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A study of the N to Δ transition form factors in full QCD

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The N to Δ transition form factors G_{M1}, G_{E2} and G_{C2} are evaluated using dynamical MILC configurations and valence domain wall fermions at three values of quark mass corresponding to pion mass 606 MeV, 502 MeV and 364 MeV on lattices of spatial size 20^3 and 28^3. The unquenched results are compared to those obtained at similar pion mass in the quenched theory.

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1. Introduction

Non-zero quadrupole strength in the N to Δ transition is interpreted as a signal for deformation in the nucleon and/or Δ. Hadron deformation is a fundamental property that depends on QCD dynamics and determining the shape of the nucleon has motivated experimental measurements of quadrupole form factors in the N to Δ transition for many years. Selection rules in the N to Δ transition allow for a magnetic dipole, M1, an electric quadrupole, E2, and a Coulomb quadrupole, C2, with the corresponding transition form factors G_{M1}, G_{E2} and G_{C2} as functions of the four-momentum transfer squared, q^2. The magnetic transition, which proceeds through a quark spin-flip, is expected to be the dominant one while non-zero quadrupole strengths can be obtained in many models by assuming a deformed nucleon and/or Δ as for example in the quark model where a D-wave admixture is assumed in the nucleon and Δ wave functions. Pion electro- and photo-production experiments near the Δ(1232) resonance have clearly established non-zero quadrupole strength in the range of a few percent with respect to the dominant M1 strength providing support for deformation [1]. In ref. [2], a precise measurement at $-q^2 = 0.127\,\mathrm{GeV}^2$ extracted the values

$$\mathrm{R_{EM}} \equiv -\frac{\mathrm{G_{E2}}}{\mathrm{G_{M1}}} = (-2.3 \pm 0.3 \pm 0.6)\,\%, \qquad \mathrm{R_{SM}} \equiv -\frac{|\mathbf{q}|}{2m_\Delta}\frac{\mathrm{G_{C2}}}{\mathrm{G_{M1}}} = (-6.1 \pm 0.2 \pm 0.5)\,\% \tag{1.1}$$

for the ratio of the electric and coulomb quadrupole to the magnetic dipole form factors, R_{EM} (or EMR) and R_{SM} (or CMR). The first uncertainty given in Eq. (1.1) is the statistical and systematic error and the second the model error. At the same time experiments at Jefferson Lab have studied the q^2 dependence of these three form factors in the range of $0.4 - 1.8\,\mathrm{GeV}^2$ [3].

Within lattice QCD one can evaluate these transition form factors starting directly from the QCD Lagrangian. The first lattice study of the N to Δ transition was carried out very early on [4] and, although the quadrupole form factors were not determined to high enough accuracy, the methodology for the calculation of this matrix element on the lattice was presented. Using the formalism laid out in Ref. [4] a more detailed study followed in Ref. [5] which established the existence of small and negative electric quadrupole strength for a given momentum transfer **q** within quenched QCD. In subsequent work [6, 7] we focus on the determination of the momentum dependence of the transition form factors. We carry out a number of improvements that enables us to extract both electric and coulomb quadrupole form factors to high enough accuracy to exclude a zero value. The improvements come from optimizing the interpolating field used for the Δ so that the three-point function is evaluated for the maximum allowed values of lattice momentum vectors for a given q^2 and from using a simultaneous overconstrained analysis of the resulting matrix elements. We obtain small negative values for the ratios R_{EM} and R_{SM} in qualitative agreement with the experimental measurements for values of $-q^2$ in the range 0.4 to 1.5 GeV^2. On the other hand, the values obtained for R_{SM} are smaller than experiment for $-q^2 < 0.4$ GeV^2. Similarly the values obtained for G_{M1} in this quenched study, after linearly extrapolated to the chiral limit, disagree with the experimental measurements raising questions about the importance of sea quark effects and the validity of the linear extrapolation of the lattice data. Recently a relativistic chiral effective-field theory calculation of the R_{EM} and R_{SM} ratios [8] shows that the linear extrapolation is incorrect at low q^2 and a stronger dependence on the pion mass sets in at a pion mass value where the Δ becomes unstable. This stronger mass dependence drives R_{SM} to more negative values resolving the discrepancy between lattice results and experiment at low q^2. This study explicitly demonstrates the importance of pion cloud contributions in the determination of these transition form factors.

In this work we attempt to study the effects of unquenching on the three transition form factors using dynamical MILC configurations for the sea quarks and employing domain wall valence fermions. Given the fact that chiral fermions are still very expensive to simulate this "hybrid" scheme appears as a reasonable compromise and has already been employed in heavy quark spectroscopy and generalized parton distributions calculations [9].

2. Lattice techniques

The matrix element for the N to Δ electromagnetic transition for on-shell nucleon and Δ states and real or virtual photons has the form [10]

$$\langle \Delta(p',s') \,|j_\mu|\, N(p,s)\rangle = i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta\, m_N}{E_\Delta(\mathbf{p}')\, E_N(\mathbf{p})}\right)^{1/2} \bar{u}_\tau(p',s')\mathscr{O}^{\tau\mu}u(p,s) \tag{2.1}$$

where p,s and p',s' denote initial and final momenta and spins and $u_\tau(p',s')$ is a spin-vector in the Rarita-Schwinger formalism. The operator $\mathscr{O}^{\tau\mu}$ can be decomposed in terms of the Sachs form factors as

$$\mathscr{O}^{\tau\mu} = G_{M1}(q^2)K^{\tau\mu}_{M1} + G_{E2}(q^2)K^{\tau\mu}_{E2} + G_{C2}(q^2)K^{\tau\mu}_{C2}\ , \tag{2.2}$$

where the exact expressions for the kinematical functions $K^{\tau\mu}$ can be found in ref. [5]. The extraction of the Sachs form factors requires the computation of the three-point function $G^{\Delta j^\mu N}_{\sigma}(t_2,t_1;\mathbf{p}\,',\mathbf{p};\Gamma)$ along with the nucleon and Δ two-point functions G^{NN} and $G^{\Delta\Delta}_{ij}$. The nucleon source is taken at time zero, the photon is absorbed by a quark at a later time t_1 and the Δ sink is at an even later time t_2. Provided the Euclidean time separations t_1 and t_2-t_1 are large enough, the time dependence and field renormalization constants will cancel in the ratio

$$R_\sigma(t_2,t_1;\mathbf{p}\,',\mathbf{p}\,;\Gamma;\mu) = \frac{\langle G^{\Delta j^\mu N}_{\sigma}(t_2,t_1;\mathbf{p}\,',\mathbf{p};\Gamma)\rangle}{\langle G^{\Delta\Delta}_{ii}(t_2,\mathbf{p}\,';\Gamma_4)\rangle}\times$$
$$\left[\frac{\langle G^{NN}(t_2-t_1,\mathbf{p};\Gamma_4)\rangle\,\langle G^{\Delta\Delta}_{ii}(t_1,\mathbf{p}\,';\Gamma_4)\rangle\,\langle G^{\Delta\Delta}_{ii}(t_2,\mathbf{p}\,';\Gamma_4)\rangle}{\langle G^{\Delta\Delta}_{ii}(t_2-t_1,\mathbf{p}\,';\Gamma_4)\rangle\,\langle G^{NN}(t_1,\mathbf{p};\Gamma_4)\rangle\,\langle G^{NN}(t_2,\mathbf{p};\Gamma_4)\rangle}\right]^{1/2} \xrightarrow{t_2-t_1\gg 1,\,t_1\gg 1} \Pi_\sigma(\mathbf{p}\,',\mathbf{p}\,;\Gamma;\mu)\,, \tag{2.3}$$

where σ denotes the vector index of the Δ field. The matrices Γ are projections for the Dirac indices

$$\Gamma_i = \frac{1}{2}\begin{pmatrix}\sigma_i & 0\\ 0 & 0\end{pmatrix}\ ,\quad \Gamma_4 = \frac{1}{2}\begin{pmatrix} I & 0\\ 0 & 0\end{pmatrix}\ . \tag{2.4}$$

We fix the Δ at rest and therefore $\mathbf{q}=\mathbf{p}'-\mathbf{p}=-\mathbf{p}$. $Q^2=-q^2$ is the Euclidean momentum transfer squared. Determining $\Pi_\sigma(\mathbf{q}\,;\Gamma;\mu)$ for given values of σ and Γ by fitting to the plateau of $R_\sigma(t_2,t_1;\mathbf{p}\,',\mathbf{p}\,;\Gamma;\mu)$ enables us to obtain the Sachs form factors. For example,

$$\Pi_\sigma(\mathbf{q}\,;\Gamma_4;\mu) = iA\varepsilon^{\sigma 4\mu j}p^j G_{M1}(Q^2) \quad , \tag{2.5}$$

with A a kinematical coefficient. Other similar relations can be found in Ref. [6]. A novel feature of Ref. [7] is the choice of linear combination of ratios $R_\sigma(t_2,t_1;\mathbf{p}\,',\mathbf{p}\,;\Gamma;\mu)$ for appropriate values of (σ,Γ) such that, for given Q^2, a maximum number of photon momentum transfers $\mathbf{q}$ give non-vanishing contributions. The optimal combination for G_{M1}, referred to as S_1-type, is

$$S_1(\mathbf{q};\mu) = \sum_{\sigma=1}^{3} \Pi_\sigma(\mathbf{q}\,;\Gamma_4;\mu) = iA\left[(p_2-p_3)\delta_{1,\mu}+(p_3-p_1)\delta_{2,\mu}+(p_1-p_2)\delta_{3,\mu}\right]G_{M1}(Q^2) \quad (2.6)$$

where all the vectors $\mathbf{q}$ related through lattice rotations participate. Similarly, the quadrupole form factors are obtained from type S_2 given by

$$\begin{aligned} S_2(\mathbf{q};\mu) &= \sum_{\sigma\neq k=1}^{3} \Pi_\sigma(\mathbf{q}\,;\Gamma_k;\mu) \\ &= B\Bigg\{\left[(p_2+p_3)\delta_{1,\mu}+(p_3+p_1)\delta_{2,\mu}+(p_1+p_2)\delta_{3,\mu}\right]G_{E2}(Q^2) \\ &- 2\frac{p_\mu}{\mathbf{p}^2}(p_1p_2+p_1p_3+p_2p_3)\left[G_{E2}(Q^2)+\frac{E_N-m_\Delta}{2m_\Delta}G_{C2}(Q^2)\right]\Bigg\}\,, \end{aligned} \quad (2.7)$$

for $\mu=1,2,3$. For $\mu=4$ we have

$$S_2(\mathbf{q};\mu=4) = \frac{6\,C}{\mathbf{p}^2}(p_1p_2+p_1p_3+p_2p_3)G_{C2}(Q^2)\,, \quad (2.8)$$

where B and C are kinematical factors. The three-point functions needed for the evaluation of S_1 and S_2 are obtained by fixing the appropriate combination of interpolating fields for the Δ and require only *one sequential* inversion for each fixed sink type. Therefore, two independent sequential inversions suffice for the complete evaluation of the form factors for all available lattice momenta $\mathbf{q}$. The full set of lattice measurements for any available μ and $\mathbf{q}$ for a given Q^2 are simultaneously analyzed and the form factors are extracted from a global χ^2-minimization procedure [7].

The dynamical configurations that we are using are generated by the MILC collaboration and are available from the NERSC archive. For the lattice of volume $20^3\times 64$ we use the ensembles with the strange quark mass fixed at $am_s=0.05$ and the light quark flavors at $am_{u,d}=0.03$ and 0.02 simulated with the improved staggered (Asqtad) action which ensures better scaling properties at the lattice spacing of $a=0.125$ fm used in the simulations. In addition we use configurations generated on a lattice of size $28^3\times 64$ i.e. physical spatial volume of $(3.5\text{ fm})^3$ at $am_s=0.05$ and $am_{u,d}=0.01$. We apply hypercubic (HYP) smearing to the configurations and use Domain wall fermions in the valence sector. The size of the fifth dimension is set to $L_5=16$ where it has been shown [9] that, for these lattices, the residual quark mass, computed from the divergence of the five-dimensional axial current, is at least an order of magnitude smaller than the bare mass. The height of the domain wall parameter is set at $am_0=1.7$ and Dirichlet boundary conditions are imposed at $t/a=32$ since only half the time extent of the lattice is needed. Smeared quark wave functions are known to increase the overlap of the interpolating fields with the hadronic states reducing the contamination from excited states. Before the calculation of the valence propagators we apply gauge invariant Wuppertal smearing to the local quark fields $q(x)$ via $q_{\text{smear}}(\mathbf{x},t)=\sum_{\mathbf{y}}(1+\alpha H)^n(\mathbf{x},\mathbf{y})q(\mathbf{y},t)$ with H the hopping matrix, $\alpha\sim 3$ and $n=30$.

The bare Domain wall quark mass parameter $(am_q)^{DW}$ has been tuned by the LHP collaboration [9] by requiring that the pion mass computed with the Domain wall valence quarks equals the mass of the lightest pion computed with the staggered fermions. The resulting quark mass parameters and corresponding pion masses obtained by the LHP collaboration are shown in Table 1. In the same Table we also give our values for the nucleon and Δ masses. As can be seen the mass of the Δ carries the largest errors.

Volume	$(am_{u,d})^{\rm sea}$	$(am_s)^{\rm sea}$	$(am_q)^{DW}$	m_π^{DW} (GeV)	m_π/m_ρ	m_N (GeV)	m_Δ (GeV)
$20^3\times 32$	0.03	0.05	0.0478	0.606(2)	0.588(7)	1.392(9)	1.662(21)
$20^3\times 32$	0.02	0.05	0.0313	0.502(4)	0.530(11)	1.255(19)	1.586 (36)
$28^3\times 32$	0.01	0.05	0.0138	0.364(1)	0.387(7)	1.196(25)	1.643(63)

Table 1: Mass parameters for the sea (Asqtad) and valence (DW) quarks and corresponding meson masses taken from [9]. In the last two columns we give our values for the nucleon and Δ mass using 125 configurations for $am_{u,d}=0.03$, 75 for $am_{u,d}=0.02$ and 38 for $am_{u,d}=0.01$.

3. Results and outlook

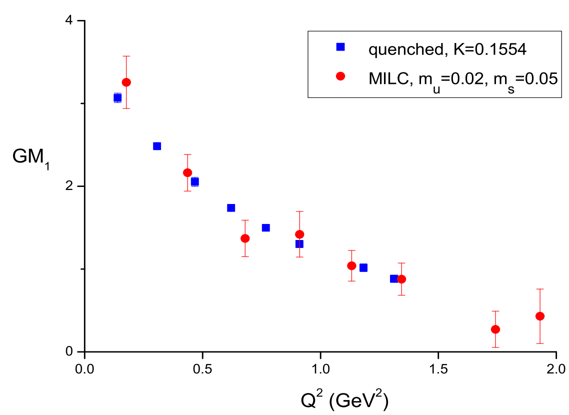

Figure 1: G_{M1} in units of $(e/2m_N)$ in quenched and $N_f=2+1$ QCD at $m_\pi\sim 0.5$ GeV.

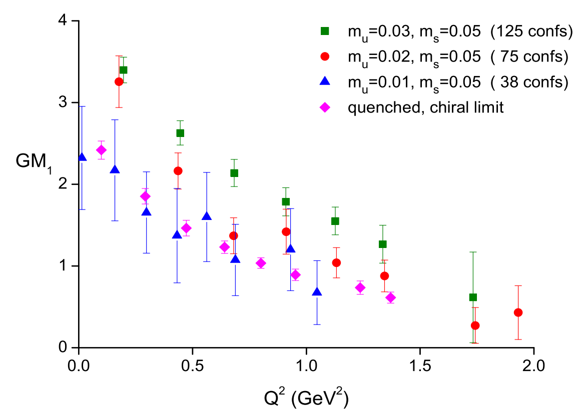

Figure 2: G_{M1} in units of $(e/2m_N)$ using MILC configurations for three quark masses and quenched results at the chiral limit.

For the $20^3\times 32$ volume we analyzed 125 configurations at $am_{u,d}=0.03$ and 75 at $am_{u,d}=0.02$ using both S_1 and S_2 types of interpolating fields. This enable us to extract the dipole and quadrupole form factors. In addition, we analyzed 38 configurations on the $28^3\times 32$ at $am_{u,d}=0.01$ lattice using just the S_1-type, providing a first indication of the magnetic dipole form factor behavior at a lighter pion mass. We employ the local current operator $\bar\psi(x)\gamma_\mu\psi(x)$, which is not conserved and therefore we need to multiply by the renormalization factor Z_V. This is evaluated non-perturbatively by evaluating the elastic nucleon electric form factor at $Q^2=0$.

In Fig. 1 we make a comparison of the magnetic dipole form factor in quenched and dynamical lattice QCD at a similar pion mass of about 0.5 GeV. No unquenching effects can be observed at this rather heavy pion mass, given the size of the statistical errors on the dynamical QCD results. The results on G_{M1} on the available lattices are presented in Fig. 2 along with the quenched results obtained by linearly extrapolating to the chiral limit [5]. The accuracy of the results in full QCD

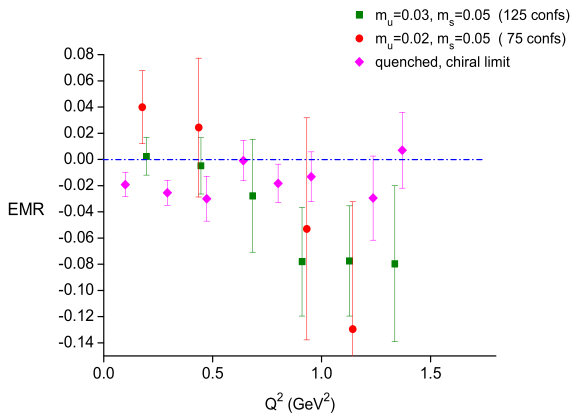

Figure 3: EMR ratio using MILC configurations at $am_{u,d} = 0.03, 0.02$ and quenched results at the chiral limit.

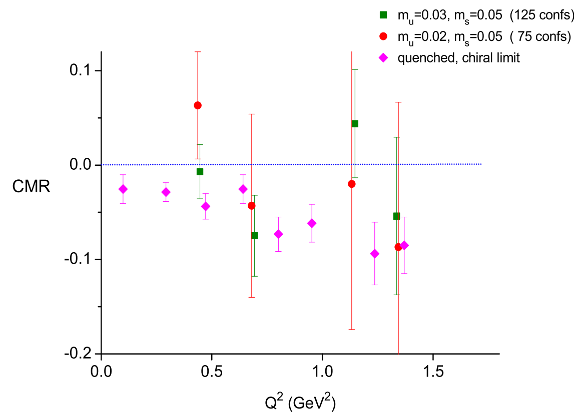

Figure 4: CMR ratio using MILC configurations at $am_{u,d} = 0.03, 0.02$ and quenched results at the chiral limit.

do not allow for a chiral extrapolation but the general trend is that they reach lower values as compared to the quenched case, a welcome fact given that the quenched results were higher than experiment [7]. On the other hand, the EMR and CMR ratios calculated on these ensembles and presented in Figs. 3 and 4 are still very noisy prohibiting a meaningful comparison to the quenched results.

In conclusion, preliminary results are presented for the N to Δ magnetic dipole transition form factor G_{M1} and the EMR and CMR ratios, evaluated using Domain wall valence quarks and dynamical MILC configurations. The statistical fluctuations are enhanced in full QCD requiring a larger number of configurations in order to reach the accuracy required for a meaningful comparison to either the quenched data or experiment.

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