

Operator product expansion and quark condensate from LQCD in coordinate space

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We perform an exploratory study of the operator product expansion of the quark propagator on the lattice at short distance in coordinate space. This permits a simple determination of the quark condensate, $\langle \bar{\psi}\psi \rangle^{\overline{\text{MS}}}(2\text{GeV}) = -(265 \pm 5_{\text{stat}} \pm 22_{\text{syst}} \text{ MeV})^3$, and of the renormalization constant of the quark field, $Z_{\psi}^{\overline{\text{MS}}}(2\text{GeV}) = 0.871 \pm 0.003_{\text{stat}} \pm 0.020_{\text{syst}}$. This new method also provides a remarkable non-perturbative test of the OPE predictions at short distance in QCD.

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1. Introduction

The chiral quark condensate (QC) $\langle \bar{\psi}\psi \rangle$ plays a central role in the non-perturbative sector of QCD. It is expected to be the order parameter which controls the spontaneous chiral symmetry breaking and sets the scale of the pion masses, provided its value is non-vanishing and its order of magnitude appropriate. To verify these basic requirements, high precision estimates of the QC, though welcome, are not necessary.

At the same time there is no easy experimental access to the QC, as some theoretical assumptions always enter. This suggests to look for different methods based on independent hypothesis which, to some extent and indirectly, can be tested in this way. Two examples are the classical measures of the QC extracted from the pion scattering length through chiral perturbation theory [1] or from the nucleon and $B^* - B$ mass-splitting sum rules [2].

On the other hand lattice QCD can give an estimate of the QC based on first principles. The simplest approach to compute the QC on the lattice uses the GMOR formula, which is based on the axial chiral Ward identity [3]. Alternative determinations of the quark condensate on the lattice have been obtained in the framework of the ε -expansion of QCD in a small volume [4] and from the study of the Goldstone pole contribution to the pseudoscalar quark Green function [5].

The method proposed in [6] and summarized in this talk starts from the operator product expansion of the quark propagator at short euclidean distance:

$$S(x^2) = \frac{1}{2\pi^2} C_I(x^2) \frac{\not{x}}{(x^2)^2} + \frac{1}{4\pi^2} C_m(x^2) \frac{m}{x^2} - \frac{1}{4N_C} C_{\bar{\psi}\psi}(x^2) \langle \bar{\psi}\psi \rangle + \dots \quad (1.1)$$

The quark propagator on the l.h.s. is computed on the lattice and the QC is extracted fitting the r.h.s. in the chiral limit. Our final result reads $\langle \bar{\psi}\psi \rangle^{\overline{\text{MS}}}(2\text{GeV}) = -(265 \pm 23 \text{MeV})^3$.

The idea is simple and provides a non-perturbative test of the hypothesis behind, namely the validity of the operator product expansion in QCD. The precision achieved in this exploratory study is not high, but can be improved in time, being mainly computer-limited. However the method itself is flexible and powerful: in principle, from the fit of eq. (1.1), the quark masses and the gluon condensate which appears at $\mathcal{O}(x^2)$ can be obtained too. Another advantage is that the renormalization procedure is rather simple: once the quark propagator is renormalized, its operator product expansion is expressed in terms of renormalized quantities too. The same holds for the $O(a)$ improvement.

2. Lattice simulation

We generated 180 gauge configurations in the quenched approximation with the $O(a)$ -improved Wilson action, on a volume $32^3 \times 70$ at $\beta = 6.45$. This value of the coupling corresponds to an inverse lattice spacing $a^{-1} = 3.87(19)\text{GeV}$, evaluated from the K and K^* meson masses with the lattice plane method. The simulated values of the hopping parameter are $\kappa = 0.1349, 0.1351, 0.1352, 0.1353$ and correspond to light quark masses in the range $m_s/2 \lesssim m \lesssim m_s$. The values of the renormalized masses used in the chiral extrapolations and in eq. (1.1) have been taken from [7]. The statistical errors are computed with the jackknife technique. The quantities directly evaluated

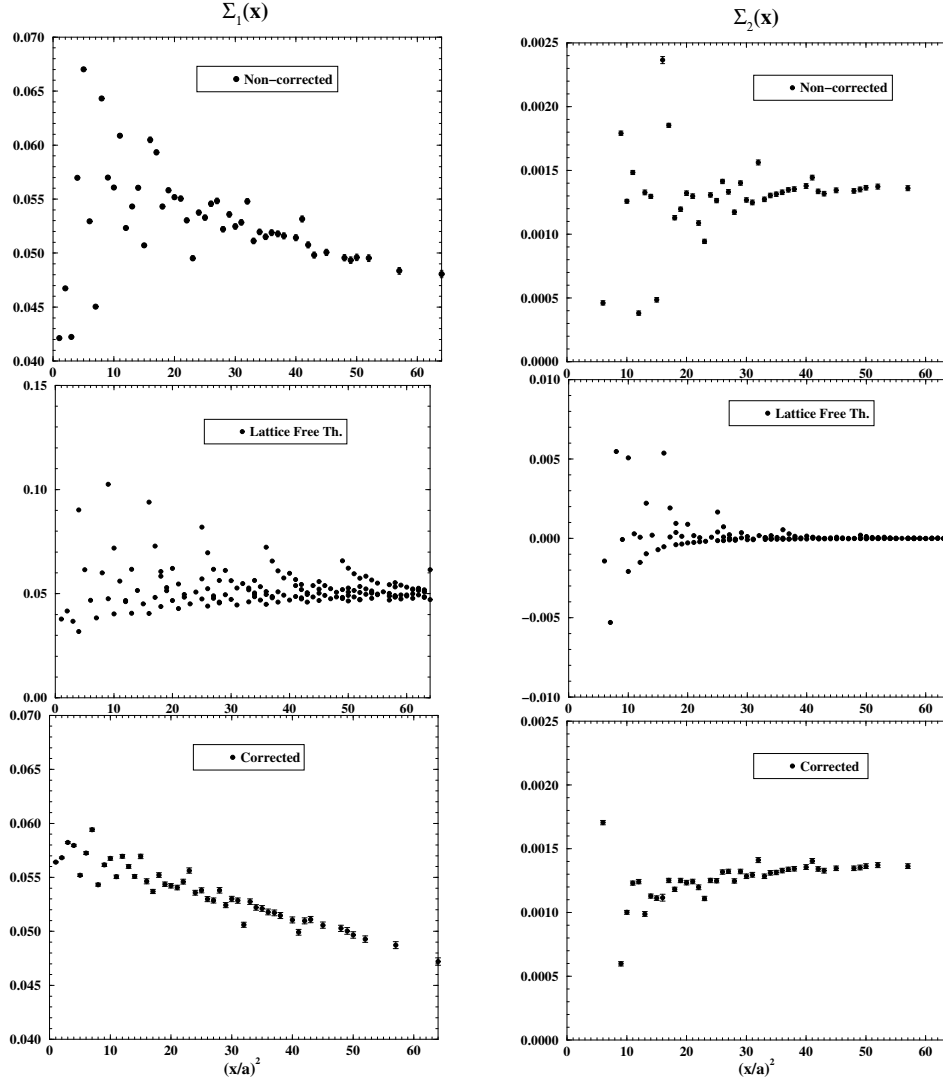


Figure 1: The bare form factors Σ_1 (left panels) and Σ_2 (right panels). From top to bottom: form factors in the interacting theory at $k=0.1349$; in the free lattice theory at infinite volume and in the chiral limit; corrected form factors (as defined in the text).

from the numerical simulation are the scalar form factors of the quark propagator, defined from

$$S(x^2) = \frac{x}{(x^2)^2} \Sigma_1(x^2) + \frac{1}{x^2} \Sigma_2(x^2). \quad (2.1)$$

We have calculated the Wilson coefficients $C_i(x^2)$ in eq. (1.1) at $\mathcal{O}(\alpha_s)$ and resummed at the NLO in the $\overline{\text{MS}}$ scheme. The NLO expressions of the coefficients and of the evolution functions can be found in [6]. From here on, x indicates the euclidean distance in lattice units squared, $(x/a)^2$.

The scalar form factors $\Sigma_{1,2}(x, \mu)$ are calculated in the Landau gauge and shown in fig. 1 (top) for $\kappa = 0.1349$ as function of x . The points show fish-bone curves due to the spread of the results obtained from different lattice sites which correspond to the same x^2 in the continuum limit, especially at short distance. Similar patterns can be observed in the form factors computed in the free lattice theory and plotted in fig. 1(center). This suggests we are observing lattice artifacts. We

can reduce the discretization errors from $\mathcal{O}(a^2)$ to $\mathcal{O}(\alpha_s a^2)$ by correcting the lattice data with the lattice artifacts evaluated in the free theory. In detail, we define

$$\Sigma_1^{corr}(x) = \left(\frac{\Sigma_{1,free}^{cont}(x)}{\Sigma_{1,free}^{lat}(x)} \right) \Sigma_1(x) \quad \Sigma_{1,free}^{cont}(x) = \frac{1}{2\pi^2}, \quad (2.2)$$

$$\Sigma_2^{corr}(x) = \Sigma_2(x) - \Sigma_{2,free}^{lat}(x) \quad \Sigma_{2,free}^{cont}(x) = 0, \quad (2.3)$$

The result of the correction is shown in fig. 1(bottom). These are the data actually used in the extraction of the QC.

3. Renormalization

As anticipated, we need to compute only the renormalization constant of the quark field. For this purpose, we use the non-perturbative renormalization scheme named X-scheme in [8]- [9] and defined by the condition

$$Z_\psi^X(\mu = 1/x) \Sigma_1(x) = \Sigma_{1,free}^{cont}(x) \quad (3.1)$$

in the Landau gauge and in the chiral limit. From eqs.(1.1) and (2.1),

$$\Sigma_1(x) = \frac{1}{2\pi^2} C_I(x) + \dots \quad (3.2)$$

In order to reach the chiral limit, linear and quadratic chiral extrapolations have been considered and the differences are included in the systematics. The result for $Z_\psi^X(\mu = 1/x)$ as obtained from eq. (3.1) is shown in fig. 2. The final estimate of the renormalization constant is obtained from a constant fit in the range [9,25]:

$$Z_\psi^X(2\text{GeV}) = 0.871 \pm 0.003_{stat} \pm 0.020_{syst} \quad (3.3)$$

This is also the value of $Z_\psi^{\overline{\text{MS}}}(2\text{GeV})$, in the Landau gauge. A fit which includes a long distance region, $x \in [9, 40]$, has been also performed in order to evaluate the systematics.

4. Determination of the quark condensate

From eqs.(1.1) and (2.1)

$$\Sigma_2^{\overline{\text{MS}}}(x, \mu) = \frac{1}{4\pi^2} C_m^{\overline{\text{MS}}}(x, \mu) m^{\overline{\text{MS}}}(\mu) - \frac{1}{4N_c} C_{\bar{\psi}\psi}^{\overline{\text{MS}}}(x, \mu) \langle \bar{\psi}\psi \rangle^{\overline{\text{MS}}}(\mu) x^2 + \dots \quad (4.1)$$

Therefore in the chiral limit the QC represents the leading term in the OPE of $\Sigma_2(x, \mu)$. A linear and a quadratic extrapolations have been performed in order to evaluate the systematics.

In order to identify directly the QC as the intercept, the fits in x are performed on the normalized quantity

$$-\frac{(\Sigma_2^{\overline{\text{MS}}}(x, \mu))^{\text{chiral}}}{C_{\bar{\psi}\psi}(x, \mu) x^2 / 4N_c} = \langle \bar{\psi}\psi \rangle^{\overline{\text{MS}}}(\mu) + \mathcal{O}(x^2) \quad (4.2)$$

Some results are collected in table 1 and the result of a constant fit in [9,25] is shown in fig. 3.

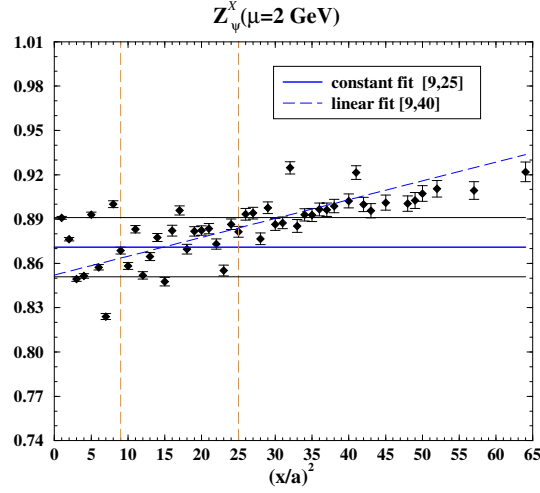


Figure 2: Values of $Z_{\psi}^X(\mu = 2\text{ GeV})$ for different values of $x \equiv (x/a)^2$. The solid lines indicate the results obtained from a constant fit in x and the estimated systematic error. The dashed vertical lines show the x -range where the constant fit is performed. The result of a linear fit is also shown (dashed line); the estimate of $Z_{\psi}^X(\mu = 2\text{ GeV})$ is given by the intercept.

Constant fit [9,25]	Linear fit [9,40]
$-(265 \pm 5)^3$	—
$-(266 \pm 4)^3$	$-(265 \pm 7)^3$

Table 1: Values of the chiral quark condensate in the $\overline{\text{MS}}$ scheme at the scale $\mu = 2\text{ GeV}$.

As a check of the result we also extract the QC from $\Sigma_2(x, \mu)$ by inverting the order of the fits: first, the fit in x including the mass term, then the chiral fit. We obtain a perfect agreement of the results with the central values given in table 1. We note, however, that in order to stabilize the results of the fit, we have not been able to treat the quark masses as free parameters, but their values have been kept fixed to the ones determined in ref.[7]. Averaging the results and adding the systematic error we obtain

$$\langle \bar{\psi}\psi \rangle^{\overline{\text{MS}}}(2\text{ GeV}) = -(265 \pm 5_{\text{stat}} \pm 22_{\text{sys}} \text{ MeV})^3. \quad (4.3)$$

The systematic uncertainty amounts to 26%, the main contribution being the spread of the points in the fitting regions (18%). The reason is that the fitting intervals have been chosen in the region $x \in [9, 40]$ in order to satisfy the condition

$$1 < \sqrt{x} \lesssim 1/(a\Lambda_{\text{QCD}}) \quad x \equiv \left(\frac{x}{a}\right)^2 \quad (4.4)$$

where the discretization errors are under control (lower limit) and the perturbative calculation of the coefficients at a scale $\mu \sim 1/x$ is reliable (upper limit). But at the value of the lattice spacing considered in the present simulation, this region is small and affected by non-negligible discretization effects, even after the correction of the lattice artifacts described in sec.2 has been applied.

We estimate that neglecting higher order terms in the OPE introduce an uncertainty of $\sim 5\%$. All the other sources of systematic error are due to the lattice uncertainties: 15% the error due

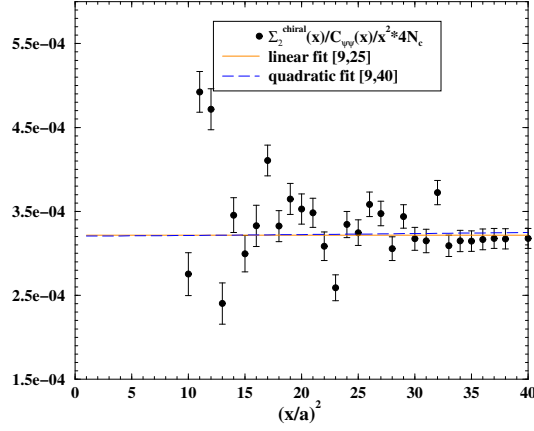


Figure 3: Values of $\frac{(\Sigma_2^{\overline{\text{MS}}}(x,\mu))^{\text{chiral}}}{C_{\psi\psi}(x,\mu)x^2/4N_c} = -\langle\bar{\psi}\psi\rangle^{\overline{\text{MS}}}(\mu) + \mathcal{O}(x^2)$ as a function of $x \equiv (x/a)^2$. A constant and a linear fit in x are shown. The absolute value of the quark condensate is given by the intercept.

to the determination of the lattice spacing, 10% the quenched estimate of Λ_{QCD} , 9% the use of linear/quadratic chiral extrapolations, 5% the choice of the fitting interval, <1% the error on renormalization constant of the quark field and the use of vector or axial definitions of the quark masses. We could not quantify the finite volume effects, which should be small in the fitting regions (see [8] for a study in the free theory), and, above all, the quenching approximation.

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