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Pseudoscalar Mass and Decay Constant in Lattice QCD with Exact Chiral Symmetry[*]

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The masses and decay constants of pseudoscalar mesons D, D_s, and K are determined in quenched lattice QCD with exact chiral symmetry. For 100 gauge configurations generated with single-plaquette action at $\beta = 6.1$ on the $20^3 \times 40$ lattice, we compute point-to-point quark propagators for 30 quark masses in the range $0.03 \leq m_q a \leq 0.80$, and measure the time-correlation functions of pseudoscalar and vector mesons. The inverse lattice spacing a^{-1} is determined with the experimental input of f_π, while the strange quark bare mass ($m_s a = 0.08$), and the charm quark bare mass ($m_c a = 0.80$) are fixed such that the masses of the corresponding vector mesons are in good agreement with $\phi(1020)$ and $J/\psi(3097)$ respectively. Our results of pseudoscalar-meson decay constant are: $f_K = 152(6)(10)$ MeV, $f_D = 235(8)(14)$ MeV, and $f_{D_s} = 266(10)(18)$ MeV [1]. The latest experimental result of f_{D^+} from CLEO [3] is in good agreement with our prediction.

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1. Introduction

The pseudoscalar-meson decay constants (e.g., f_D, f_{D_s}, f_B and f_{B_s}) play an important role in extracting the CKM matrix elements (e.g., the leptonic decay width of $D_s^+ \to l^+ \nu_l$ is proportional to $f_{D_s}^2 |V_{cs}|^2$), which are crucial for testing the flavor sector of the standard model via the unitarity of CKM matrix. Experimentally, precise determination of f_D and f_{D_s} will result from the high-statistics program of CLEO-c, however, the determination of f_B and f_{B_s} remains beyond the reach of current experiments.

Theoretically, lattice QCD provides a solid framework to compute the masses and decay constants of pseudoscalar mesons (as well as other physical observables) nonperturbatively from the first principles of QCD. Thus reliable lattice QCD determinations of f_B and f_{B_s} are of fundamental importance, in view of their experimental determinations are still lacking. Obviously, the first step for lattice QCD is to check whether the lattice determinations of f_D and f_{D_s} will agree with those coming from the high-statistics charm program of CLEO-c. This motivated our study in Ref. [1]. It turns out that our *predictive* value of $f_D = 235(8)(14)$ MeV (posted on June 26) is in good agreement with the experimental result $f_{D^+} = (223 \pm 16^{+7}_{-9})$ MeV announced by CLEO-c at Lepton-Photon Symposium (LP2005) on July 1 [2]. (Note that the latest value of f_{D^+} from CLEO-c has been updated to $f_{D^+} = (222.6 \pm 16.7^{+2.8}_{-3.4})$ MeV [3].)

In this talk, we review our results reported in Ref. [1], and also present our results for the ratios f_K/f_π and f_{D_s}/f_D, which may be of interest from several viewpoints.

Here we briefly outline our computations, and refer to Ref. [1] (and references therein) for further details. First, we compute quenched quark propagators for 30 quark masses in the range $0.03 \le m_q a \le 0.80$, in the framework of optimal domain-wall fermion proposed by Chiu [4]. Then we determine the inverse lattice spacing $a^{-1} = 2.237(75)$ GeV from the pion time-correlation function, with the experimental input of pion decay constant $f_\pi = 131$ MeV. The strange quark bare mass $m_s a = 0.08$ and the charm quark bare mass $m_c a = 0.80$ are fixed such that the corresponding masses extracted from the vector meson correlation function agree with $\phi(1020)$ and $J/\psi(3097)$ respectively. Then the masses and decay constants of any hadrons containing c, s, and $u(d)$ quarks[1] are predictions of QCD from the first principles, with the understanding that chiral extrapolation to physical $m_{u,d} \simeq m_s/25$ (or equivalently $m_\pi = 135$ MeV) is required for any observables containing $u(d)$ quarks.

The observable we measure is the time-correlation function for pseudoscalar meson ($\bar{q}Q$)

$$C_P(t) = \left\langle \sum_{\vec{x}} \mathrm{tr}\{\gamma_5 (D_c + m_Q)^{-1}_{x,0} \gamma_5 (D_c + m_q)^{-1}_{0,x}\} \right\rangle_U \tag{1.1}$$

where the subscript U denotes averaging over gauge configurations. Here $C_P(t)$ is measured for the following three categories: (i) Symmetric masses $m_Q = m_q$, (ii) Asymmetric masses with fixed $m_Q = m_s = 0.08a^{-1}$, (iii) Asymmetric masses with fixed $m_Q = m_c = 0.80a^{-1}$, where m_q is varied for 30 masses in the range $0.03 \le m_q a \le 0.80$. Note that for mesons composed of strange and/or charm quarks, their masses and decay constants can be measured directly without chiral extrapolation.

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[1] In this paper, we work in the isospin limit $m_u = m_d$.

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The decay constant f_P for a charged pseudoscalar meson P is defined by

$$\langle 0|A_\mu(0)|P(\vec{q})\rangle = f_P q_\mu$$

where $A_\mu = \bar{q}\gamma_\mu\gamma_5 Q$ is the axial-vector part of the charged weak current after a CKM matrix element $V_{qq'}$ has been removed. Using the formula $\partial_\mu A_\mu = (m_q + m_Q)\bar{q}\gamma_5 Q$, one obtains

$$f_P = (m_q + m_Q)\frac{|\langle 0|\bar{q}\gamma_5 Q|P(\vec{0})\rangle|}{m_P^2} \tag{1.2}$$

where the pseudoscalar mass $m_P a$ and the decay amplitude $z \equiv |\langle 0|\bar{q}\gamma_5 Q|P(\vec{0})\rangle|$ can be obtained by fitting the pseudoscalar time-correlation function $C_P(t)$ to the usual formula

$$\frac{z^2}{2m_P a}[e^{-m_P a t} + e^{-m_P a(T-t)}] \tag{1.3}$$

The outline of this paper is as follows. We present our results of m_K and f_K in section 2, m_D, m_{D_s}, f_D, and f_{D_s} in section 3, and the ratios f_K/f_π and f_{D_s}/f_D in section 4. In section 5, we summarize and conclude with some remarks.

2. f_K and m_K

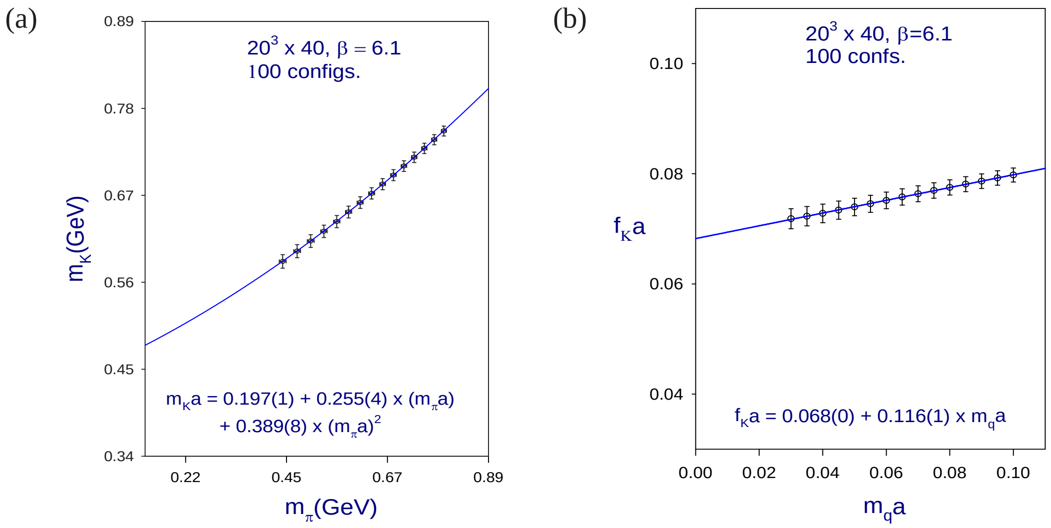

Figure 1: (a) The kaon mass m_K versus the pion mass m_π for 15 bare quark masses in the range $0.03 \le m_q a \le 0.10$. The solid line is the quadratic fit. (b) The kaon decay constant $f_K a$ versus the bare quark mass $m_q a$. The solid line is the linear fit.

We measure the time-correlation function of kaon $C_K(t)$ (1.1) with m_Q fixed at $m_s = 0.08a^{-1}$, while m_q is varied for 30 masses in the range $0.03 \le m_q a \le 0.80$. Then the data of $C_K(t)$ is fitted by the formula (1.3) to extract the kaon mass $m_K a$ and the kaon decay constant $f_K a$.

In Fig. 1a, the kaon mass m_K is plotted versus m_π, for 15 quark masses in the range $0.03 \le m_q a \le 0.10$. The data of $m_K a$ can be fitted by $m_K a = 0.197(1) + 0.255(4)(m_\pi a) + 0.389(8)(m_\pi a)^2$.

At the physical limit $m_\pi = 135$ MeV, it gives $m_K = 478(16)$ MeV, in good agreement with the experimental value of kaon mass (495 MeV).

In Fig. 1b, $f_K a$ is plotted versus bare quark mass $m_q a$. The data is well fitted by the straight line $f_K a = 0.068(0) + 0.116(1) \times (m_q a)$. At $m_q a = 0$, it gives $f_K = 152(6)$ MeV, in agreement with the value $f_{K^+} = 159.8 \pm 1.4 \pm 0.44$ MeV complied by PDG [5].

3. f_D, f_{D_s}, m_D, and m_{D_s}

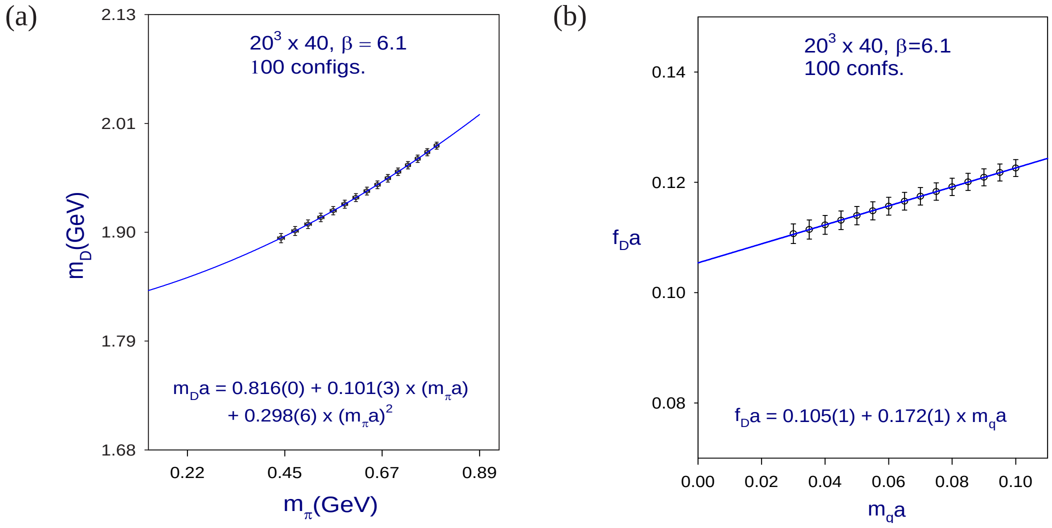

Figure 2: (a) The *D*-meson mass m_D versus the pion mass m_π for 15 bare quark masses in the range $0.03 \le m_q a \le 0.10$. The solid line is the quadratic fit. (b) The D-meson decay constant $f_D a$ versus the bare quark mass $m_q a$. The solid line is the linear fit.

Now we turn to charmed pseudoscalar mesons. We measure the time-correlation function $C_D(t)$ (1.1) with m_Q fixed at $m_c = 0.80a^{-1}$, while m_q is varied for 30 different masses in the range $0.03 \le m_q a \le 0.80$. Then the data of $C_D(t)$ is fitted by the formula (1.3) to extract the mass $m_D a$ and decay constant $f_D a$.

In Fig. 2a, $m_D a$ is plotted versus $m_\pi a$, for 15 quark masses in the range $0.03 \le m_q a \le 0.10$. The data of $m_D a$ can be fitted by $m_D a = 0.816(0) + 0.101(3)(m_\pi a) + 0.298(6)(m_\pi a)^2$. At $m_\pi = 135$ MeV, it gives $m_D = 1842(15)$ MeV, in good agreement with the mass of D meson (1865 MeV). In Fig. 2b, the decay constant $f_D a$ is plotted versus bare quark mass $m_q a$. The data is well fitted by the straight line $f_D a = 0.105(1) + 0.172(1) \times (m_q a)$. At $m_q a = 0$, it gives $f_D = 235(8)$ MeV, which serves as a prediction of lattice QCD with exact chiral symmetry.

The pseudoscalar meson of $\bar{s}c$ or $s\bar{c}$ corresponds to $m_Q a = m_c a = 0.80$ and $m_q a = m_s a = 0.08$. Its mass and decay constant are extracted directly from the time-correlation function, which are plotted as the eleventh data point (counting from the smallest one) in Fig. 2. The results are $m_{D_s} a = 0.878(2)$ and $f_{D_s} a = 0.119(2)$. The mass gives $m_{D_s} = 1964(5)$ MeV, in good agreement with the mass of $D_s(1968)$. The decay constant gives $f_{D_s} = 266(10)$ MeV, which agrees with the value $f_{D_s^+} = 267 \pm 33$ MeV complied by PDG [5].

4. Ratios f_K/f_π and f_{D_s}/f_D

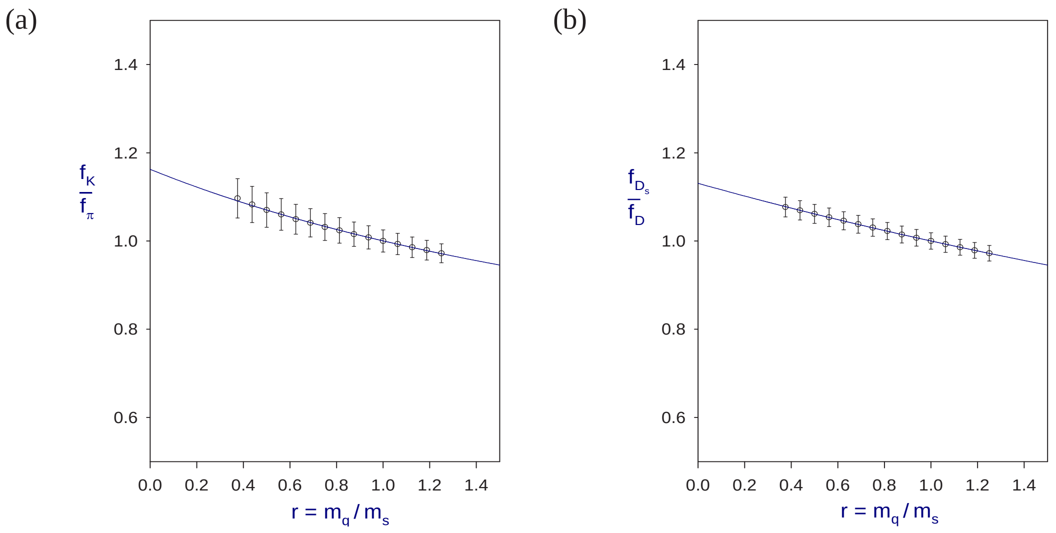

Figure 3: (a) The ratio f_K/f_π versus $r = m_u/m_s$ for 15 bare quark masses in the range $0.03 \le m_q a \le 0.10$. The solid line is the fit (4.1). (b) The ratio f_{D_s}/f_D versus m_u/m_s. The solid line is the fit (4.2).

At this point, it is instructive to obtain the ratios f_K/f_π and f_{D_s}/f_D, and to see how well they agree the experimental values. In Fig. 3a, the ratio f_{K^+}/f_π is plotted versus $r = m_u/m_s$ for 15 bare quark masses in the range $0.03 \le m_q a \le 0.10$. The data of f_{K^+}/f_π can be fitted by

$$0.669(22) + \frac{0.931(116)}{1.813(163)+r} \tag{4.1}$$

At physical limit $m_u/m_s = 1/26$, it gives $f_{K^+}/f_\pi = 1.17(6)$, in good agreement with the experiemntal value ~ 1.22 (PDG) [5]. With $f_\pi = 131$ MeV as the input, it gives $f_{K^+} = 153(8)$ MeV, in good agreement with the experimental value $f_{K^+} = 159.8 \pm 1.4 \pm 0.44$ MeV complied by PDG [5].

Similarly, in Fig. 3b, the data of $f_{D_s^+}/f_{D^+}$ can be fitted by

$$\frac{8.658(27)}{7.657(26)+r} \tag{4.2}$$

At physical limit $r = 1/26$, it gives $f_{D_s^+}/f_{D^+} = 1.13(2)$, which serves as a theoretical prediction.

5. Summary and Concluding Remarks

In Ref. [1], we have determined the masses and decay constants of pseudoscalar mesons K, D and D_s, in quenched lattice QCD with exact chiral symmetry. Our results are:

$$m_K = 478 \pm 16 \pm 20 \text{ MeV}, \quad m_D = 1842 \pm 15 \pm 21 \text{ MeV}, \quad m_{D_s} = 1964 \pm 5 \pm 10 \text{ MeV},$$
$$f_K = 152 \pm 6 \pm 10 \text{ MeV}, \quad f_D = 235 \pm 8 \pm 14 \text{ MeV}, \quad f_{D_s} = 266 \pm 10 \pm 18 \text{ MeV},$$

where in each case, the first error is statistical, while the second is our crude estimate of combined systematic uncertainty. Note that at the time when our results were posted at arXiv:hep-ph/0506266 on June 26, the preliminary result of f_{D^+} from CLEO-c was $f_{D^+} = 202(41)(17)$ MeV [6]. So our value of f_D was not in good agreement with the preliminary result of CLEO. However, with higher statistics at CLEO-c, the experimental value of f_{D^+} turns out to be $f_{D^+} = (222.6 \pm 16.7^{+2.8}_{-3.4})$ MeV [3], in good agreement with our prediction of f_D. Now it remains to see whether the value of f_{D_s} coming from the high-statistics charm program of CLEO-c would agree with our value determined by lattice QCD with exact chiral symmetry.

A salient feature of our calculation is that all quarks (no matter heavy or light) are treated on an equal footing, and they are fully relativistic with exact chiral symmetry on the lattice. Also, we have not used any heavy quark approximations for the charm quark. Even though our results are obtained in the quenched approximation, we suspect that for lattice QCD with exact chiral symmetry, the quenching error in f_D, and f_{D_s} is less than 5%, in view of the good agreement between our result of f_K and the experimental value.

In closing, we note that there are several quenched/unquenched lattice QCD results for f_D and f_{D_s} in the literature [7, 8, 9], as well as reported in this conference [10, 11]. For the unquenched staggered quark with $n_f = 2+1$ [9, 10], its prediction of $f_{D^+} = (201 \pm 3 \pm 17)$ MeV only agrees with the latest CLEO experimental result [3] at 45% confidence level, and is 10% smaller than the experimental result. Note that this unquenched lattice QCD calculation relies on the so-called fourth-root trick to reduce the four tastes of each staggered quark to one taste, in computing the fermion determinant. Since this prescription for staggered quark has not achieved a high-precision prediction for f_{D^+}, it may be an indication of the failure of the fourth-root trick in practice.

References

[1] T. W. Chiu, T. H. Hsieh, J. Y. Lee, P. H. Liu and H. J. Chang, Phys. Lett. B **624**, 31 (2005) [hep-ph/0506266].

[2] M. Artuso, plenary talk given at 22nd International Symposium on Lepton-Photon Interactions at High Energy, Uppsala, Sweden, 30 Jun - 5 Jul 2005.

[3] M. Artuso *et al.* [CLEO Collaboration], hep-ex/0508057.

[4] T. W. Chiu, Phys. Rev. Lett. **90**, 071601 (2003); Phys. Lett. B **552**, 97 (2003); hep-lat/0303008; Nucl. Phys. Proc. Suppl. **129**, 135 (2004).

[5] S. Eidelman *et al.* [Particle Data Group Collaboration], Phys. Lett. B **592**, 1 (2004).

[6] G. Bonvicini *et al.* [CLEO Collaboration], Phys. Rev. D **70**, 112004 (2004)

[7] D. Becirevic, P. Boucaud, J. P. Leroy, V. Lubicz, G. Martinelli, F. Mescia and F. Rapuano, Phys. Rev. D **60**, 074501 (1999)

[8] L. Lellouch and C. J. D. Lin [UKQCD Collaboration], Phys. Rev. D **64**, 094501 (2001)

[9] C. Aubin *et al.*, "*Charmed meson decay constants in three-flavor lattice QCD*", hep-lat/0506030.

[10] J. Simone, "*The determination of decay constants f_D and f_{D_s} in three flavor lattice QCD*",poster presented at Lattice 2005.

[11] S.-J. Dong, "*D_s meson decay constant f_{D_s} from quenched lattice QCD with overlap fermions*", talk presented at Lattice 2005.