

Mass Spectra of Pentaquarks – Overlap versus Wilson Fermions*

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We investigate the mass spectra of $\Theta^+(udud\bar{s})$ in quenched lattice QCD, with overlap and Wilson fermions respectively. Using three different interpolating operators, we measure their correlation matrix, and extract the mass spectra in even and odd parity channels, for 100 gauge configurations generated with single plaquette action at $\beta = 6.1$ on the $20^3 \times 40$ lattice. The lowest $1/2^-$ state agrees with the KN s-wave scattering state, for both fermion schemes. On the other hand, for the lowest $1/2^+$ state, it is different from any hadron scattering states for the overlap fermion, while it seems to agree with KN^* s-wave for the Wilson fermion.

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1. Introduction

The experimental observation of exotic baryon $\Theta^+(1540)$ (with quantum numbers of K^{+n}) by LEPS collaboration at Spring-8 and subsequent confirmation from some experimental groups has become one of the most interesting topics in hadron physics. The remarkable features of $\Theta^+(1540)$ are its strangeness $S = +1$, and its exceptionally narrow decay width (< 15 MeV) even though it is about 100 MeV above the KN threshold. Its strangeness $S = +1$ immediately implies that it cannot be an ordinary 3-quark baryon. Its minimal quark content is $udud\bar{s}$. Nevertheless, there are quite a number of experiments which so far have not observed $\Theta^+(1540)$. Further, some experiments which had claimed positive evidence of Θ have switched to negative when they attain higher statistics. This almost denies the existence of $\Theta^+(1540)$.

To study this problem in lattice QCD, one has to construct an interpolating operator which has a significant overlap with the pentaquark state. Then one computes its time-correlation function to extract the masses of its even and odd parity states. However, any $udud\bar{s}$ operator must couple to any hadronic states with the same quantum numbers (e.g., KN scattering states). To disentangle the lowest-lying pentaquark state from the various KN scattering states, we use three different interpolating operators for $\Theta^+(udud\bar{s})$ to form a 3×3 correlation matrix,

$$C_{ij}^{\pm}(t) = \left\langle \sum_{\vec{x}} \text{tr} \left[\frac{1 \pm \gamma_4}{2} \langle O_i(\vec{x}, t) \bar{O}_j(\vec{0}, 0) \rangle_f \right] \right\rangle_U \quad (1.1)$$

and extract the masses for both \pm parity states from its eigenvalues. These three operators are

$$(O_1)_{x\alpha} = [\mathbf{u}^T C \gamma_5 \mathbf{d}]_{xc} \{ \bar{s}_{x\beta e}(\gamma_5)_{\beta\eta} \mathbf{u}_{x\eta e}(\gamma_5 \mathbf{d})_{x\alpha c} - \bar{s}_{x\beta e}(\gamma_5)_{\beta\eta} \mathbf{d}_{x\eta e}(\gamma_5 \mathbf{u})_{x\alpha c} \} \quad (1.2)$$

$$(O_2)_{x\alpha} = [\mathbf{u}^T C \gamma_5 \mathbf{d}]_{xc} \{ \bar{s}_{x\beta e}(\gamma_5)_{\beta\eta} \mathbf{u}_{x\eta c}(\gamma_5 \mathbf{d})_{x\alpha e} - \bar{s}_{x\beta e}(\gamma_5)_{\beta\eta} \mathbf{d}_{x\eta c}(\gamma_5 \mathbf{u})_{x\alpha e} \} \quad (1.3)$$

$$(O_3)_{x\alpha} = \varepsilon_{cde} [\mathbf{u}^T C \gamma_5 \mathbf{d}]_{xc} [\mathbf{u}^T C \mathbf{d}]_{xd} (C \bar{s}^T)_{x\alpha e} \quad (1.4)$$

where C is the charge conjugation which satisfies $C\gamma_\mu C^{-1} = -\gamma_\mu^T$ and $(C\gamma_5)^T = -C\gamma_5$, and the diquark operator is defined as $[\mathbf{u}^T \Gamma \mathbf{d}]_{xa} \equiv \varepsilon_{abc} (\mathbf{u}_{x\alpha b} \Gamma_{\alpha\beta} \mathbf{d}_{x\beta c} - \mathbf{d}_{x\alpha b} \Gamma_{\alpha\beta} \mathbf{u}_{x\beta c})$, where $\Gamma_{\alpha\beta} = -\Gamma_{\beta\alpha}$. Thus the diquark transforms like a spin singlet (1_s), color anti-triplet ($\bar{3}_c$), and flavor anti-triplet ($\bar{3}_f$). For $\Gamma = C\gamma_5$, it transforms as a scalar, while for $\Gamma = C$, it transforms like a pseudoscalar.

We generate 100 gauge configurations with Wilson gauge action at $\beta = 6.1$ on the $20^3 \times 40$ lattice. Then we compute the point-to-point quark propagators for both overlap fermion and Wilson fermion, with both periodic and anti-periodic boundary conditions in the time direction, respectively. Then we measure the 3×3 correlation matrix from the *averaged* quark propagators. For the Wilson fermion, we compute quark propagators for $\kappa = 0.149, 0.150, 0.151, 0.152, 0.153$, and 0.1534 , with stopping criteria 10^{-11} for the CG (conjugate gradient) loops. For the overlap fermion, we use the nested and 2-pass CG algorithms to compute quark propagators. With $m_0 = 1.3$, $N_s = 128$, and 16 low-lying eigenmodes of $|H_w|$ projected, the Zolotarev coefficients and poles are fixed with $0.18 \leq \lambda(|H_w|) \leq 6.3$ for all gauge configurations. We compute quark propagators for 30 bare quark masses in the range $0.03 \leq m_q a \leq 0.8$, with stopping criteria 10^{-11} and 2×10^{-12} for the outer and inner CG loops, respectively. The precision of chiral symmetry and the norm of the residue vector for each column of quark propagators are:

$$\sigma = \left| \frac{Y^\dagger S^2 Y}{Y^\dagger Y} - 1 \right| < 10^{-14}, \quad \|(D_c + m_q)Y - \mathbf{1}\| < 2 \times 10^{-11} \quad (1.5)$$

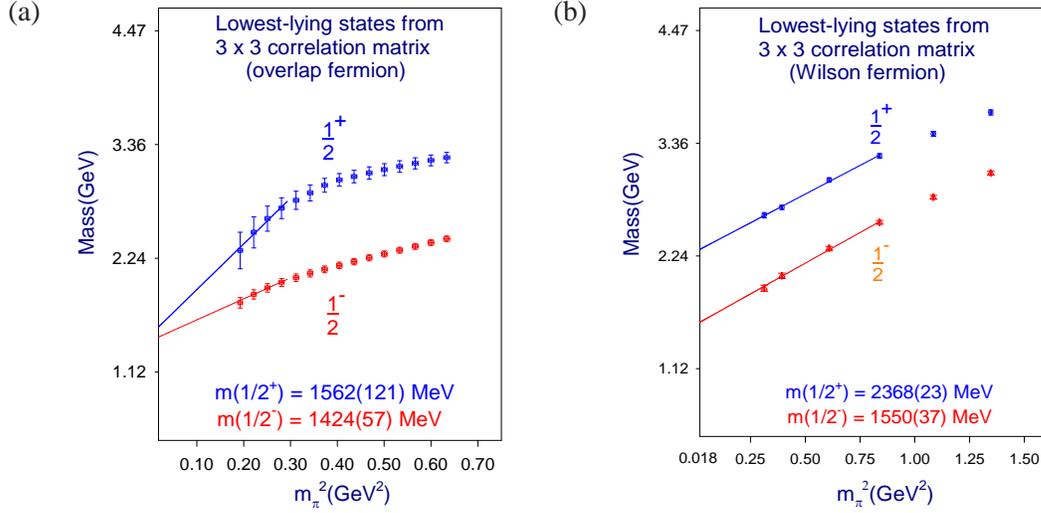


Figure 1: Comparison of the lowest-lying states from the 3×3 correlation matrix, for (a) overlap, and (b) Wilson fermions respectively.

To determine a^{-1} , we measure the pion time correlation function with overlap quark propagators, and extract pion mass ($m_\pi a$) and decay constant ($f_\pi a$). With the experimental input $f_\pi = 131$ MeV, we determine $a^{-1} = 2.237(76)$ GeV. To determine m_s (for overlap fermion) and κ_s (for Wilson fermion), we extract the vector meson mass from the correlation function

$$C_V(t) = \frac{1}{3} \sum_{\mu=1}^3 \sum_{\vec{x}} \text{tr} \{ \gamma_\mu (D_c + m_q)_{x,0}^{-1} \gamma_\mu (D_c + m_q)_{0,\vec{x}}^{-1} \} \quad (1.6)$$

For overlap quark, at $m_q a = 0.08$, it gives $M_V a = 0.4601(41)$, which amounts $M_V = 1029(10)$ MeV, in good agreement with $\phi(1020)$. Thus we fix $m_s a = 0.08$, and we have 10 quark masses smaller than m_s . Similarly, for Wilson quark, we fix $\kappa_s = 0.151$, and we have three κ 's $> \kappa_s$.

2. Results

In Fig. 1, we plot the masses of the lowest-lying states extracted from the 3×3 correlation matrix, for both parity channels, and for overlap and Wilson fermions respectively. The solid lines are chiral extrapolation (linear in m_π^2) using the smallest four masses. At $m_\pi = 135$ MeV, they give (a) Overlap fermion: $m(1/2^-) = 1424(57)$ MeV, $m(1/2^+) = 1562(121)$ MeV; (b) Wilson fermion: $m(1/2^-) = 1550(37)$ MeV, $m(1/2^+) = 2368(23)$ MeV.

Evidently, for any parity channel, its behavior with the overlap fermion is quite different from that with the Wilson fermion. In particular, for the $1/2^+$ state with the overlap fermion, it experiences a rapid decrease in the regime $m_u a \leq 0.045 \simeq 0.56 m_s a$. We interpret this as a manifestation of the *diquark correlations* when m_u becomes sufficiently small, since the overlap fermion preserves the exact chiral symmetry on the lattice. On the other hand, Wilson fermion breaks chiral symmetry explicitly and the bare quark mass is not well-defined, thus could not capture the diquark correlations except in the continuum limit.

Next we compare the lowest-lying states extracted from the 3×3 correlation matrix with the KN and KN^* (s-wave) scattering states, as shown in Fig. 2. Naively, assuming the interaction

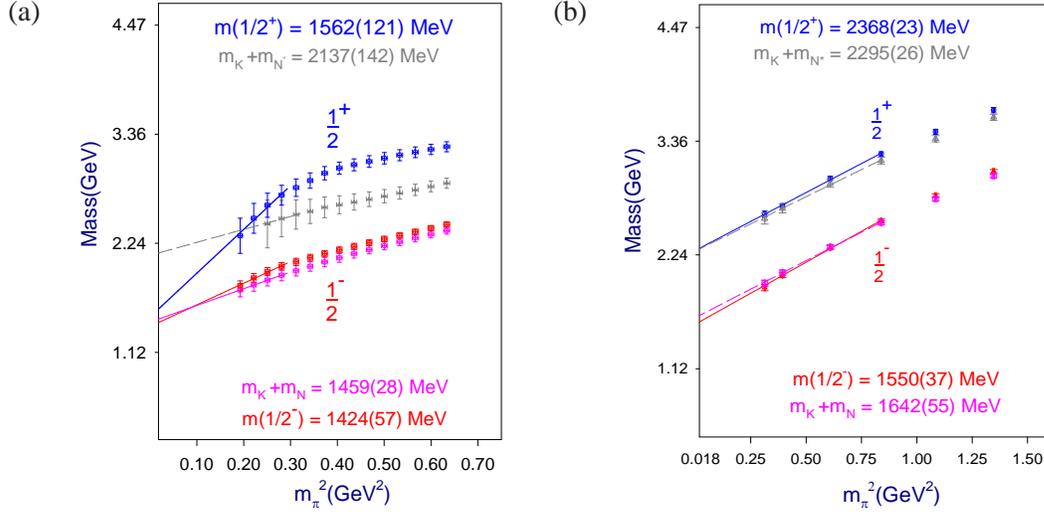


Figure 2: The lowest-lying states of the 3×3 correlation matrix versus KN and KN^* (s-wave) scattering states, for (a) overlap, and (b) Wilson fermions respectively.

between the Kaon and the Nucleon in these (s-wave) scattering states is very weak, then their masses can be estimated as $m_K + m_N$ and $m_K + m_{N^*}$ respectively. For the $1/2^-$ state, its mass agrees with the KN s-wave, for overlap and Wilson fermions respectively. On the other hand, for the $1/2^+$ state, it agrees with the KN^* s-wave for Wilson fermion, but it is different from the KN^* s-wave for overlap fermion. This reveals an important fact that one cannot identify the nature of any excited state by simply comparing its mass with those of scattering states.

To obtain a more reliable estimate of the mass spectra of KN scattering states, we measure the correlation function of KN operator without any exchange of quarks between the propagators of K and N , i.e.,

$$C_{KN}^{\pm}(t) = \left\langle \sum_{\vec{x}} \text{tr} \left[\frac{1 \mp \gamma_4}{2} \left\langle N(\vec{x}, t) \bar{N}(\vec{0}, 0) \right\rangle_f \left\langle K(\vec{x}, t) \bar{K}(\vec{0}, 0) \right\rangle_f \right] \right\rangle_U \quad (2.1)$$

Then we compare its mass spectrum with the naive estimates, $\sqrt{m_K^2 + (2\pi/L)^2} + \sqrt{m_N^2 + (2\pi/L)^2}$ (p-wave), and $m_K + m_N$ (s-wave), as shown in Fig. 3. For the $1/2^-$ state, it agrees with $m_K + m_N$ very well, for overlap and Wilson fermions. On the other hand, for the $1/2^+$ state, it disagrees with the naive estimate for both fermion schemes. This suggests that the KN p-wave (in the quenched approximation) in a finite torus is more complicated than just two free particles with opposite momenta $\vec{p}_K = -\vec{p}_N = 2\pi\hat{e}_i/L$. Comparing the mass spectra of the KN scattering states with those from 3×3 correlation matrix, as in Fig. 4, we identify the lowest $1/2^-$ state of $\Theta(udud\bar{s})$ with the KN s-wave, and also rule out the possibility that the lowest $1/2^+$ state could be the KN p-wave.

Next we consider another hadron scattering state which has the same quantum numbers of the $1/2^+$ state of $\Theta(udud\bar{s})$, namely, the s-wave scattering state of $KN\eta'$, where η' is an artifact of quenched approximation. As shown in Fig. 5, the lowest $1/2^+$ state is different from the s-wave of $KN\eta'$, for both fermion schemes.

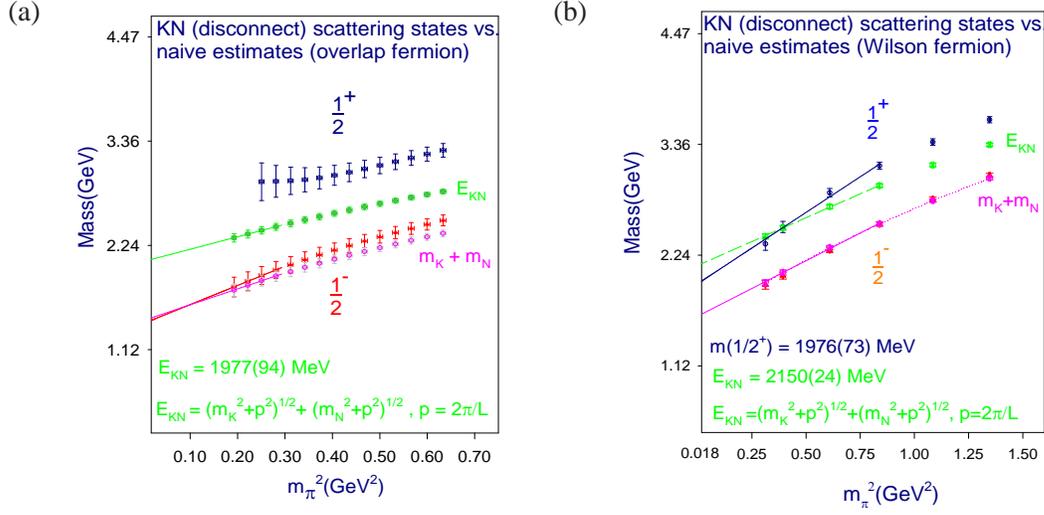


Figure 3: The mass spectra of KN (disconnected) scattering states versus naive estimates, for (a) overlap, and (b) Wilson fermions respectively.

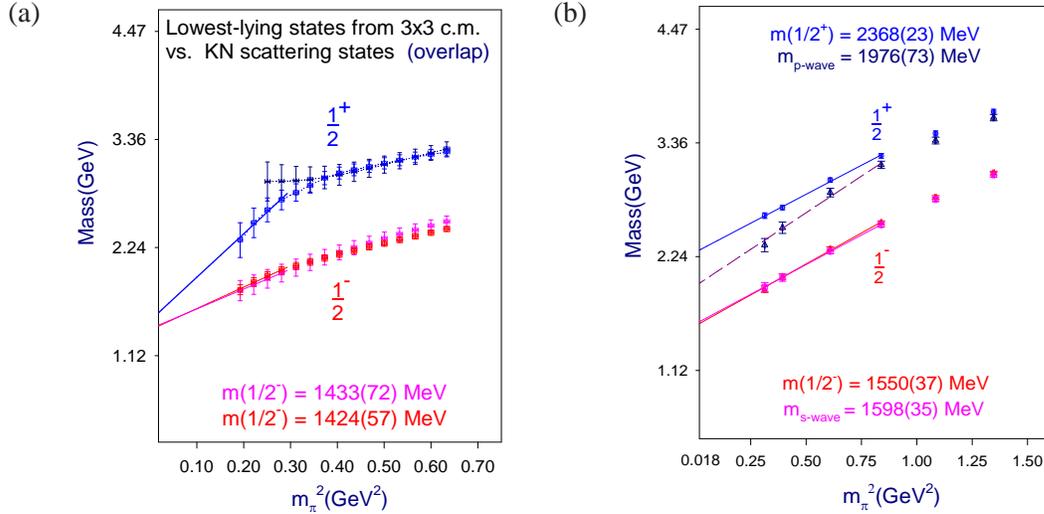


Figure 4: The lowest-lying states from 3×3 correlation matrix versus KN scattering states, for (a) overlap, and (b) Wilson fermions respectively.

3. Summary and Discussion

For both fermion schemes, the lowest-lying $1/2^-$ state agrees with the KN s-wave scattering state. However, for the lowest-lying $1/2^+$ state, it is different from any hadron scattering states with the same quantum numbers (i.e., KN p-wave, $KN\eta'$ s-wave, and KN^* s-wave) for the overlap fermion, while it seems to agree with KN^* s-wave for the Wilson fermion. Obviously, one cannot clarify the nature (resonance or scattering state) of any hadronic state by simply comparing its mass with those of scattering states. Further tests (e.g., volume test) are required to resolve the issues pertaining to the lowest $1/2^+$ state. In any case, it is vital to preserve exact chiral symmetry on

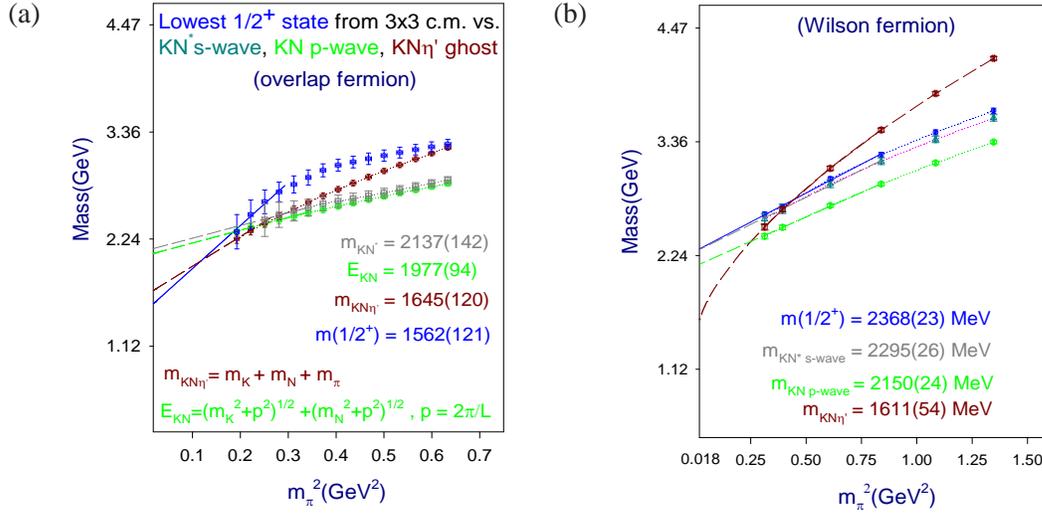


Figure 5: The lowest $1/2^+$ state from 3×3 correlation matrix versus KN^* s-wave, KN p-wave, and $KN\eta'$ s-wave, for (a) overlap, and (b) Wilson fermions respectively.

the lattice, otherwise, the diquark correlations (if any) could not be captured. In closing, we note that, so far, none of the exploratory studies in lattice QCD [1]-[9] has reached an unambiguous answer to the question *whether the spectrum of QCD possesses any resonance around 1540 MeV with $S = +1$.*

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