PROCEEDINGS OF SCIENCE

Combining Quark and Link Smearing to Improve Extended Baryon Operators

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The effects of Gaussian quark-field smearing and analytic stout-link smearing on the correlations of gauge-invariant extended baryon operators are studied. Gaussian quark-field smearing substantially reduces contributions from the short wavelength modes of the theory, while stout-link smearing significantly reduces the noise from the stochastic evaluations. The use of gauge-link smearing is shown to be crucial for baryon operators constructed of covariantly-displaced quark fields. Preferred smearing parameters are determined for a lattice spacing $a_s \sim 0.1$ fm.

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1. Introduction

One goal of the Lattice Hadron Physics Collaboration (LHPC) is to calculate the low-lying hadron spectrum in QCD[1]. Determining the spectrum requires extracting excited-state energies from our Monte Carlo computations, necessitating the evaluation of correlation matrices of sets of operators. To extract such energies, operators which couple strongly to the low-lying states of interest and weakly to the high-lying states must be used. The need for extended three-quark operators to capture both the radial and orbital structures of baryons has been emphasized and described in Ref. [1] (see Fig. 1).

For single-site (local) hadron operators, it is well known that the use of spatially-smeared quark fields is crucial. For extended baryon operators, one expects quark-field smearing to be equally important, but the relevance and interplay of link-field smearing is less well known. Thus, we decided that a systematic study of both quark-field and link-variable smearing was warranted.

2. The Smearing Procedures

Damping out couplings to the short-wavelength, high-momentum modes is the crucial feature which any effective smearing prescription[2, 3] must have. Gaussian suppression of the high-momentum modes is perhaps the simplest method one can use. Since a Gaussian in momentum space remains a Gaussian in coordinate space, we decided to employ a gauge-covariant smearing scheme[4] in which the smeared quark field is defined at a given site as a Gaussian-weighted average of the surrounding sites on the same time-slice:

$$\widetilde{\Psi}(\mathbf{x}) \sim \int d^3r \; e^{-\mathbf{r}^2/(4\sigma_s^2)} \; \Psi(\mathbf{x}+\mathbf{r}) \sim e^{+\sigma_s^2\Delta/4} \; \Psi(\mathbf{x}). \tag{2.1}$$

In practice, this expression must be approximated by

$$\tilde{\Psi}(x) = \left(1 + \frac{\sigma_s^2}{4n_\sigma}\Delta\right)^{n_\sigma} \Psi(x), \tag{2.2}$$

$$\Delta\Psi(x) = \sum_{k=\pm1,\pm2,\pm3} \left(U_k(x)\Psi(x+\hat{k}) - \Psi(x)\right), \tag{2.3}$$

where Δ denotes the three-dimensional gauge-covariant Laplacian. The two parameters to tune in this smearing procedure are the smearing radius σ_s and the integer number of iterations n_σ.

APE smearing[5] is the most commonly-used means of smearing the gauge-field link-variables. However, to avoid the abrupt projection back onto the gauge group needed by this prescription, we

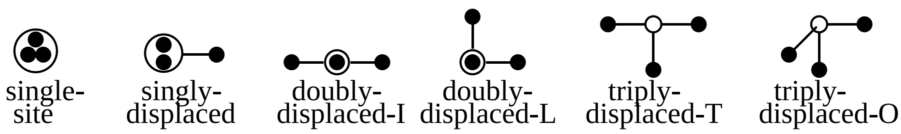

Figure 1: The spatial arrangements of the extended three-quark baryon operators. Smeared quark-fields are shown by solid circles, line segments indicate gauge-covariant displacements, and each hollow circle indicates the location of a Levi-Civita color coupling. For simplicity, all displacements have the same length in an operator. Results presented here used displacement lengths of $3a_s$ (~ 0.3 fm).

decided instead to use the analytic smearing scheme known as stout-link smearing[6] defined by

$$U \rightarrow U^{(1)} \rightarrow U^{(2)} \rightarrow \cdots \rightarrow U^{(n_\rho)}, \tag{2.4}$$

$$U_k^{(n+1)}(x) = \exp\left(i\rho\Theta_\mu^{(n)}(x)\right) U_k^{(n)}(x), \tag{2.5}$$

$$\Theta_k(x) = \frac{i}{2}\left(\Omega_k^\dagger(x) - \Omega_k(x)\right) - \frac{i}{2N}\mathrm{Tr}\left(\Omega_k^\dagger(x) - \Omega_k(x)\right) \tag{2.6}$$

$$\Omega_k(x) = C_k(x)U_k^\dagger(x) \qquad \text{(no summation over } k) \tag{2.7}$$

$$C_k(x) = \sum_{i\neq k}\left(U_i(x)U_k(x+\hat{\imath})U_i^\dagger(x+\hat{k}) + U_i^\dagger(x-\hat{\imath})U_k(x-\hat{\imath})U_i(x-\hat{\imath}+\hat{k})\right). \tag{2.8}$$

Only the spatial links are smeared, and $C_k(x)$ is a sum of spatial staples. The two parameters to tune in this smearing procedure are the staple weight ρ and the integer number of iterations n_ρ. The quark-field and link-field smearing schemes preserve the gauge invariance of hadron operators.

3. Systematic Study of the Smearing Procedures

In order to study the effects of the smearing procedures on the correlations of extended baryon operators, we first focused on three particular nucleon operators: a single-site operator O_{SS} in the G_{1g} irreducible representation of the cubic point group, a singly-displaced operator O_{SD} with a particular choice of each Dirac index, and a triply-displaced-T operator O_{TDT} with a specific choice of each Dirac index (see Fig. 1).

As usual, the effective mass associated with the correlation function $C_{ii}(t) = \langle O_i(t)\overline{O}_i(0)\rangle$ is defined by $M_i(t) = \ln\left(C_{ii}(t)/C_{ii}(t+a_t)\right)$. To compare the effectiveness of different values of the quark-field smearing parameters, we compared the effective mass $M_i(t = 4a_t)$ for each of the three operators at a particular temporal separation $t = 4a_t$. Results using 50 quenched configurations on a $12^3 \times 48$ anisotropic lattice using the Wilson action with $a_s \sim 0.1$ fm and $a_s/a_t \sim 3.0$ are shown in Fig. 2. Without gauge-link smearing, the displaced operators were found to be excessively noisy,

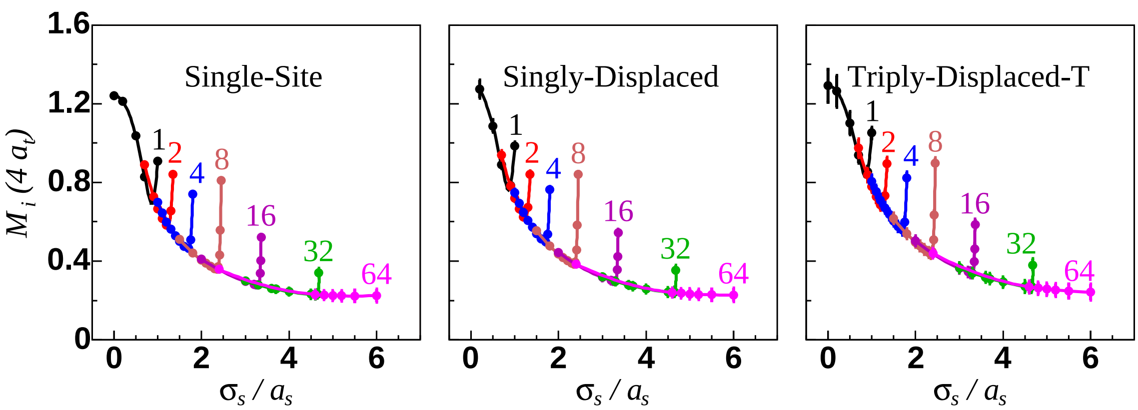

Figure 2: $M_i(4a_t)$ for the operators O_{SS}, O_{SD}, O_{TDT} against smearing radius σ_s for $n_\sigma = 1,2,4,8,16,32,64$. The gauge field is smeared using $n_\rho = 16$ and $n_\rho\rho = 2.5$. Results are based on 50 quenched configurations on a $12^3 \times 48$ anisotropic lattice using the Wilson action with $a_s \sim 0.1$ fm and $a_s/a_t \sim 3.0$. The quark mass is such that the mass of the pion is approximately 700 MeV.

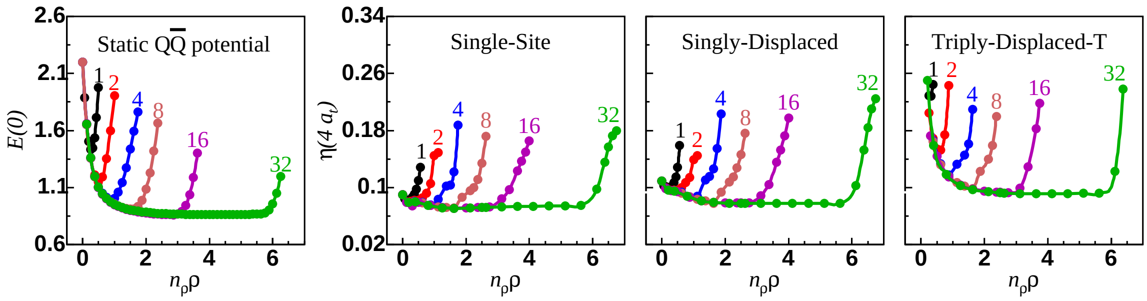

Figure 3: Leftmost plot: the effective mass $E(0)$ for $t = 0$ corresponding to the static quark-antiquark potential at spatial separation $R = 5a \sim 0.5$ fm against $n_\rho \rho$ for $n_\rho = 1, 2, 4, 8, 16, 32$. Results are based on 100 configurations on a 16^4 isotropic lattice using the Wilson gauge action with $\beta = 6.0$. Right three plots: the relative jackknife error $\eta(4a_t)$ of effective masses $M_i(4a_t)$ of the three nucleon operators O_{SS}, O_{SD}, O_{TDT} for $n_\sigma = 32$, $\sigma_s = 4.0$ against $n_\rho \rho$ for $n_\rho = 1, 2, 4, 8, 16, 32$. Results are based on 50 quenched configurations on a $12^3 \times 48$ anisotropic lattice using the Wilson action with $a_s \sim 0.1$ fm, $a_s/a_t \sim 3.0$.

making a meaningful comparison impossible. For this reason, the results shown in Fig. 2 include gauge-field smearing with $n_\rho = 16$ and $n_\rho \rho = 2.5$. One sees that $M_i(t = 4a_t)$ is independent of n_σ for sufficiently small σ_s. For each value of n_σ, $M_i(t = 4a_t)$ first decreases as σ_s is increased, until the approximation to a Gaussian eventually breaks down, signaled by a sudden rapid rise in $M_i(t = 4a_t)$. This rapid rise occurs at larger values of σ_s for larger values of n_σ.

Next, we studied the effect of changing the gauge-field smearing parameters. First, the effective mass $E(0)$ associated with the static quark-antiquark potential at a spatial separation $R = 5a_s \sim 0.5$ fm and at a particular temporal separation $t = 0$ was used to compare the effectiveness of different values of ρ and n_ρ. The results are shown in the leftmost plot in Fig. 3. The behavior is qualitatively similar to that observed in Fig. 2. One sees that the $t = 0$ effective mass is independent of the product $n_\rho \rho$ for sufficiently small values of $n_\rho \rho$. For each value of n_ρ, $E(0)$ decreases as $n_\rho \rho$ increases, until a minimum is reached and a rapid rise occurs. The onset of the rise occurs at larger values of $n_\rho \rho$ as n_ρ increases. Note that $E(0)$ does not decrease appreciably as $n_\rho \rho$ increases above 2.5. Hence, $n_\rho \rho = 2.5$ with $n_\rho = 16$ are our preferred values for the link smearing at lattice spacing $a_s \sim 0.1$ fm, based on the static quark-antiquark potential.

Somewhat surprisingly, we found that changing the link-smearing parameters did not appreciably affect the mean values of the effective masses of our three nucleon operators. However, the effect on the variances of the effective masses was dramatic. The relative jackknife error $\eta(4a_t)$ of $M(4a_t)$ is shown against $n_\rho \rho$ in the right three plots in Fig. 3, and amazingly, this error shows the same qualitative behavior as in Fig. 2 and the leftmost plot in Fig. 3. One key point learned here is that the preferred link-smearing parameters determined from the static quark-antiquark potential produce the smallest error in the extended baryon operators.

The effective masses shown in Fig. 4 also illustrate these findings. The top row shows that applying only quark-field smearing to the three selected nucleon operators significantly reduces couplings to higher-lying states, but the displaced operators remain excessively noisy. The second row illustrates that including only link-field smearing substantially reduces the noise, but does not appreciably alter the effective masses themselves. The bottom row shows dramatic improvement

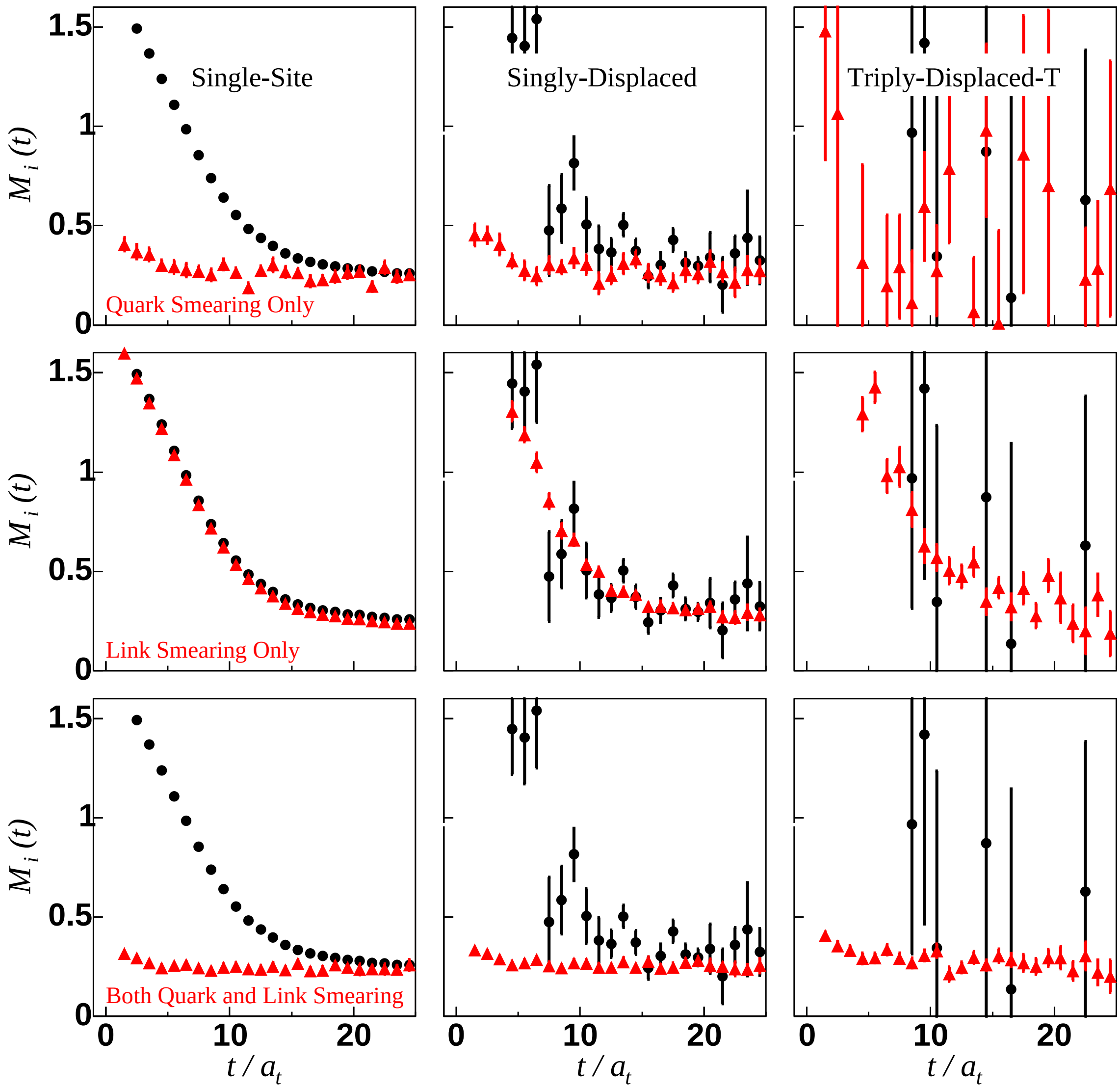

Figure 4: Effective masses $M(t)$ for unsmeared (black circles) and smeared (red triangles) operators O_{SS}, O_{SD}, O_{TDT}. Top row: only quark-field smearing $n_\sigma = 32$, $\sigma_s = 4.0$ is used. Middle row: only link-variable smearing $n_\rho = 16$, $n_\rho \rho = 2.5$ is applied. Bottom row: both quark and link smearing $n_\sigma = 32$, $\sigma_s = 4.0$, $n_\rho = 16$, $n_\rho \rho = 2.5$ are used, dramatically improving the signal for all three operators. Results are based on 50 quenched configurations on a $12^3 \times 48$ anisotropic lattice using the Wilson action with $a_s \sim 0.1$ fm, $a_s/a_t \sim 3.0$.

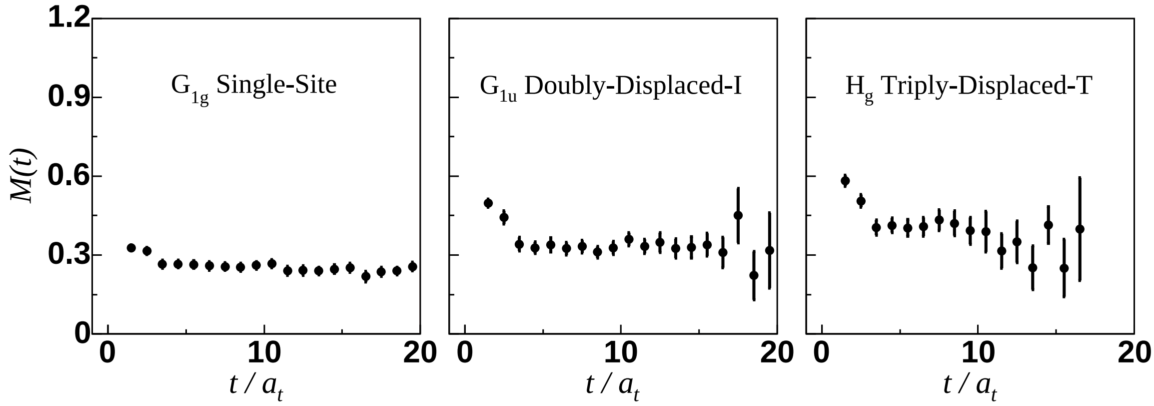

Figure 5: Effective masses for three selected nucleon operators: a single-site operator in the G_{1g} channel (left), a doubly-displaced-I operator in the G_{1u} channel (center), and a triply-displaced-T operator in the H_g channel. The smearing parameters used were $n_\sigma = 32,\ \sigma_s = 4.0,\ n_\rho = 32,\ n_\rho \rho = 2.5$. Results are based on 25 quenched configurations on a $12^3 \times 48$ anisotropic lattice using the Wilson action with $a_s \sim 0.1$ fm and $a_s/a_t \sim 3.0$. We have used opposite-parity time-reversed averaging [1] to increase statistics.

from reduced couplings to excited states and dramatically reduced noise when both quark-field and link-field smearing is applied, especially for the extended operators. The effectiveness of the smearing schemes used is further illustrated in Fig. 5.

4. Conclusion

Incorporating both quark-field and link-variable smearing is crucial for extracting the baryon spectrum using gauge-invariant extended three-quark operators. Gaussian quark-field smearing dramatically diminishes couplings to the short wavelength modes of the theory, whereas stout-link smearing drastically reduces the noise in operators with displaced quarks. Preferred smearing parameters $\sigma_s = 4.0,\ n_\sigma = 32,\ n_\rho \rho = 2.5,\ n_\rho = 16$ were found for a lattice spacing $a_s \sim 0.1$ fm and were independent of the baryon operators chosen. Two issues which remain are to determine the effects of smearing on the low-lying excited-states, and to determine the dependence of the preferred smearing parameters on the quark mass. This work was supported by the U.S. National Science Foundation through grants PHY-0099450 and PHY-0300065, and by the U.S. Department of Energy under contracts DE-AC05-84ER40150 and DE-FG02-93ER-40762.

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