck size", and "reasonably homogeneous" means that the gradient term in energy density is somewhat smaller than the potential term. Under these assumptions, one may consider both metric and scalar field as homogeneous and isotropic, which means that metric has the FRW form, and the scalar field does not depend on spatial coordinates. Then the the scalar field equation is

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -\frac{\partial V}{\partial \phi} \tag{7.3}$$

To complete the system of equations, we have to write the Friedmann equation. For homogeneous scalar field, the energy density is

$$\rho \equiv T_{00} = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

Hence, the Friedmann equation has the following form,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_{Pl}^2} \left( \frac{1}{2}\dot{\phi}^2 + V(\phi) \right) \tag{7.4}$$

The vacuum-like state occurs when the energy density is almost constant in time. This is possible when the friction in eq. (7.3) is so strong that the scalar field barely evolves. Its kinetic energy

$(1/2)\dot{\phi}^2$ is small compared to the potential energy $V(\phi)$, the latter stays almost constant in time. This is called slow roll regime. The conditions for slow roll are

$$\frac{\dot{a}}{a}\dot{\phi} \gg \ddot{\phi} \tag{7.5}$$

and

$$V(\phi) \gg \dot{\phi}^2 \tag{7.6}$$

If these conditions are satisfied, then the system of equations (7.3) and (7.4) simplifies; one has instead

$$3\frac{\dot{a}}{a}\dot{\phi} = -\frac{\partial V}{\partial \phi} \tag{7.7}$$

and

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_{Pl}^2}V(\phi) \tag{7.8}$$

It follows from eq. (7.8) that at large enough $V(\phi)$, the Hubble parameter $\dot{a}/a$ is large, and then it follows from eq. (7.7) that the field $\phi$ indeed rolls down slowly. The potential $V(\phi)$ indeed remains almost constant, and the Universe expands exponentially. Thus, once the slow roll regime is ensured, inflation occurs automatically.

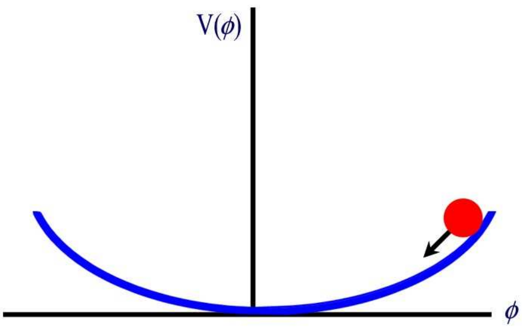


**Figure 12:** Evolution of the inflaton field.

Let us study whether the slow roll conditions may be satisfied. Making use of eq. (7.7), and then eq. (7.8), and dropping numerical factors of order 1, we write

$$\begin{aligned}\dot{\phi}^2 &\sim \left(\frac{\partial V}{\partial \phi}\right)^2 \cdot \frac{1}{(\dot{a}/a)^2} \\ &\sim \left(\frac{\partial V}{\partial \phi}\right)^2 \frac{M_{Pl}^2}{V}\end{aligned} \tag{7.9}$$

Therefore, the slow roll condition (7.6) takes the form

$$\frac{\partial V}{\partial \phi} \ll \frac{V}{M_{Pl}}$$

For potentials which have power-law behavior at large $\phi$ (i.e., $V \propto \phi^2$, $V \propto \phi^4$, etc.) one has

$$\frac{\partial V}{\partial \phi} \sim \frac{V}{\phi}$$

so the slow roll condition (7.6) is satisfied at

$$\phi \gg M_{Pl}$$

It is straightforward to see that the first slow roll condition (7.5) is also satisfied for power-law potentials at $\phi \gg M_{Pl}$. Inflation occurs whenever the value of the scalar field is larger than the Planck mass.

It is worth noting that when considering the field values of order and larger than the Planck mass one makes quite an extrapolation. One may suspect that there may be contributions to the scalar potential, which are generated by gravitational effects and have the form

$$\sum_N C_N \frac{\phi^{N+4}}{M_{Pl}^N} \tag{7.10}$$

with coefficients $C_N$ of the order of one. Such a behavior would destroy the slow roll; in fact, one can place strong bounds on $C_N$ from the analysis of density perturbations (see below): we will see that correct amplitude of density perturbations is obtained for very flat scalar potentials. This brings up an issue of the nature of inflaton field and a mechanism forbidding contributions like (7.10).

Furthermore, one may worry that even with sufficiently flat scalar potentials, the classical analysis of the evolution of the Universe is not applicable at $\phi \gg M_{Pl}$. This is not the case, since at so large values of $\phi$, energy density may still be well below the Planck value, $M_{Pl}^4$. For example, consider quartic potential, $V(\phi) = \lambda\phi^4$, where $\lambda$ is a dimensionless coupling constant. The energy density at the Planck value of the scalar field is smaller than the Planck energy density, provided $\lambda \ll 1$,

$$V(\phi \sim M_{Pl}) \sim \lambda M_{Pl}^4 \ll M_{Pl}^4 \ , \quad \text{for } \lambda \ll 1$$

We will see that correct amplitude of density perturbations is obtained when $\lambda$ is very small indeed, $\lambda \sim 10^{-10}$. Taking this value, we see that inflation occurs well below the Planck energies, and our classical analysis makes sense. Furthermore, we will point out that there is direct observational evidence that towards the end of inflation, its energy scale is well below the Planck scale,

$$H < 10^{-4} M_{Pl} \ , \quad \text{end of inflation} \tag{7.11}$$

This bound has to do with the generation of gravitational waves at inflation and their effect on CMB anisotropy.

One's best guess about the initial value of the scalar field $\phi_b$ is that the energy density in the beginning of inflation is of the order of the Planck energy density,

$$V(\phi_b) \sim M_{Pl}^4$$

For quartic potential this means

$$\phi_b \sim \lambda^{-1/4} M_{Pl} \gg M_{Pl} \tag{7.12}$$

Starting from this value, the field slowly rolls down the potential, until it reaches the value $\phi_e \sim M_{Pl}$. Inflation occurs during all this period of time. Once the field becomes of order $\phi_e \sim M_{Pl}$, the slow roll regime terminates, the field quickly rolls down to $\phi \approx 0$, and then oscillates around its minimum $\phi = 0$. Field oscillations generally lead to particle creation from vacuum, so the oscillations get damped, and the Universe begins to be filled with particles. This is a reheating process, which ends up when the coherent oscillations of the scalar field terminate, the classical scalar field settles down to the minimum of the scalar potential, and particles created by the oscillations get in thermal equilibrium. After the system thermalizes, it is described by the Hot Big Bang theory.

The reheating process is quite complex, and may occur in several stages. We will not discuss it here; on general grounds it is clear that the outcome of this process is thermal state anyway. The end of reheating is at the same time the beginning of Hot Big Bang. Most naive (and, in fact, unrealistic) estimate of the temperature of the Universe after reheating is obtained if one assumes that reheating takes of order one Hubble time. Under this assumption, all energy density of the inflaton field $V(\phi_e)$ transforms into heat, and the energy density does not get substantially diluted, because of the expansion of the Universe, during reheating. This picture implies that the Hubble parameters at the end of inflation and in the beginning of Hot Big Bang are of the same order,

$$H_{end\ of\ inflation} \sim \frac{T^2}{M_{Pl}^*}$$

where we made use of the standard formula for the Hubble parameter at the radiation-dominated stage. According to this estimate, the temperature of the Universe at the beginning of the Hot Big Bang may be quite high: a model-independent bound comes from eq. (7.11), which gives

$$T < 10^{-2}\sqrt{M_{Pl}M_{Pl}^*} \sim 10^{16}\ \text{GeV}$$

More realistic estimates give the reheat temperature which is several orders of magnitude lower, because reheating takes more than one Hubble time. In fact, one can design inflationary models with arbitrarily low reheat temperature.

Let us see that the inflationary stage naturally lasts long enough, so that the scale factor increases a lot during inflation. In this way we make sure that the three problems of the Hot Big Bang theory (horizon, flatness and entropy) are naturally solved. During inflation, the scale factor increases by

$$\begin{aligned} \frac{a_e}{a_b} &= \mathrm{e}^{N_{e-folds}} \\ N_{e-folds} &= \int_{t_b}^{t_e} H dt \end{aligned} \tag{7.13}$$

where subscripts $b$ and $e$ refer to the beginning and end of inflation. We obtain from eqs. (7.7) and (7.8) that the number of e-foldings may be written as follows (again omitting factors of order one),

$$\begin{aligned} N_{e-folds} &\sim \int_{\phi_e}^{\phi_b} d\phi\, \frac{H^2}{\partial V/\partial\phi} \\ &\sim \int_{\phi_e}^{\phi_b} d\phi\, \frac{V(\phi)}{M_{Pl}^2(\partial V/\partial\phi)} \end{aligned} \tag{7.14}$$

For power-law potentials, and $\phi_b \gg \phi_e$ one estimates

$$N_{e-folds} \sim \int_{\phi_e}^{\phi_b} \frac{\phi d\phi}{M_{Pl}^2} \sim \frac{\phi_b^2}{M_{Pl}^2} \quad (7.15)$$

We have seen that at the beginning of inflation, the scalar field is naturally very large, $\phi_b \gg M_{Pl}$. Therefore, the number of e-foldings is indeed naturally very large. As an example, for quartic potential we have an estimate (7.12), and taking $\lambda \sim 10^{-10}$ we estimate

$$N_{e-folds} \sim 10^5$$

The size of the Universe after inflation in this case is of order

$$a \sim \mathrm{e}^{N_{e-folds}} \sim 10^{100000} \quad (7.16)$$

(it does not matter in which units!); this is more than enough to solve the Hot Big Bang problems.

One remark is in order. When discussing the Hot Big Bang theory, we saw that the horizon size was of the order of the Hubble distance, $H^{-1}$. If there was inflation before the Hot Big Bang, this estimate is no longer valid: the actual horizon size is much larger, as the size of the causally connected region of space increased during inflation by a factor $\mathrm{e}^{N_{e-folds}}$. In particular, the present Hubble volume of the size of $10^4$ Mpc makes only a small fraction of the true horizon volume. Yet it is the distance of order $10^4$ Mpc from which the earliest electromagnetic radiation — relic photons — reaches us. In this sense the distance $10^4$ Mpc is still the size of the visible part of the Universe even in theories with inflationary stage.

One more point is that the estimates like (7.16) show that inflation naturally predicts that the spatial curvature of the Universe is extremely small today, i.e., $\rho = \rho_c$ with extremely high precision. At some point many cosmologists believed that the Universe was open, with $\rho \approx 0.3\rho_c$. This was a problem for simple inflationary models. The situation cleared up due to the data on CMB anisotropy, which tell that $\rho = \rho_c$ within 2 %, and show no evidence for spatial curvature.

Finally, we should mention that one field inflation studied in this subsection is not at all the only model of inflation (and it was not the first historically). Gross features of inflationary scenario are quite similar in various models. Their detailed predictions for density perturbations and gravitational waves differ, however, so there is hope that future detailed measurements of the properties of CMB and large scale structure will make it possible to discriminate between various inflationary models.

## 8. GENERATION OF PRIMORDIAL PERTURBATIONS AT INFLATION

### 8.1 Fluctuations of inflaton field

In inflationary scenario, density perturbations are generated from vacuum fluctuations of the inflaton field, for reviews see Refs. [12, 13, 49]. In this subsection, we illustrate the main idea and obtain the basic formulas for the scalar field perturbations. In effect, we will be dealing with quantum field theory of free scalar field in time-dependent background, and study pair creation.



PoS(RTN2005)003

It is amazing that such a simple mechanism is likely to be in the origin of all the structure of our Universe.

We begin with dividing the quantum inflaton field $\hat{\phi}$ into its classical part and quantum fluctuation operator,

$$\hat{\phi}(\mathbf{x},t) = \phi(t) + \varphi(\mathbf{x},t)$$

where the classical part $\phi(t)$ is precisely the object we discussed in the previous subsection, and $\varphi$ is the operator describing small perturbations. We are going to develop linear theory in $\varphi$. Furthermore, we are going to neglect the curvature of the scalar potential, as it is small at inflationary stage. Finally, we will approximate the Hubble parameter $H$ at inflation by a constant, i.e., neglect its dependence on time. Introducing weak dependence of $H$ on time is not difficult; we will comment in appropriate places on the effects coming from the time-dependence of $H$.

Under these assumptions, the action for perturbations coincides with the action (2.35), and the field equation is precisely (2.36). Thus, we can make use of the solutions (2.38), (2.39) and (2.40). Unlike in the case of the radiation dominated or matter dominated Universe, the Hubble parameter stays (almost) constant at inflation, while the physical momentum gets redshifted. Thus, a mode of given $k$ is *first* subhorizon and then superhorizon, which is just the opposite to the situation in matter/radiation regime[23]. This is what inflation is about: short scales are stretched beyond the Hubble radius. For a mode of given $k$, the expansion of the Universe is effectively adiabatic at early times, so this mode experiences vacuum fluctuations like in Minkowski space-time. These fluctuations get frozen in (up to a mode (2.40) that rapidly decays away) at the time when this mode exits the horizon. At the radiation or matter dominated epoch this mode re-enters the horizon and starts to oscillate again; at this moment its amplitude is much greater than the amplitude of vacuum fluctutaions at the same frequency.

In fact, our analysis may not apply to the period after inflation, as the classical part $\phi$ is zero at that time, and the curvature of the scalar potential may not be negligible. However, we will need the behavior of the fluctuations at the inflationary stage only.

In Minkowski space-time, the amplitude of vacuum fluctuations of massless field, whose typical momentum is $p$, is of order $\varphi \sim p$. This can be seen either on dimensional grounds (the scalar field $\varphi(x)$ has dimension of mass), or by requiring that the energy of a zero-pont fluctuation of frequency $p$ in volume $p^{-3}$, which is roughly $E \sim \dot{\varphi}^2 \, d^3x \sim p^2\varphi^2 p^{-3} \sim p^{-1}\varphi^2$, be of order $\omega/2 \sim p$. For subhorizon modes, the amplitudes of vacuum fluctuations are also of order $\varphi \sim p$; the larger the wavelength, the smaller the amplitude. The point is that when $p$ becomes of order $H$ at inflationary stage, the field amplitude freezes in at $\phi \sim p \sim H$, and no longer decreases even though the physical wavelength continues to increase. Relative to vacuum, the amplitude increases like $a(t)$, and is thus enhanced by a huge factor. We immediately obtain an estimate for the amplitudes of perturbations of superhorizon modes, created from vacuum,

$$\varphi \sim H \, , \quad p \ll H$$

and infer that these amplitudes are independent of wavelengths (flat, Harrison–Zeldovich spectrum).

[23]The term "horizon" here refers to de Sitter horizon, not to be confused with the cosmological horizon. In fact, what matters both here and in section 2.7 is the length scale $H^{-1}$.

To obtain quantitative estimates, one proceeds as follows. One introduces a field $\chi$ instead of $\varphi$, such that

$$\varphi(\mathbf{x},t) = a^{-1}(\eta)\chi(\mathbf{x},\eta)$$

In terms of Fourier harmonics $\chi_k(\eta)$, the field equation takes the form

$$-\partial_\eta^2\chi_k - k^2\chi_k + \frac{\partial_\eta^2 a}{a}\chi_k = 0 \qquad (8.1)$$

where $\eta$ is conformal time, see (2.3). Assuming[24] that the Hubble parameter at inflation is time-independent, $H = \text{const}$, one finds that at inflation

$$\eta = -\frac{1}{H}\mathrm{e}^{-Ht}$$

The conformal time $\eta$ runs from large negative values (beginning of inflation) to smaller negative values; the inflation ends at some small $|\eta|$. The scale factor at inflation is

$$a(\eta) = -\frac{1}{H\eta}$$

so the field equation (8.1) is explicitly

$$-\partial_\eta^2\chi_k - k^2\chi_k + \frac{2}{\eta^2}\chi_k = 0$$

Its solutions are

$$\chi_k^{(\pm)}(\eta) = \mathrm{e}^{\pm ik\eta}\left(1 \pm \frac{i}{k\eta}\right)$$

The behavior of these solutions is in accord with our previous discussion: they oscillate at $k/(aH) = |k\eta| \gg 1$ and are proportional to $a \propto 1/|\eta|$ at $k/(aH) \ll 1$.

To quantize the field $\chi$, one notices that $\chi_k^{(-)}$ and $\chi_k^{(+)}$ are negative- and positive frequency exponents at large negative times $\eta$. Hence, we immediately write for the quantized field (at early times, $\chi$ is a canonically normalized field whose action coincides with the action for massless scalar field in Minkowski space-time)

$$\chi(\mathbf{x},\eta) = \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2k}}\left(\mathrm{e}^{i\mathbf{kx}}\chi_k^{(-)}(\eta)\hat{b}_k + \mathrm{e}^{-i\mathbf{kx}}\chi_k^{(+)}(\eta)\hat{b}_k^+\right) \qquad (8.2)$$

where $\hat{b}_k$ and $\hat{b}_k^+$ are creation and annihilation operators obeying the standard commutational relations,

$$[\hat{b}_k, \hat{b}_{k'}^+] = \delta(\mathbf{k} - \mathbf{k}')$$

After crossing out the horizon, one has

$$\chi_k^{(\pm)} = \pm\frac{i}{k\eta}$$

[24]This is in fact not a strong assumption. It is important to solve the field equation near the time of the horizon crossing only, since at early times the solution is adiabatic, see (2.38), while at late times $\varphi$ stays constant. This explains, that in fact the relevant value of $H$ entering the formulas in this and next section is the value of the Hubble parameter at the time when the mode $k$ exits the horizon at inflation. This value slightly depends on $k$.

and, in terms of $\varphi$,

$$\varphi_k^{(\pm)} = \mp \frac{iH}{k}$$

Thus, at inflationary epoch, the contribution of superhorizon modes to the field operator is

$$\varphi(\mathbf{x},\eta) = \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2}} \frac{iH}{k^{3/2}} (\mathrm{e}^{i\mathbf{kx}}\hat{b}_k - \mathrm{e}^{-i\mathbf{kx}}\hat{b}_k^+)$$

Making use of this expression, one can calculate all correlation functions involving superhorizon modes. One finds that these are the Gaussian fluctuations with flat spectrum. For example,

$$\begin{aligned} \langle\varphi^2\rangle &= \frac{H^2}{2}\int \frac{d^3k}{(2\pi)^3k^3} \\ &= \frac{H^2}{(2\pi)^2}\int \frac{dk}{k} \end{aligned} \qquad (8.3)$$

This corresponds to fluctuations of amplitude $H/(2\pi)$ in a decimal interval of wavelengths.

Two remarks are in order. First, the Hubble parameter entering the fluctuation spectrum (8.3) is the Hubble parameter at inflation. In fact, this is the Hubble parameter towards the end of inflation, as relevant fluctuations of the scalar filed cross out the horizon towards the end of inflation. The spectrum is exactly flat only if the Hubble parameter is constant in time; if this is not so, the spectrum is slightly tilted (there is extra dependence on $k$). The reason is that fluctuations of different momenta $k$ cross out the horizon at different times, and the relevant value of the Hubble parameter is the value at the time when a fluctuation of a given wavenumber crosses out the horizon. We will not discuss the tilt in any detail in these lectures.

Second, when writing (8.2) we made an implicit assumption that the mode $k$ is described by the usual quantum field theory at very early times. This is certainly questionable, if "very early times" mean, say, the beginning of inflation, $t_b$. Indeed, at that time the physical wavelength of every interesting mode was extremely short,

$$\lambda(t_b) = \lambda_0 \frac{a(t_b)}{a_0}$$

(superscript 0 still means "present"). In view of estimates like (7.16), the wavelength $\lambda(t_b)$ is naturally many orders of magnitude smaller than, e.g., the Planck length. It is certainly of interest to understand how robust are the predictions of inflation with respect to possible new effects at so short distances. For a review of the work in this direction see, e.g., Ref. [13].

### 8.2 Density perturbations

How does the above discussion relate to density perturbations? An intuitive way to understand the creation of density perturbations by fluctuations of the inflaton field is as follows. Let us consider a region of the Hubble size towards the end of inflation. In this region, the actual value of the scalar field is

$$\phi_{act} = \phi + \varphi$$

where $\phi$ is the unperturbed value determined by the classical field equations; $\phi$ is homogeneous over the whole inflating Universe. Now, $\varphi$ is a linear combination of scalar field fluctuations of superhorizon size. Because of the second term, the scalar field is different in different regions of the Hubble size; if $\varphi > 0$, the field is larger than the average over the whole Universe, and vice versa. The Hubble regions evolve in time independently, so a region where $\varphi > 0$ exits from inflation later than on average. The delay time is

$$\delta t = \frac{\varphi}{\dot{\phi}}$$

This delay leads to higher energy density, after inflation, in the region we look at: in other regions there is more time for the density to get diluted due to the expansion. We obtain the density perturbation

$$\delta\rho = \dot{\rho}\delta t$$

where

$$\dot{\rho} \sim H\rho$$

Combining all factors, we get at scales which at the end of inflation exceed the inflationary Hubble size

$$\frac{\delta\rho}{\rho}(\mathbf{x}) = \frac{H}{\dot{\phi}}\varphi(\mathbf{x})$$

This means that primordial density perturbations are proportional to fluctuations of the inflaton field; they occur as random (Gaussian) field whose amplitude in a decimal interval of wavelengths is (see eq. (8.3))

$$\frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot{\phi}}$$

If $H$ were constant in time, this would be flat, Harrison–Zeldovich spectrum with no preferred length scale; as we discussed in the end of the previous subsection, the spectrum is in fact slightly tilted, the tilt being model-dependent.

To obtain the correct magnitude for primordial density perturbations,

$$\frac{\delta\rho}{\rho} \sim 10^{-5} \tag{8.4}$$

one has to tune the parameters of the inflaton potential. As an example, let us consider one field inflation with quartic potential, discussed in previous section. In the slow roll approximation, one has

$$H^2 = \frac{8\pi}{3M_{Pl}^2}V(\phi)$$

and

$$\dot{\phi} = -\frac{1}{3H}\frac{\partial V}{\partial \phi}$$

This gives

$$\frac{\delta\rho}{\rho} \sim \left[\frac{V^{3/2}}{M_{Pl}^3|\partial V/\partial\phi|}\right]_{\text{end of inflation}}$$

Inflation ends when $\phi$ is about $M_{Pl}$, so for the quartic potential $V = \lambda\phi^4$ one has

$$\frac{\delta\rho}{\rho} \sim \sqrt{\lambda}$$

The right magnitude of the density perturbations, eq. (8.4), is obtained for

$$\lambda \sim 10^{-10}$$

This tiny value of the self-coupling is not a peculiarity of the quartic potential: generally speaking, small density perturbations require flat inflaton potential.

These primordial density perturbations stay constant until they re-enter the horizon at radiation dominated or matter dominated stage. After that they make sound waves. The initial conditions for these sound waves are precisely the ones we found natural in section 2.7: there are no modes growing towards would-be singularity. As we outlined in that section, the modes start growing after re-entering the horizon, and finally produce structures in the Universe. This picture, together with the flatness of the spectrum of density perturbations, immediately implies that small structures (galaxies) form earlier than larger structures (clusters): shorter wavelengths re-enter the horizon earlier, and hence smaller structures start to develop earlier. This general prediction is in accord with observational data on the structure in the Universe.

Inflationary prediction for nearly flat spectrum of density perturbations is in agreement with both measurements of the CMB anisotropy and observations of structures in the Universe, see fig. 13 for illustration. In fact, the existing data start to rule out some models of inflation which predict considerable tilt.

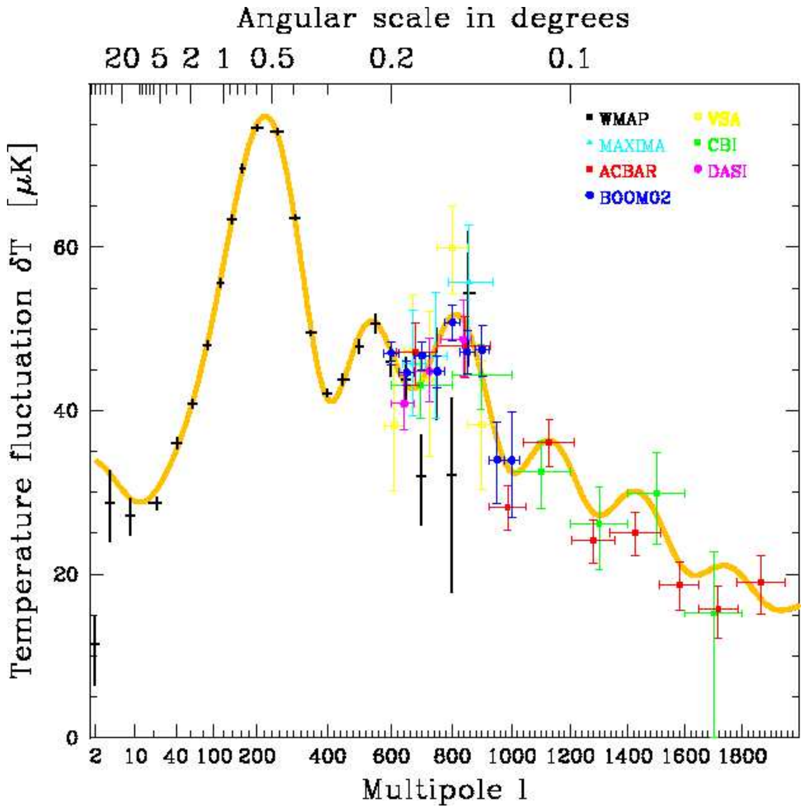


**Figure 13:** Fit to data on CMB anisotropy, assuming flat primordial spectrum and no contribution from gravitational waves.

### 8.3 Gravity waves: the scale of inflation

Inflation creates not only scalar field perturbations, but also gravitational waves. The mechanism is precisely the same as that outlined in section 8.1, and, in fact, the equation for gravitational waves (tensor perturbations) is literally the same as the equation for scalar perturbations. The action for metric perturbations, however, contains an extra factor $1/G = M_{Pl}^2$, so the canonically normalized field differs from metric perturbation by the factor $M_{Pl}$. With this qualification, we make use of eq. (8.3) to obtain the amplitude of gravitational waves in a decimal interval of wavelengths,

$$h = \frac{H}{2\pi M_{Pl}}$$

where $H$ is the Hubble parameter towards the end of inflation. The primordial spectrum of the gravitational waves is flat, like the spectrum of the scalar field fluctuations, see eq. (8.3).

Primordial gravitational waves make contribution to the CMB anisotropy. This contribution is potentially important at large angular scales, $\Delta\theta >$ (a few)$^o$. At these scales,

$$\left[\frac{\delta T}{T}\right]_{grav.\ waves} \sim h$$

The very fact that the CMB anisotropy $\delta T/T$ does not exceed $10^{-4}$ at large angular scales implies the bound on the Hubble parameter near the end of inflation,

$$H < 10^{-4} M_{Pl}\ , \quad \text{end of inflation}$$

We already discussed the significance of this bound in section 7.

In most inflationary models, the contribution of gravitational waves into the CMB anisotropy is smaller (sometimes much smaller) than the contribution of density perturbations, even at large angular scales. This expectation is confirmed by the data, which are very well fit under the assumption that the contribution of gravitational waves is negligible, see fig. 13. Yet there is a chance to detect the contribution of gravitational waves, and discriminate it from the contribution of density perturbations, in future measurements of the CMB polarization. In this way one would be able to find the scale of inflation, and possibly even reconstruct part of the inflaton potential [50].

## 9. CONCLUSIONS

At first sight, our Universe appears infinitely complex. Yet, with not so many parameters, a coherent picture of the present and past Universe emerges, which has already passed precision tests of CMB anisotropy, Big Bang Nucleosynthesis, structure formation, etc. Even more precise measurements are due to come, which makes the whole field lively and fascinating.

Even the gross features of cosmology are "orthogonal" to the Standard Model of particle physics:

– Most of energy in the present Universe is in unknown forms. Furthermore, cosmology requires the existence of *both* new stable particles (clumped non-baryonic dark matter) *and* dark energy. The latter is the most fundamental and mysterious of all aspects of cosmology, as we know it today.



PoS(RTN2005)003

– Inflation needs inflaton, a new scalar field with very flat potential. No such field exists in the Standard Model, neither it emerges naturally in the simplest extensions of the Standard Model.

– Baryogenesis needs new sources of CP-violation and mechanisms of baryon and/or lepton number violation.

Cosmology certainly has its own intrinsic problems, some of which have been mentioned in these lectures. We all know that the Standard Model has its intrinsic problems too. Experiments and theory in particle physics and cosmology still have a lot to tell about micro- and macro-world, as well as about the interconnections between them.

## References

[1] A. D. Linde, "Particle Physics And Inflationary Cosmology," *Chur, Switzerland: Harwood (1990) 362 p. (Contemporary concepts in physics, 5)*.

[2] E. W. Kolb and M. S. Turner, "The Early Universe," *Redwood City, USA: Addison-Wesley (1990) 547 p. (Frontiers in physics, 69)*.

[3] G. G. Raffelt, "Stars As Laboratories For Fundamental Physics: The Astrophysics Of Neutrinos, Axions, And Other Weakly Interacting Particles," *Chicago, USA: Univ. Pr. (1996) 664 p*.

[4] J. A. Peacock, "Cosmological Physics," *Cambridge, UK: Univ. Pr. (1999) 682 p*.

[5] M. S. Turner and J. A. Tyson, Rev. Mod. Phys. **71** (1999) S145 [arXiv:astro-ph/9901113].

[6] M. Shaposhnikov, "Cosmology and astrophysics," *Proc. 2000 School of High-Energy Physics, Caramulo, Portugal, 20 Aug - 2 Sep 2000*.

[7] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75** (2003) 559 [arXiv:astro-ph/0207347].

[8] W. L. Freedman and M. S. Turner, Rev. Mod. Phys. **75** (2003) 1433 [arXiv:astro-ph/0308418].

[9] L. M. Krauss, "The standard model, dark matter, and dark energy: From the sublime to the ridiculous," arXiv:astro-ph/0406673.

[10] I. I. Tkachev, "Astroparticle physics," arXiv:hep-ph/0405168.

[11] A. de Oliveira-Costa, G. F. Smoot and A. A. Starobinsky, "Constraining topology with the CMB," arXiv:astro-ph/9705125;
W. N. Colley and J. R. I. Gott, Mon. Not. Roy. Astron. Soc. **344** (2003) 686 [arXiv:astro-ph/0303020].
N. G. Phillips and A. Kogut, "Constraints On The Topology Of The Universe From The WMAP First-Year Sky Maps," arXiv:astro-ph/0404400.

[12] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rept. **215** (1992) 203.

[13] R. H. Brandenberger, "Theory of cosmological perturbations and applications to superstring cosmology," arXiv:hep-th/0501033.

[14] D. P. Schneider *et al.* [SDSS Collaboration], Astron. J. **123**, 567 (2002), `http:www.sdss.org/dr1/algorithms/edrpaper.html#fig-ZhistQso`

[15] S. Shectman et. al., Astrophys. J. **470** (1996) 172.

[16] A. V. Filippenko and A. G. Riess, Phys. Rept. **307** (1998) 31 [arXiv:astro-ph/9807008].

[17] A. V. Filippenko and A. G. Riess, "Evidence from Type Ia Supernovae for an Accelerating Universe," arXiv:astro-ph/0008057.

[18] A. G. Riess *et al.* [Supernova Search Team Collaboration], Astrophys. J. **607** (2004) 665 [arXiv:astro-ph/0402512].

[19] E. Gawiser and J. Silk, Phys. Rept. **333**, 245 (2000) [arXiv:astro-ph/0002044].

[20] D. N. Spergel *et al.*, Astrophys. J. Suppl. **148**, 175 (2003).

[21] T. J. Pearson *et al.*, Astrophys. J. **591**, 556 (2003).

[22] C. l. Kuo *et al.* [ACBAR collaboration], Astrophys. J. **600** (2004) 32.

[23] L. Y. Chiang, P. D. Naselsky, O. V. Verkhodanov and M. J. Way, Astrophys. J. **590** (2003) L65 [arXiv:astro-ph/0303643];
C. G. Park, Mon. Not. Roy. Astron. Soc. **349** (2004) 313 [arXiv:astro-ph/0307469];
P. Vielva, E. Martinez-Gonzalez, R. B. Barreiro, J. L. Sanz and L. Cayon, Astrophys. J. **609** (2004) 22 [arXiv:astro-ph/0310273];
P. Mukherjee and Y. Wang, "Wavelets and WMAP non-Gaussianity," arXiv:astro-ph/0402602;
J. D. McEwen, M. P. Hobson, A. N. Lasenby and D. J. Mortlock, "A 6 sigma detection of non-Gaussianity in the WMAP 1-year data using directional spherical wavelets," arXiv:astro-ph/0406604.

[24] S. Eidelman *et al.* [Particle Data Group Collaboration], Phys. Lett. B **592** (2004) 1.

[25] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Rev. D **59** (1999) 086004 [arXiv:hep-ph/9807344].

[26] V. A. Rubakov, Uspekhi Fiz. Nauk **171** (2001) 913 [arXiv:hep-ph/0104152.]

[27] U. Seljak *et al.*, "Cosmological parameter analysis including SDSS Ly-alpha forest and galaxy arXiv:astro-ph/0407372.

[28] V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D **9** (2000) 373 [arXiv:astro-ph/9904398].

[29] K. G. Begeman, A. H. Broeils and R. H. Sanders, Mon. Not. Roy. Astron. Soc. **249** (1991) 523.

[30] S. Weinberg, Rev. Mod. Phys. **61** (1989) 1.

[31] S. Weinberg, "The cosmological constant problems," arXiv:astro-ph/0005265.

[32] L. Susskind, "The anthropic landscape of string theory," arXiv:hep-th/0302219.

[33] C. Wetterich, Nucl. Phys. B **302** (1988) 645.

[34] P. J. E. Peebles and B. Ratra, Astrophys. J. **325** (1988) L17.

[35] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37** (1988) 3406.

[36] I. Zlatev, L. M. Wang and P. J. Steinhardt, Phys. Rev. Lett. **82** (1999) 896 [arXiv:astro-ph/9807002];
P. J. Steinhardt, L. M. Wang and I. Zlatev, Phys. Rev. D **59** (1999) 123504 [arXiv:astro-ph/9812313].

[37] A. Nicolis and R. Rattazzi, JHEP **0406** (2004) 059 [arXiv:hep-th/0404159].

[38] V. A. Kuzmin, Phys. Part. Nucl. **29** (1998) 257 [Fiz. Elem. Chast. Atom. Yadra **29** (1998) 637] [arXiv:hep-ph/9701269].

[39] A. Bottino and N. Fornengo, "Dark matter and its particle candidates," arXiv:hep-ph/9904469.

[40] J. R. Ellis, J. L. Feng, A. Ferstl, K. T. Matchev and K. A. Olive, "Prospects for detecting supersymmetric dark matter at post-LEP benchmark points," arXiv:astro-ph/0110225.

[41] M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.

[42] V. A. Rubakov and M. E. Shaposhnikov, Usp. Fiz. Nauk **166** (1996) 493 [Phys. Usp. **39** (1996) 461] [arXiv:hep-ph/9603208].

[43] A. D. Dolgov, “Baryogenesis, 30 years after,” arXiv:hep-ph/9707419.

[44] A. Riotto and M. Trodden, Ann. Rev. Nucl. Part. Sci. **49** (1999) 35 [arXiv:hep-ph/9901362].

[45] W. Buchmuller and M. Plumacher, “Neutrino masses and the baryon asymmetry,” arXiv:hep-ph/0007176.

[46] A. D. Linde, Phys. Rept. **333** (2000) 575.

[47] A. Albrecht, “Cosmic inflation,” arXiv:astro-ph/0007247.

[48] G. Lazarides, “Inflationary cosmology,” arXiv:hep-ph/0111328.

[49] R. H. Brandenberger, “Particle physics aspects of modern cosmology,” arXiv:hep-ph/9701276.

[50] J. E. Lidsey, A. R. Liddle, E. W. Kolb, E. J. Copeland, T. Barreiro and M. Abney, Rev. Mod. Phys. **69** (1997) 373 [arXiv:astro-ph/9508078].