Towards a chiral gauge theory by deconstruction in AdS5

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We describe an implementation of a deconstructed gauge theory with charged fermions defined on an interval in five dimensional AdS space. The four dimensional slices are Minkowski, and the end slices support four dimensional chiral zero modes. In such a theory, the energy scales warp down as we move along the fifth dimension. If we augment this theory with localized neutral 4-dimensional Majorana fermions on the low energy end, and implement a Higgs mechanism there, we can arrange the theory such that the lightest gauge boson mode and the chiral mode on the wall at the high energy end are parametrically lighter than all the other states in the theory. If this semiclassical construction does not run into problems at the quantum level, this may provide an explicit construction of a chiral gauge theory. Instanton effects are expected to make the gauge boson heavy only if the resulting effective theory is anomalous.

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1. Introduction

Chiral gauge theory of weak interactions forms an important ingredient of the standard model, but so far chiral gauge theories have defied a definition beyond perturbation theory. These in fact form the only class of perturbatively defined field theories with no non-perturbative formulation. This problem is gaining practical import since a number of proposed extensions to the standard model invoke strongly interacting chiral dynamics, and a quantitative analysis requires the evaluation of non-perturbative condensates in these theories. The only known non-perturbative regulator in four-dimensions is the lattice formulation and we investigate it here.

A fundamental problem in realizing a chiral gauge theory on the lattice is encapsulated in the Nielsen Ninomiya theorem [1]. According to this theorem, under mild conditions of locality and analyticity of the propagator, a translationally invariant fermion theory that preserves the continuum chiral symmetry exactly on the lattice has paired left and right chiral modes in the continuum limit. Since these modes are related by boosts in the discretized theory, they cannot be differently charged under any symmetry preserved by the discretization, and any naïve attempt at obtaining an unbroken chiral gauge theory must fail. This result is also expected based on our knowledge that chiral gauge theories are undefined if anomalies do not cancel. The anomaly cancellations can occur, as in the standard model, between fermion species that interact only through the gauge fields. This cancellation is difficult to preserve in any naïve discretization.

A way out of this conundrum was suggested by Kaplan [2] by extending fermionic gauge theories to five dimensions. Theories on a five dimensional interval typically have *edge* states that are chiral under the four dimensional Euclidean group acting on the edge of the region. These states still come in chiral pairs, but one can arrange the parameters to locate the left and right chiral modes on the different edges, and hence separate them in the fifth dimension. Since the fifth dimension is unphysical, one is free to add interactions that are not translationally invariant along this direction. Previous attempts to realize a chiral gauge theory by confining the gauge interactions to only a part of the five dimensional space were, however, unsuccessful. [3].

In this paper we consider an alternate formulation with a truly five dimensional gauge field [4]. The translational invariance of this gauge interaction is broken by invoking a Higgs' mechanism using extra matter fields propagating only along one edge. This Higgs' phenomenon makes the fermion states at that edge heavy, but has negligible effect on states located elsewhere in the five dimensional world. A classical analysis reveals that in the presence of background five-dimensional space-time curvature we can take a limit in which a four-dimensional gauge boson and one chiral fermion remain massless, whereas all the other states in the theory become infinitely heavy and decouple. If the resulting gauge theory has an uncancelled triangle anomaly then this construction of a chiral gauge theory is easily seen to fail at the quantum level. A complete quantum analysis remains beyond our reach because even perturbation theory is not valid in some regions in this five dimensional space.

2. The flat space analogue

Before constructing the full model, it is instructive to study an analogue construction in flat five-dimensional space. We consider this world bounded by flat Euclidean slices separated by
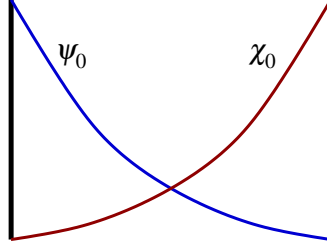
Figure 1: The exponentially localized edge states of the fermions, $\psi_0 \propto \exp -Mz$ and $\bar{\chi}_0 \propto \exp Mz$.

distance $R' - R$. A free massive fermion in this spacetime is described by the action

$$S = \int d^4 x \int_R^{R'} dz \{ -i\bar{\psi}\partial_\mu\bar{\sigma}^\mu\psi - i\chi\partial_\mu\sigma^\mu\bar{\chi} + \psi\partial_z\chi - \bar{\chi}\partial_z\bar{\psi} \\ + M\psi\chi + M\bar{\chi}\bar{\psi} \},$$

where ψ and χ are two-component fermion fields of opposite chirality, σ^μ are the Pauli matrices (or identity for the time direction), M is the five dimensional mass, and the overbar represents conjugation. A Kaluza-Klein decomposition gives modes χ_n and ψ_n that satisfy

$$(\partial_z - M)\bar{\chi}_n = m_n\bar{\chi}_n \qquad\qquad -(\partial_z + M)\psi_n = m_n\psi_n.$$

The edge states are the states with $m_0 = 0$ and are exponentially localized at the left and right walls (Fig. 1). The mass scaleof the rest of the states in the theory, $\pi/2(R' - R)$, is set by the length of the fifth dimension.

A gauge theory is obtained by changing the derivatives above to gauge covariant ones and adding a kinetic term for the gauge bosons

$$\int d^4 x \int_R^{R'} dz \frac{1}{4g_5^2} \left(F_{\mu\nu}F^{\mu\nu} + 2F_{\mu5}F^{\mu5} \right).$$

In such a theory the left and right chiral modes, ψ_0 and χ_0, have equal charge, and one needs to decouple one of them (by making it heavy), say χ_0 on the right wall, to obtain a chiral gauge theory. To this end, we introduce an Higgs' mechanism localized on the right wall, and use the Higgs' field H to mix χ with a neutral fermions S_L with large Majorana mass, m:[1]

$$y\chi H S_L + m S_L S_L + \text{h.c.}$$

where y is the Yukawa coupling. This provides a mass of order $y\langle H\rangle$ to χ_0. Since Yukawa couplings are infrared free it is difficult to make y much larger than the gauge coupling g. This is problematic since for small values of $\langle H\rangle$ the gauge boson also acquires a mass of order $g\langle H\rangle$, and it appears

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[1]The addition of a neutral right handed fermion S_R at R' along with its Yukawa coupling to the bulk field $\bar{\psi}$ has, however, very little impact on the zero mode ψ_0 exponentially localized at R. Such an addition is necessary in the actual construction but is dropped in this exposition for brevity.

one cannot take a limit where the unwanted mode χ_0 decouples, but the gauge boson stays in the spectrum.

The situation is qualitatively different when the vacuum expectation value is large. The odd Kaluza-Klein modes of the gauge boson, which vanish at the boundary, do not feel the effect of the Higgs' mechanism at the classical level and retain a mass, $\tilde{m}_1$, of the order of $\pi/2(R' - R)$. The even Kaluza-Klein modes pick up large masses by this Higgs' mechanism. So, if the length of the fifth dimension is large, we do find light gauge bosons and a massless chiral mode, ψ_0 in the spectrum[2] as required to construct a chiral gauge theory.

This construction, however, fails since *all and not only the lightest* of the odd Kaluza-Klein modes of the gauge field are controlled by the same scale $\pi/2(R' - R)$. Thus in the limit that the extra dimension becomes large, the theory reverts to being five dimensional.

3. The Model in AdS_5

It has been known for a while [5] that if we consider an interval in a curved five dimensional space, the gravitational acceleration towards one wall can give rise to an effective four dimensional theory *even when the length of the extra dimension is large.* Accordingly, we consider a Euclidean version of a slice of AdS_5 bounded by four-dimensional flat Minkowski slices separated by a proper distance denoted by $\ln R'/R$. AdS_5 is a homogeneous five-dimensional space with a constant negative radius of curvature, which is also denoted by $-R$. The comparison with our flat space analysis is easiest in the metric

$$ds^2 = \left(\frac{R}{z} \right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right) ,$$

with the chosen interval being (R, R'). In this background, the odd Kaluza-Klein modes of the gauge boson have masses given by

$$\tilde{m}_1^2 = O\left(\frac{1}{R'^2 \ln(R'/R)} \right) , \qquad\qquad \tilde{m}_n^2 \approx O\left(\frac{1}{R'^2} \right) ,$$

whereas the fermion modes are not significantly affected for large M. This lets us take the limit

$$R' \to 0 \qquad\qquad\qquad R'^2 \ln(R'/R) \to \infty$$

which decouples all the Kaluza-Klein modes leaving behind a massless gauge boson interacting with the chiral fermion ψ_0.

4. Deconstruction

Five dimensional theories may not be renormalizable, and it may not be possible to interpret the Lagrangian parameters as running couplings at a certain scale. We, therefore, need to be careful in drawing conclusions based on the tree level constructions provided in the previous sections. To study this systematically we choose to deconstruct [6] the five dimensional theory into a stack

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[2]The masses of the Kaluza-Klein modes χ_n, controlled by the scale M and not by the size of the extra dimension, stay heavy.

of four dimensional continuum slices placed at discrete positions along the fifth dimension. To maintain as many of the space time symmetries of the AdS_5 space as possible, we choose the positions of the slices to be

$$z_i = (1+a)^{i-1}R, \qquad i = 1\ldots N, \qquad a = \exp\left(\frac{1}{N-1}\ln\frac{R}{R'}\right) - 1$$

where we refer to the small dimensionless number a as the "lattice spacing". The modes at each of these four dimensional slices can be reinterpreted as those of fields transforming under a separate four dimensional gauge group. It is easy to check that the gauge coupling of the ith gauge group is given by

$$\frac{1}{g_i^2} = \frac{aR}{g_5(z_i)^2},$$

where we have generalized the model to allow for a variation of g_5 along the fifth dimension.

The link fields in the fifth dimension, $\mathscr{P}\exp\int_{z_i}^{z_{i+1}} iA_5\,dz$ can then be interpreted as bifundamental scalars (transforming according to the fundamental (anti-fundamental) representation under the gauge theory at z_i (z_{i+1})). With this identification, the original theory, before we add the Higgs mechanism at R', can be seen to be invariant under the product of the gauge groups at each site but realized in the Higgs phase with the bifundamentals acquiring a vacuum expectation value related to the lattice spacing. This view of the five-dimensional theory is called deconstruction. The lowest gauge boson mode is, not surprisingly, the discretization of the lowest Kaluza-Klein mode: the equal superposition of the gauge fields on all the slices, and its effective coupling is given by

$$\frac{1}{g_4^2} \approx \sum_{i=1}^{N} \frac{1}{g_i^2}.$$

The continuum AdS_5 symmetry requires all lengths to scale in proportion to the z coordinate as we move in the fifth dimension. To maintain a discrete version of this we can choose the renormalized couplings to satisfy

$$g_i^R(\mu_i) = constant \qquad\qquad \text{if} \qquad\qquad \mu_i \propto 1/z_i.$$

A one loop calculation, assuming $\mu \ll 1/R'$, then yields

$$\frac{1}{g_4^2(\mu)} \approx \frac{N}{g_1^2(1/aR)} + \frac{\beta_0}{8\pi^2}\ln aR\mu$$

whence we can obtain the Λ parameter of the theory associated with the lowest mode to be

$$aR\Lambda = \exp\frac{-8\pi^2 N}{\beta_0 g_1^2(1/aR)}.$$

To obtain a chiral gauge theory we need to take the limit

$$\frac{m_{KK}}{\Lambda} \to \infty, \qquad\qquad\qquad \frac{m_1}{\Lambda} \to 0,$$

where m_{KK} is the typical Kaluza-Klein state and m_1 is the lightest gauge mode. Using

$$m_{KK}^2 R^2 \sim (1+a)^{2N}, \qquad\qquad\qquad \frac{m_1^2}{m_{KK}^2} \sim \frac{1}{N\log(1+a)},$$

one finds that the limit can be realized with the choice

$$N \to \infty \qquad\qquad g_1^2(\frac{1}{aR}) \sim \frac{8\pi^2}{\beta_0 a},$$

with *a* held fixed.

The arguments used in deriving these results, however, relied on the wavefunctions on the lattice being a discretization of the continuum tree-level wave functions. This property holds only if the lattice spacing *a* is much smaller than unity, and this in turn implies we need to choose the lattice couplings to be large. As a result, the one-loop results presented here can only be treated as suggestive.[3]

5. Potential problem

It is well known that the massless limit of a vector field theory is singular, and often involves strong couplings. It is instructive to note that even in our construction, the 'longitudinal' mode of the gauge field becomes strongly interacting with an effective Yukawa like interaction with the fermions. This interaction, of strength

$$y(z) \propto \frac{z \ln(z/R)}{R' \ln(R'/R)},$$

pushes the unwanted chirality fermion away from the wall with the Higgs' mechanism, and can potentially make it light. Explicit evaluation in one-loop perturbation theory, however, shows that the fermion decouples even in the presence of this interaction. Unfortunately, the large gauge coupling precludes a pertubative resolution of this question. To settle this issue a non-perturbative calculation is required.

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[3]A more detailed argument shows that it is indeed the coupling at the scale $1/aR$ which controls the validity of the required one-loop calculation.