PROCEEDINGS OF SCIENCE

Nuclear Physics and Lattice QCD

Martin J. Savage[*†]

University of Washington

E-mail: savage@phys.washington.edu

Lattice QCD is progressing toward being able to impact our understanding of nuclei and nuclear processes. I discuss areas of nuclear physics that are becoming possible to explore with lattice QCD, the techniques that are currently available and the status of numerical explorations.

XXIIIrd International Symposium on Lattice Field Theory
25-30 July 2005
Trinity College, Dublin, Ireland

[*]Speaker.

[†]I would like to thank Daniel Arndt, Silas Beane, Paulo Bedaque, William Detmold, Kostas Orginos, Elisabetta Pallante and Assumpta Parreño who are collaborators in this work.

© Copyright owned by the author(s) under the terms of the Creative Commons Attribution-NonCommercial-ShareAlike Licence.

PoS(LAT2005)020

1. Introduction

Nuclear physics is a well-established and mature field of study. A large number of experiments have been performed during the past decades that have led to a great understanding of nuclei and nuclear processes. We are currently moving into an exciting time for the field, one in which, for the first time, properties of nuclei will be rigorously determined directly from QCD. This will first occur through numerical calculations of quantities using lattice QCD, matching onto effective field theories (EFTs) that have been developed during the last decade or so, and using the EFT's to compute the properties of the light nuclei. The first lattice QCD calculation of the deuteron will be a milestone for nuclear physics. It is this scenario that has motivated a number of us to leave the relative comfort of our "analytic" offices, in which we developed the EFT's and fretted over the ever increasing number of counterterms, and to attempt calculations of multi-nucleon systems with lattice QCD.

2. Motivations

There are a number of compelling reasons to use lattice QCD to perform nuclear physics calculations. While it is important to explore and understand the building-blocks of nuclei, the proton and neutron, a greater challenge lies in understanding the structure and interactions of nuclei.

2.1 The Emergence of Nuclear Physics from QCD

One of the deep issues in our field, one for which we currently have little or no understanding, is how nuclear physics emerges from QCD. The Lagrangian describing QCD is simple: there is a gauge-covariant kinetic term for the quarks, a mass-term for the quarks and the Yang-Mills term describing the glue. There are a small number of fundamental constants that appear in this Lagrangian: $\Lambda_{\rm QCD}$ and the six quark masses. This Lagrangian and these seven constants (along with the electroweak interactions) give rise to all of nuclear physics. We wish to establish a rigorous pathway from QCD to nuclei. As mentioned previously, this will be accomplished by performing lattice QCD calculations of some quantities, determining counterterms in the appropriate low-energy EFT from these calculations, and then using the EFT to compute quantities of interest in relatively simple systems. This EFT will be subsequently matched onto the many-body machinery that has been developed and used by nuclear physicists, which will then be used to compute the properties of more complex systems [1].

As a simple example, consider the classic nuclear process $np \to d\gamma$. This is a process in which a low-energy neutron is captured by a proton to form a deuteron, emitting M1 radiation in the process. As the energy increases from threshold the E1 amplitude rapidly increases and soon dominates the M1 amplitude. A cartoon of the process at the quark-diagram level is shown in fig. 1. The Maiani-Testa theorem [5] prevents direct "computation of this process" on the lattice. Instead, one must compute other quantities on the lattice and use these quantities to obtain the matrix elements of interest. This will be accomplished by matching a lattice calculation to an EFT calculation. For the process at hand, one can either use the pionless theory, in which only

[1] This may involve a lattice calculation of the effective field theory itself, e.g. Ref. [1, 2, 3, 4], without reference to QCD.

PoS(LAT2005)020

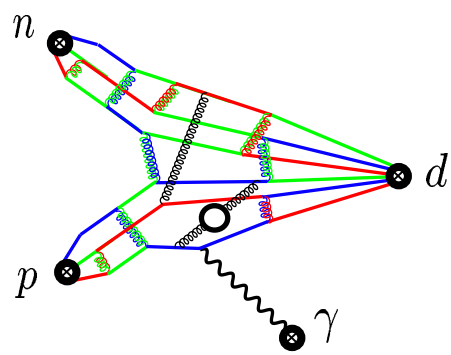

Figure 1: A cartoon of the QCD calculation of the simplest nuclear radiative capture process $np \to d\gamma$.

nucleons and photons are dynamical degrees of freedom, or the pionful theory, in which nucleons, pions and photons are dynamical, see Ref. [6]. The pionless theory is very useful for processes at momentum much below the pion mass, and this theory has been explored out to several orders in the power-counting. The pionful theory is somewhat less developed, but if one is interested in the quark-mass dependence, as will be the case for the initial calculations performed at quark-masses greater than those of nature, the pionful theory will be used. In either theory, there are contributions that are unrelated to the NN scattering amplitude, described by gauge-invariant operators involving nucleons and the electromagnetic field strength tensor. The coefficients of the operator(s) will be determined by matching to a lattice calculation. For this process ($np \to d\gamma$) there are experimental constraints that provide a very precise determination of these counterterms, and a lattice calculation will provide (other than the quark mass dependence) nothing more than a check of the method.

Ultimately, moving toward more complex systems one will need to match these EFTs, the zero, one and higher nucleon EFTs to the many-body techniques that have recently been developed. The nuclear shell-model continues to move toward a rigorous underpinning based on the renormalization group. In principle, one starts with a very large model space, as defined by the number of levels of the harmonic oscillator (HO) (for instance) and matches chiral perturbation theory and NN EFT's to observables in this model space. One then systematically reduces the model space, by integrating out the high-levels of the HO. This is exactly analogous to the renormalization group methods we are used to dealing with in particle physics, except this is now constructed for bound states. As states are integrated out, the Hamiltonian and operators are renormalized. Eventually one is left with a small, low-energy space –the shell-model space–and a large excluded space. The operators and Hamiltonian evolve in such a way that the energy-levels and matrix elements in the shell-model space reproduce those computed in the full-space, see Ref. [7].

2.2 Calculations of Processes where Experiments are not Possible

Lattice QCD will have significant impact on nuclear physics by computing quantities for which there is little or no data. One such example of this is to perform calculations that impact our understanding of dense nuclear matter. I do not mean asymptotically dense, but at the densities that arise in the collapse of stars, that are a few times nuclear matter density. A second example is the calculation of matrix elements of operators that probe physics beyond the standard model, such as $\beta\beta$-decay.

PoS(LAT2005)020

2.2.1 Supernova, Neutron Stars and Black Holes

The nature of the remnant of supernova 1987A is still undetermined. It is unknown if the remnant is a neutron star or a black hole (and it is unknown if it underwent spherical collapse or aspherical collapse). At the heart of the matter is nuclear physics, and in particular the compressibility of nuclear matter at densities that are a few times that of ordinary nuclear matter. To resolve this issue, one needs to know the composition of nuclear matter in this regime, and this is where lattice QCD can contribute.

As one compresses nuclear matter composed of neutrons, protons and electrons, the Fermi-energy of the electronic component increases rapidly compared to that of the nucleons. At some point it becomes energetically favorable for the system to increase the density of neutrons via the weak interaction, rather than to further increase the density of electrons and protons, despite the proton-neutron mass-splitting. Hence, the name neutron star. Thus it is the neutrons and their interactions that primarily dictate the compressibility of nuclear matter at densities higher than nuclear matter densities. This discussion ignores the possibility that other hadronic components can become energetically favored. In 1986 Kaplan and Nelson [8, 9] pointed out that the K^-n interaction should significantly reduce the mass of the K^- in neutron matter, and as stressed by Bethe and Brown [10], if the K^- mass falls below the electron Fermi energy, the formation of a condensate of K^-'s, $\langle K^- \rangle \neq 0$ will occur. Nuclear matter would then be composed of neutrons, protons and a kaon condensate which has a softer equation of state than pure neutron matter. The nK^- scattering amplitude is poorly known, and thus there is considerable uncertainty in the density dependence of the K^- mass. This ambiguity can be resolved by a careful lattice QCD study.

A competing process arises from the density dependence of the Σ^- mass. If the mass of the Σ^- drops below the electron chemical potential, then nuclear matter will be composed of neutrons, protons and Σ^-'s. For obvious reasons, this system will have a softer equation of state than pure neutron matter. The $n\Sigma^-$ scattering amplitude is also poorly known, only accessible through the properties of hypernuclei, and so one can presently only speculate as to the existence of such matter.

Lattice QCD can have significant impact on determining the equation of state of hadronic matter at high densities. The two processes discussed above, kaon-condensation and hyperonic matter can be refined by a calculation of the neutron-kaon and neutron-hyperon scattering amplitudes.

2.2.2 $\beta\beta$-Decay

The last decade has seen remarkable progress in our understanding of the neutrino sector. The measurements by SuperK and SNO demonstrating that neutrinos have non-zero mass, and that the mass eigenstates are not the flavor eigenstates, confirmed what had been speculated for many years based upon the results of the Davies chlorine experiment. It remains to be determined if the neutrinos are Dirac or Majorana. In the later case, lepton number is not conserved, as the neutrino is its own anti-particle. Observation of a lepton number violating process, such as neutrinoless $\beta\beta$-decay, would be yet another sensational discovery in the neutrino sector.

Nature has provided a limited number of nuclei that undergo $\beta\beta$-decay, and the constraints on building detectors to observe $\beta\beta$-decay allow for only a handful of elements to be considered. Perhaps the most widely known is Germanium, where one looks for a peak in the e^-e^- invariant mass spectrum resulting from $^{76}Ge \rightarrow ^{76}Se\, e^-e^-$, sitting on top of the background from the lepton

PoS(LAT2005)020

number conserving process $^{76}Ge \to ^{76}Se\ e^-\overline{\nu}_e e^-\overline{\nu}_e$ as a signal of lepton-number violation. As the simplest baryonic process that can contribute to this process is of the form $pp \to nne^-e^-$, and higher body operators are expected to be suppressed, it is clear that a rigorous calculation of the rate of such decay is not simple. There are complications at the many-body level, where the calculation of the nuclear rate from the two-nucleon operator (local or non-local) is notoriously difficult. Further, for a massive neutrino, or where the process is dominated at short-distances, matching from QCD onto the nuclear EFT suffers from issues similar to those that arise in the calculation of nonleptonic weak matrix elements [11, 12]. Lattice QCD is the only way to rigorously explore these matrix elements.

2.3 The Dependence of Nuclei and Nuclear Processes on the fundamental constants of Nature

Before we can be content with our understanding of nuclei and nuclear processes we must know and understand how they depend upon the fundamental constants of nature. The recent suggestion that the electromagnetic coupling is time dependent, based upon the splitting between atomic absorption-lines in gas clouds at different red-shifts (along the line of sight to quasars) [13], renewed interest in such a possibility. I look forward to an independent verification of this exciting result.

If we consider a nuclear process that is of central importance to the development of life, e.g. $3\alpha \to ^{12}C$, we should be able to fully understand how the rate for this process, and the rate for the production of carbon in stars depends upon the mass of the light-quarks, the scale of the strong interaction and the value of the electromagnetic coupling constant. At this point in time we have very little control over any aspect of this calculation. One of the reasons for this is that this process is highly fine-tuned, but also we have only a limited knowledge about how nuclear physics depends upon the quark masses (even in the absence of fine-tunings). Clearly it is not going to be easy to map out the spectrum of ^{12}C as a function of the quark masses, however, this is a worthy goal for nuclear physics.

2.4 Fine-Tunings in Nuclear Physics

During the last few years the role of fine-tunings in physics has become a central focus of particle physics. The question of why the cosmological constant, $\Lambda \sim 10^{-47}$ GeV^4, is so small and yet nonzero has sparked great interest. However, fine-tunings also play a central role in the strong interactions.

The nucleon-nucleon potential can be dissected into roughly three length scales. The long-distance part is well described by one pion exchange (OPE). Performing a phase shift analysis of NN scattering data while treating the mass of the pion as a free parameter, along with its couplings, recovers the physical pion mass and its axial coupling. What is more impressive is that a recent analysis of NN scattering [14] has shown that the two-pion exchange contribution computed in chiral perturbation theory provides a better fit than the traditional "σ-meson" exchange. What's really impressive is that the values of the local counterterms that enter at this order are consistent with those extracted from the single-nucleon sector. Therefore both the long-distance and intermediate-distance component of the NN-potential is well described by chiral perturbation theory. While

PoS(LAT2005)020

many "old-school" NN potentials have a one meson exchange short-distance repulsion, the modern potentials have a short-distance component that is optimized in shape and strength, consistent with EFT power-counting, to best fit the data.

The truly fascinating aspect of this, is that while the NN potential has three quite distinct distance scales, the s-wave wavefunction resulting from solving the Schrödinger equation for the two-nucleon system is approximately horizontal at the edge of the potential, i.e. the scattering lengths are unnaturally large, ~ -24 fm in the $^1S_0$-channel and $\sim +5.4$ fm in the $^3S_1$-channel (the channel containing the deuteron, which is bound by ~ 2.2 MeV). There is clearly a fine-tuning among the regions of the potential in order for this to be true. Clearly, this is all controlled by the parameters of QCD, but in nature these parameters have conspired to place us very near an infrared fixed-point in the hadronic sector (if the scattering lengths were infinite the system would exhibit scale-invariance and would be at an infrared, unstable fixed-point) [2]. Proximity to this fixed point means that the EFT used to describe such systems does not follow the usual power-counting developed for systems that have natural scattering lengths, where it is simply a matter of counting engineering dimensions. The four-nucleon operators contribute at the same order in the counting as the chiral limit of OPE, while deviations from the chiral limit are suppressed [16]. In order to have states near threshold, the coefficients of the four-nucleon operators are fine-tuned to achieve the delicate cancellation between kinetic and potential energy that exists.

The $3\alpha \rightarrow {}^{12}C$ process that was discussed previously, is also finely-tuned. In order for this process to proceed with any appreciable rate there are two important fine-tunings that have occurred. First, the ground state of ^8Be is barely unbound, and in the stellar interior one has $2\alpha \rightarrow {}^8Be$, and the ^8Be state lives for an unnaturally long period of time, enough time for a third α to interact to form ^{12}C through radiative capture. However, the rate would still be insufficient except for the fact that there is a level in ^{12}C at precisely the right energy to significantly enhance the capture rate. It is quite easy for me to believe that we can only understand this set of fine-tunings via the anthropic principle, as without carbon we would not be having this discussion. However, I have been unable to reconcile these fine-tunings with those in the NN-sector, as I have no reason to believe that the form of the NN interaction required to greatly enhance the triple-alpha process would also put us close to a fixed-point in the NN-sector. There has been no work to try to reconcile these features, and it is something that should be understood.

3. The Quark-Mass Dependence of Nucleon-Nucleon Scattering

The development of EFTs for nuclear physics during the last 15 years since Weinberg's pioneering papers on the subject, have allowed us to rigorously explore the quark-mass dependence of simple nuclear systems. We have learned that simple analytic dependences on the quark masses are incorrect and the actual quark mass dependence is quite complex. This is compounded by the fine-tunings discussed in the previous section.

During the last couple of years, the quark-mass dependence of the two-nucleon sector has been explored [17, 18, 19]. The scattering lengths as a function of the pion mass are shown in fig. 2. At the first non-trivial order, in addition to the quark mass dependence from OPE, there are also

[2] The fact that both channels, and not just one, are close to the fixed point is understood in the large-N_c limit of QCD [15] leaving us to understand why the system as a whole is near the fixed-point

PoS(LAT2005)020

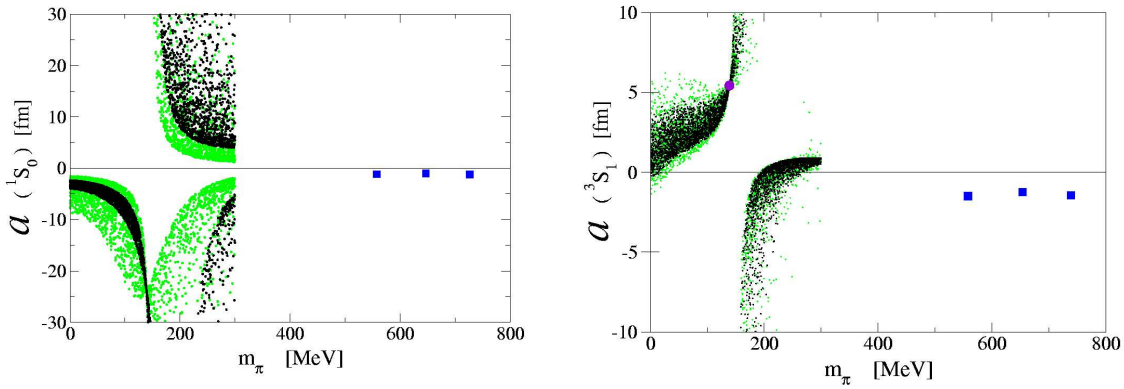

Figure 2: The NN scattering lengths in the s-wave channels as a function of the pion mass at NLO in the NN EFT. The two regions defi ned by the points (green and black) correspond to two different choices of the (sets of) coeffi cients of the m_q-dependent four-nucleon operators [18]. The three (blue) solid squares are the scattering lengths resulting from a quenched lattice QCD calculation [20].

contributions from four-nucleon operators that have a single insertion of the quark mass matrix with coefficients, the D_2's, that are currently unknown. To determine the reasonable ranges of scattering length at a given quark mass dimensional analysis has been used to constrain the coefficients of these operators. Fig 2 suggests that it is likely that the di-neutron remains unbound as one moves toward the chiral limit, while it could be bound or unbound as the mass is increased. In the deuteron channel, the deuteron binding energy moves toward its natural value for decreasing quark mass, but again, it could be bound or unbound at higher masses. In fig. 2 I have shown the only lattice QCD calculation of this system [20]. This pioneering lattice calculation is quenched and at large pion masses, outside the range of the EFT's that currently exist, and therefore, unfortunately, does not shed light on the question at hand.

The program to undertake is to perform a lattice QCD calculation of the scattering lengths (or of the deuteron) at the larger masses that are currently accessible, but inside the range of validity of the EFT, match to the D_2's, and use the EFT to make the chiral extrapolation to map out the entire region of interest.

4. Two Hadrons on the Lattice

Motivated by the physics described in the previous sections two collaborations have emerged: NPLQCD and StellaQCD, see fig. 3 [3]. The current focus of these two efforts is to extract the scattering lengths, and more generally, the phase shifts associated with low-energy nucleon-nucleon, nucleon-hyperon and hyperon-hyperon scattering.

4.1 The Maiani-Testa Theorem (The End of the Innocence) and Lüscher's Method

The first hurdle that one encounters when considering the extraction of a scattering ampli-

[3]The participating institutions are University of Washington, University of New Hampshire, Lawrence Berkeley Laboratories, College of William and Mary, University of Groningen and University of Barcelona.

PoS(LAT2005)020

Figure 3: The logos of NPLQCD and StellaQCD.

tude from a lattice QCD calculation is the Maiani-Testa theorem. Essentially no-one outside of the conference attendees appreciates the implications of this theorem, and typically the impression that most have is that one will be able to calculate everything directly from lattice QCD. Unfortunately, this is false. The Maiani-Testa theorem states that one cannot extract two-body (and higher) S-matrix elements from Euclidean space Green functions at infinite-volume except at kinematic thresholds. Obviously, this has huge implications for the computation of nuclear processes, even the simple ones like $np \to d\gamma$, with lattice QCD.

In the two-body sector, Lüscher was able to arrive at a method to circumvent this constraint [21, 22]. Below inelastic threshold the volume dependence of the two-particle energy levels is determined by the scattering amplitude. Thus determining the energy-levels of two particles in a finite-volume allows for a determination of the scattering amplitude (at the location of the energy-levels). Lüscher's formula relating the scattering amplitude to the location of two-particle states at finite-volume is

$$p\cot\delta(p) \;=\; \frac{1}{\pi L}\,\mathbf{S}\left(\left(\frac{Lp}{2\pi}\right)^2\right) \quad , \tag{4.1}$$

where

$$\mathbf{S}(\eta) \;\equiv\; \sum_{\mathbf{j}}^{\Lambda_j} \frac{1}{|\mathbf{j}|^2-\eta} \;-\; 4\pi\Lambda_j \quad , \tag{4.2}$$

and it is understood that the UV cut-off, $\Lambda_j \to \infty$. In particular limiting cases one can arrive at analytic expressions for both the continuum states and bound states. For $a/L \to 0$, the lowest energy state is located at (we are using the nuclear physics definition of the scattering length)

$$E_0 \;=\; +\frac{4\pi a}{ML^3}\left[1 \;-\; c_1\frac{a}{L} \;+\; c_2\left(\frac{a}{L}\right)^2 \;+\; ...\right] \;+\; \mathscr{O}(L^{-6}) \quad , \tag{4.3}$$

where the coefficients are $c_1 = -2.837297$, $c_2 = +6.375183$, and the bound state, if one exists, is located at

$$E_{-1} \;=\; -\frac{\gamma^2}{M}\left[1 \;+\; \frac{12}{\gamma L}\,\frac{1}{1-2\gamma(p\cot\delta)'}\,e^{-\gamma L} \;+\; ...\right] \quad , \tag{4.4}$$

PoS(LAT2005)020

where $(p\cot\delta)' = \frac{d}{dp^2}\, p\cot\delta$ evaluated at $p^2 = -\gamma^2$. The quantity γ is the solution of

$$\gamma \,+\, p\cot\delta|_{p^2=-\gamma^2} \;=\; 0 \quad . \tag{4.5}$$

In the case of $a/L \to \infty$ (more precisely $Lp\cot\delta \to 0$), of relevance to the NN system, one finds that

$$\tilde{E}_0 \;=\; \frac{4\pi^2}{ML^2}\,[\, d_1 \,+\, d_2\, Lp\cot\delta_0 \,+ \ldots\,] \quad , \tag{4.6}$$

where the coefficients are $d_1 = -0.095901$, $d_2 = +0.0253716$ and where $p\cot\delta_0$ is evaluated at an energy $E = \frac{4\pi^2}{ML^2}\, d_1$.

4.2 The Present and the Future for NN Scattering

One of the exciting aspects of studying two-nucleons on the lattice is that rigorous calculations can be performed today, at unphysical pion masses, even if the systems have scattering lengths that are much larger than the lattices. The reason for this is that it is the range of the interaction that is relevant, and not the scattering length. The range of the NN potential is dictated by the mass of the pion, and while the physical pion mass requires volumes larger than $\gtrsim 5$ fm, a pion of mass ~ 350 MeV requires volumes larger than $\gtrsim 2.5$ fm, which means that meaningful calculations can be performed on the available MILC lattices.

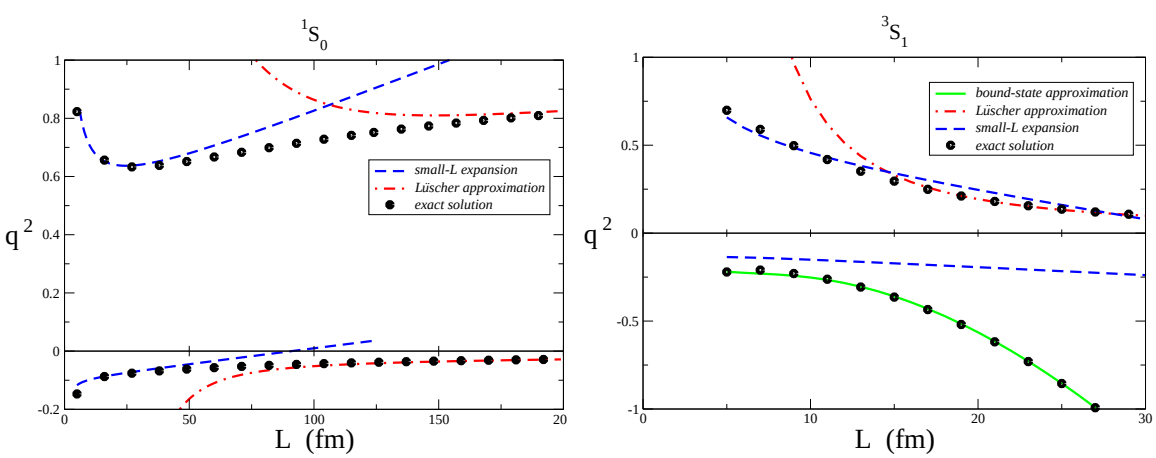

Figure 4: The position of energy levels of two nucleons in a cubically-symmetric finite volume of spatial dimension L at the physical value of the pion mass. The y-axis is $q^2 \sim E_n L^2$. The points correspond to exact solution of eq. (4.1), while some of the curves correspond to the approximations in eq. (4.3), eq. (4.4), and eq. (4.6). As the interactions between nucleons in the two channels are similar, the spectra in small volumes are also similar. Further, as the scattering lengths are large in both channels (a fine-tuning between potential and kinetic energy), the spectra in large volumes are similar, except for the bound state in the channel with positive scattering length.

The simplest tool for analyzing lattice results is the pionless EFT, which is appropriate below the pion inelastic threshold. Beane, Bedaque, Parreño and I [23] explored what would be found in a lattice QCD calculation of the NN system at finite volume at the physical value of the pion mass using the well known NN scattering amplitude. The results are shown in fig. 4. The energy

PoS(LAT2005)020

of levels associated with two nucleons in the $^1S_0$ and the mixed $^3S_1 - {}^3D_1$ channels were computed, and solutions to eq. (4.1) (containing both the continuum states and bound state) are shown by the solid circles in fig. 4. As expected the bound state solution asymptotes at large volume to the deuteron binding energy, while the lowest continuing levels move toward vanishing energy.

To explore NN interactions in smaller volumes, one needs to solve the pionful theory in a finite-volume with periodic boundary conditions (or whatever boundary conditions one chooses, for instance, twisted boundary conditions as first suggested by Bedaque [24] and explored further in Refs. [25, 26]). This has not yet been performed, and given that the pionful NN EFT calculations are performed to lower orders than the pionless calculations, this is going to require some serious efforts by the NN EFT community.

4.3 Electroweak Interactions

Perhaps the simplest non-trivial process in nuclear physics involving electroweak interactions is the radiative capture process $np \to d\gamma$. This is a classic nuclear physics process, and provides a clean demonstration of meson-exchange-currents (MEC's) (electroweak interactions that are not related by gauge symmetry to the NN scattering amplitude). Lattice QCD is going to be able to compute this process by matching onto the low-energy EFT (either pionless or pionful), determining the coefficients of the gauge-invariant operators that are not related to the NN scattering amplitude, and using the EFT to compute the amplitudes of interest. Detmold and I explored what could be learned from lattice QCD calculations of processes such as this using background-field techniques [27]. We found an expression for the energy-levels of two nucleons in a finite volume in the presence of a background magnetic field, and also a background Z^0 field (for the weak-dissociation of the deuteron). The background fields mix the $^1S_0$ and the $^3S_1$ states, and the spectrum is somewhat complicated. However, there is at least one level in the spectrum (and the challenge is to find lattices for which it is the ground state or a low lying excitation) that is sensitive to the coefficient of the gauge-invariant operator. This sensitivity comes about because the deuteron and the near bound state are finely-tuned, and placing the system in a finite volume disturbs this fine-tuning. The contribution of the local operator can restore the fine-tuning in the presence of the background field. It is in this scenario that the level becomes sensitive to the value of the coefficient.

For $np \to d\gamma$, the energy of two nucleons in a background field (in a cubically symmetric lattice are)

$$\left[p\cot\delta_1 - \frac{S_1+S_2}{2\pi L}\right]\left[p\cot\delta_3 - \frac{S_1+S_2}{2\pi L}\right] = \left[\frac{eB_0 l_1}{2} + \frac{S_1-S_2}{2\pi L}\right]^2 \quad , \tag{4.7}$$

where

$$S_1 \;=\; S(\tilde{p}^2+\tilde{u}_1^2) \;\;,\;\; S_2 \;=\; S(\tilde{p}^2-\tilde{u}_1^2) \;\;,\;\; \tilde{u}_1^2 \;=\; \frac{L^2}{4\pi^2}\, eB_0\, \kappa_1 \quad , \tag{4.8}$$

$\kappa_1 = (\kappa_p - \kappa_n)/2$ and $p\cot\delta_1 = -\frac{1}{a_1} + \frac{r_1}{2}p^2 + \ldots$ is the $^1S_0$ effective range expansion. The $\tilde{u}_1$ contributions result from the one-nucleon interactions with the background field, while the two-nucleon–background field interactions are described by l_1. It turns out to be desirable to work with asymmetric volumes to suppress the Landau-level structure, and fig. 5 shows the levels for particular values of the parameters. For $np \to dZ^{0*}$, the relation becomes

PoS(LAT2005)020

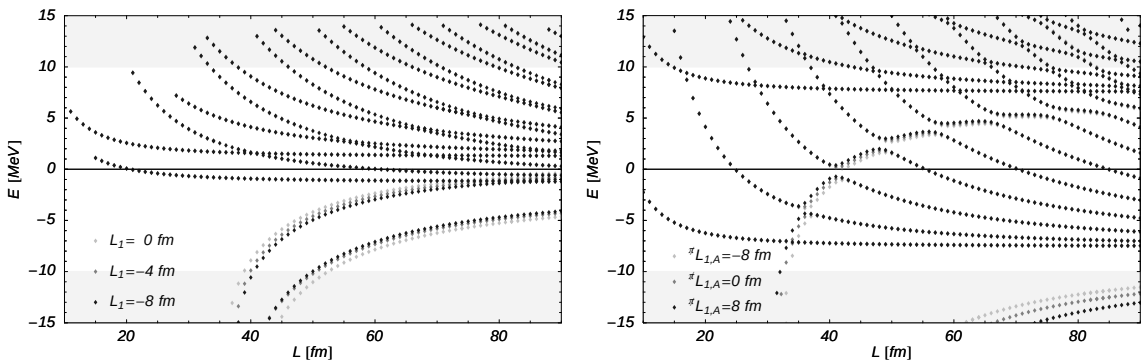

Figure 5: NN energy levels in an asymmetric volume in the presence of a background magnetic field (left panel) and background Z^0-field (right panel) [27]. There is at least one level that is sensitive to the value of the gauge-invariant operator involving two nucleons and the background field.

$$\left[\, p\cot\delta_1 - \frac{S_1+S_2}{2\pi L}\right]\left[\, p\cot\delta_3 - \frac{S_1+S_2}{2\pi L}\right] \;=\; \left[\frac{gW\; l_{1,A}}{4} - \frac{S_1-S_2}{2\pi L}\right]^2 \quad , \tag{4.9}$$

where

$$S_1 \;=\; S(\tilde{p}^2+\tilde{w}_1^2) \;\;,\;\; S_2 \;=\; S(\tilde{p}^2-\tilde{w}_1^2) \;\;,\;\; \tilde{w}_1^2 \;=\; -\frac{L^2}{4\pi^2}\, gW\; g_A\, M \quad . \tag{4.10}$$

The energy-levels in an asymmetric volume are shown in fig. 5.

4.4 The $\Lambda_Q\Lambda_Q$ Potential

An observable that may impact our understanding of nuclear forces is the potential between two Λ_Q baryons in the heavy-quark limit. The potential between to B-mesons in the heavy-quark limit has been explored previously [28, 29, 30, 31], both in quenched and dynamical calculations. This interaction also resembles the nuclear force, including OPE. For two Λ_Q baryons, isospin symmetry forbids contributions from OPE, and the leading long-distance behavior arises from the exchange of two pions (TPE). While TPE leads to a better description of the NN scattering phase-shift data than heavy meson exchange, one would like to better understand this component of the interaction. The TPE contribution to the Λ_Q potential is only part of the TPE contribution to the NN potential, as the Weinberg-Tomazawa term is absent, the axial matrix elements are different, and the spectrum of the $\Lambda_Q - \Sigma_Q$ sector differs from the $N-\Delta$ sector. However, one will be able to directly explore what TPE "looks like" in this system, for instance, if this part of TPE well-described by perturbation theory. Arndt, Beane and I computed this potential [32] in χPT and also in partially-quenched χPT. As expected it "falls like a rock", as the relevant mass-scale is $2m_\pi$, a plot of which can be found in fig. 6.

5. Nonleptonic Hyperon Decays

As the nonleptonic decays $\Lambda \to p\pi^-$, $\Sigma^- \to n\pi^-$, $\Sigma^+ \to n\pi^+$, $\Xi^- \to \Lambda\pi^-$ and their isospin partners have been well studied for an extensive period of time, the experimental data and theoretical efforts to understand the data can be found in many textbooks. Recently we emphasized that

PoS(LAT2005)020

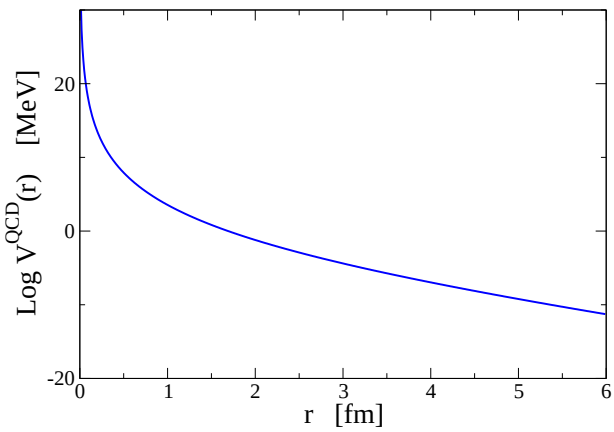

Figure 6: Logarithm of the potential between two Λ_Q's.

this is a problem worthy of exploration with lattice QCD and examined the amplitudes using SU(2) symmetry [33].

While the axial matrix elements $g_A \sim 1.27$ and $g_{\Lambda\Sigma} \sim 0.60$ are well-known experimentally, only an upper limit currently exists for $\Sigma^- \to \Sigma^0 e\overline{\nu}$, and thus there is no direct measurement of the coupling $g_{\Sigma\Sigma}$, which contributes to the Σ-decay p-wave amplitudes. Orginos has been awarded time by SciDAC to compute this matrix element, and the other octet-baryon axial matrix elements.

Inserting numerical values for the couplings and masses into the $S-$wave and $P-$wave amplitudes gives the numerical values for the nonleptonic amplitudes shown in table 1. At leading order,

Table 1: Weak Amplitudes in $SU(2)$ χPT at LO ($g_{\Sigma\Sigma} = 0.30 \to 0.55$)

Decay	$\mathscr{A}^{(S)}$ Theory	$\mathscr{A}^{(S)}$ Expt	$\mathscr{A}^{(P)}$ Theory	$\mathscr{A}^{(P)}$ Expt
$\Lambda \to p\pi^-$	1.42 (input)	1.42 ± 0.01	0.56	0.52 ± 0.02
$\Sigma^- \to n\pi^-$	1.88 (input)	1.88 ± 0.01	$-0.50 \to -0.14$	-0.06 ± 0.01
$\Sigma^+ \to n\pi^+$	0.0	0.06 ± 0.01	$+0.42 \to +0.08$	1.81 ± 0.01

the $P-$wave amplitude in Λ decays is well predicted, as it is in the three-flavor theory. However, we do not expect significant modifications to this result from higher orders in the two-flavor theory. Further, we see that the $P-$wave amplitude for $\Sigma^- \to n\pi^-$ is quite sensitive to the value of $g_{\Sigma\Sigma}$, which is presently quite uncertain. At the upper limit of the allowed range for $g_{\Sigma\Sigma}$, the $P-$wave amplitude is close to what is observed. Finally, we see that the $P-$wave amplitude for $\Sigma^+ \to n\pi^+$ is not close to the experimental value for any reasonable value of $g_{\Sigma\Sigma}$, and theory underestimates the experimental value by ~ 4 in the best case.

6. NPLQCD

The NPLQCD collaboration applied for, and was granted, time from SciDAC for exploratory investigations of the NN, NY and YY systems (Y=hyperon), and also to extract the σ-term and

PoS(LAT2005)020

strong isospin breaking in the nucleon mass. We were awarded $\sim 8\%$ of JLab resources, $\sim$ 40 Gflop – yrs to perform computations with domain-wall-fermions[34, 35] on the available MILC lattices, using Chroma/QDP++ developed by Edwards and his team at JLab [36]. We have analyzed $I = 2$ $\pi\pi$ scattering and have computed the correlation functions for $\Lambda\Lambda$. In what follows we present the results obtained by performing contractions with DWF propagators generated by the LHPC collaboration, who very kindly allowed us to use them in our work.

6.1 $I = 2$ $\pi\pi$-Scattering

Before tackling the two baryon systems we thought it wise to explore $I = 2$ $\pi\pi$-scattering [37]. Not only is this a good "warm-up" exercise, but it is also interesting physics in itself.

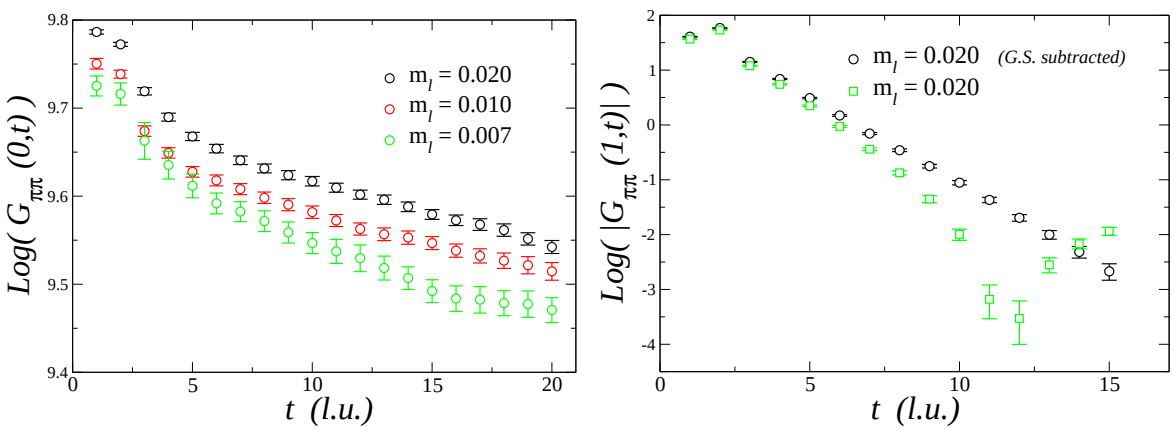

Figure 7: Logarithm of the ratio of the $I = 2$ $\pi\pi$ correlation function to the square of the π correlation function projected onto the momentum $|\mathbf{p}| = 0$ state (left panel) and the $|\mathbf{p}| = 1$ state (right panel). For the excited state (right panel) we have shown the function with and without subtracting the ground state contribution.

The correlation functions shown in fig. 7 were used to extract the ground state and first excited state energies of $\pi^+\pi^+$ on $20^3 \times 64$ staggered MILC lattices, with a physical dimension ~ 2.5 fm and lattice spacing ~ 0.125 fm. The physical pion masses from these calculations are $m_\pi \sim 290, 350,$ and 480 MeV.

In fig. 8 we show the dimensionless quantity $m_\pi a_2$ versus the dimensionless quantity m_π/f_π. The lowest mass CP-PACS point computed with Wilson fermions is also shown. It is remarkable how well the tree-level result from chiral perturbation theory describes the lattice calculation. Also, shown is the residual after removing the tree-level amplitude and multiplying by $\sim f_\pi^2/m_\pi^2$. The lattice data is not precise enough to observe the anticipated chiral logs that enter at next order, however, at these quark masses the counterterm and the chiral log essentially cancel. Clearly, a more precise study of this system is required in order to identify the chiral logs that are predicted to be present, and to allow for a significantly more precise chiral extrapolation to compare with experiment. Our extrapolated value is

$$m_\pi a_2 \;=\; -0.0426 \pm 0.0006 \pm 0.0003 \pm 0.0018 \quad , \tag{6.1}$$

PoS(LAT2005)020

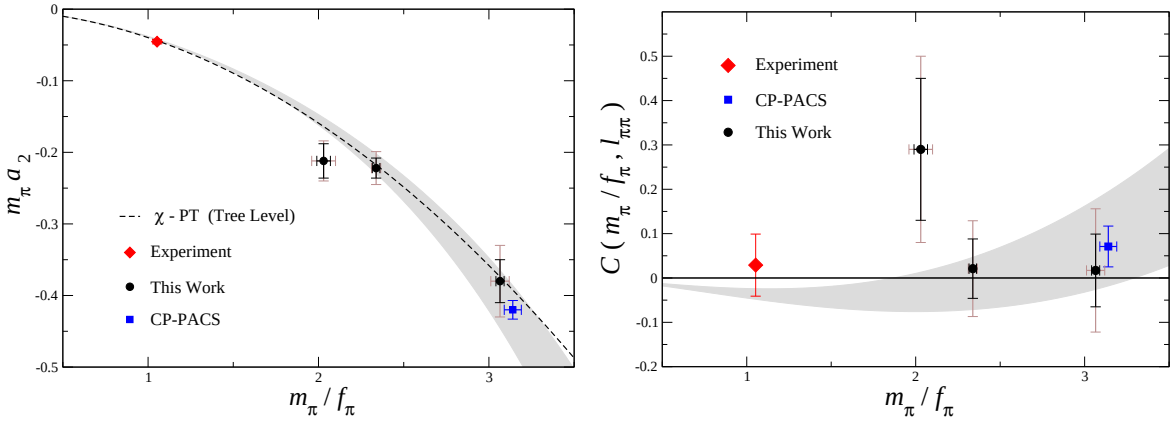

Figure 8: The results of our lattice QCD calculation of $m_\pi a_2$ as a function of m_π / f_π (ovals) [37]. Also shown are the experimental value from Ref. [38] (diamond) and the lowest quark mass result of the dynamical calculation of CP-PACS [39] (square). The gray band corresponds to a weighted fi t to our three data points using the one-loop χ-PT formula.

where the first error is statistical, the second is the systematic associated with the data analysis and the third is the systematic associated with the chiral extrapolation.

6.2 $\Lambda\Lambda$-Scattering

The simplest two-baryon system to write code for is $\Lambda\Lambda$, but this is also quite complex from a physics standpoint due to inelastic channels that become degenerate in the SU(3) limit. One should view this is an interpolating field for the $I = 0$, $S = 2$ channel. In fig. 9 we show the (preliminary) correlation functions we have obtained from some of the MILC lattices. The lightest quark mass correlator is quite noisy, and it will be difficult to extract a scattering amplitude from this data cleanly. It is clear that an increase in statistics by relatively small factors will allow for a extraction of the scattering lengths with uncertainties much less than ± 1 fm at pion masses below ~ 500 MeV.

7. Conclusions

I am very excited by the fact that lattice QCD studies of nuclei and multi-hadron systems are an important part of the future of nuclear physics. However, lattice QCD calculations alone are not sufficient, and it is clear that a parallel development of lattice QCD and effective field theory efforts in this area is required for significant progress to be made. Despite the large length scales that characterize nuclear interactions, the presently available lattices allow for investigations into these systems, and we are attempting to do this presently.

I would like to thank my collaborators for the many exciting discussions that generated this work.

PoS(LAT2005)020

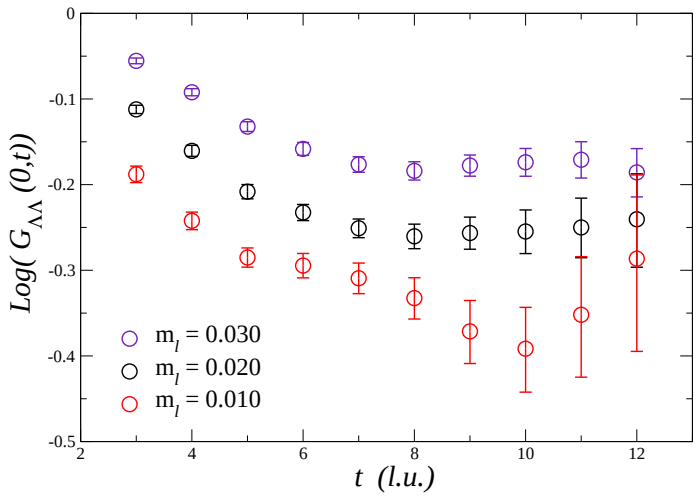

Figure 9: Preliminary data: the logarithm of the ratio of the $\Lambda\Lambda$ correlation function to the square of the Λ correlation function. (Each has been shifted vertically by an arbitrary amount for the sake of clarity.)

References

[1] H. M. Muller and R. Seki, *Given at Caltech / INT Mini Workshop on Nuclear Physics with Effective Field Theories, Pasadena, CA, 26-27 Feb 1998*

[2] H. M. Muller, S. E. Koonin, R. Seki and U. van Kolck, Phys. Rev. C **61**, 044320 (2000) [arXiv:nucl-th/9910038].

[3] T. Abe, R. Seki and A. N. Kocharian, Phys. Rev. C **70**, 014315 (2004) [Erratum-ibid. C **71**, 059902 (2005)] [arXiv:nucl-th/0312125].

[4] D. Lee and T. Schafer, arXiv:nucl-th/0412002.

[5] L. Maiani and M. Testa, *Phys. Lett.* **B245**, 585 (1990).

[6] S. R. Beane, P. F. Bedaque, W. C. Haxton, D. R. Phillips and M. J. Savage, arXiv:nucl-th/0008064.

[7] W. C. Haxton and C. L. Song, Phys. Rev. Lett. **84**, 5484 (2000) [arXiv:nucl-th/9907097].

[8] D. B. Kaplan and A. E. Nelson, Phys. Lett. B **175** (1986) 57.

[9] A. E. Nelson and D. B. Kaplan, Phys. Lett. B **192**, 193 (1987).

[10] G. E. Brown and H. Bethe, Astrophys. J. **423**, 659 (1994).

[11] M. J. Savage, Phys. Rev. C **59**, 2293 (1999) [arXiv:nucl-th/9811087].

[12] G. Prezeau, M. Ramsey-Musolf and P. Vogel, Phys. Rev. D **68**, 034016 (2003) [arXiv:hep-ph/0303205].

[13] M. T. Murphy *et al.*, Mon. Not. Roy. Astron. Soc. **327**, 1208 (2001) [arXiv:astro-ph/0012419].

[14] M. C. M. Rentmeester, R. G. E. Timmermans and J. J. de Swart, Phys. Rev. C **67**, 044001 (2003) [arXiv:nucl-th/0302080].

[15] D. B. Kaplan and M. J. Savage, Phys. Lett. B **365**, 244 (1996) [arXiv:hep-ph/9509371].

PoS(LAT2005)020

[16] S. R. Beane, P. F. Bedaque, M. J. Savage and U. van Kolck, Nucl. Phys. A **700**, 377 (2002) [arXiv:nucl-th/0104030].

[17] S. R. Beane and M. J. Savage, Nucl. Phys. A **713**, 148 (2003) [arXiv:hep-ph/0206113].

[18] S. R. Beane and M. J. Savage, Nucl. Phys. A **717**, 91 (2003) [arXiv:nucl-th/0208021].

[19] E. Epelbaum, U. G. Meissner and W. Gloeckle, Nucl. Phys. A **714**, 535 (2003) [arXiv:nucl-th/0207089].

[20] M. Fukugita, Y. Kuramashi, M. Okawa, H. Mino and A. Ukawa, Phys. Rev. D **52**, 3003 (1995) [arXiv:hep-lat/9501024].

[21] M. Lüscher, *Commun. Math. Phys.* **105** 153 (1986).

[22] M. Lüscher, *Nucl. Phys.* **B354**, 531 (1991).

[23] S. R. Beane, P. F. Bedaque, A. Parreño and M. J. Savage, Phys. Lett. B **585**, 106 (2004) [arXiv:hep-lat/0312004].

[24] P. F. Bedaque, Phys. Lett. B **593**, 82 (2004) [arXiv:nucl-th/0402051].

[25] C. T. Sachrajda and G. Villadoro, Phys. Lett. B **609**, 73 (2005) [arXiv:hep-lat/0411033].

[26] P. F. Bedaque and J. W. Chen, Phys. Lett. B **616**, 208 (2005) [arXiv:hep-lat/0412023].

[27] W. Detmold and M. J. Savage, Nucl. Phys. A **743**, 170 (2004) [arXiv:hep-lat/0403005].

[28] C. Michael and P. Pennanen [UKQCD Collaboration], Phys. Rev. D **60**, 054012 (1999) [arXiv:hep-lat/9901007].

[29] P. Pennanen, C. Michael and A. M. Green [UKQCD Collaboration], Nucl. Phys. Proc. Suppl. **83**, 200 (2000) [arXiv:hep-lat/9908032].

[30] M. S. Cook and H. R. Fiebig, arXiv:hep-lat/0210054.

[31] M. S. Cook and H. R. Fiebig, arXiv:hep-lat/0509025.

[32] D. Arndt, S. R. Beane and M. J. Savage, Nucl. Phys. A **726**, 339 (2003) [arXiv:nucl-th/0304004].

[33] S. R. Beane, P. F. Bedaque, A. Parreño and M. J. Savage, Nucl. Phys. A **747**, 55 (2005) [arXiv:nucl-th/0311027].

[34] D. B. Kaplan, Phys. Lett. B **288**, 342 (1992) [arXiv:hep-lat/9206013].

[35] Y. Shamir, Nucl. Phys. B **406**, 90 (1993) [arXiv:hep-lat/9303005].

[36] R. G. Edwards and B. Joo [SciDAC Collaboration], arXiv:hep-lat/0409003.

[37] S. R. Beane, P. F. Bedaque, K. Orginos and M. J. Savage [NPLQCD Collaboration], arXiv:hep-lat/0506013.

[38] S. Pislak *et al.*, Phys. Rev. D **67**, 072004 (2003) [arXiv:hep-ex/0301040].

[39] T. Yamazaki *et al.* [CP-PACS Collaboration], Phys. Rev. D **70**, 074513 (2004) [arXiv:hep-lat/0402025].

PoS(LAT2005)020