

The chiral transition of $N_f = 2$ QCD with fundamental and adjoint fermions

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We study QCD with two staggered Dirac fermions both in the fundamental (QCD) and the adjoint representation ($aQCD$) near the chiral transition. The aim is to find the universality class of the chiral transition and to verify Goldstone effects below the transition. We investigate $aQCD$, because in that theory the deconfinement and the chiral transitions occur at different temperatures $T_d < T_c$. Here, we show that the scaling behaviour of the chiral condensate in the vicinity of β_c is in full agreement with that of the $3d O(2)$ universality class. In the region $T_d < T < T_c$ we confirm the quark mass dependence of the chiral condensate which is expected due to the existence of Goldstone modes like in $3d O(N)$ spin models. For fundamental QCD we use the $p4$ -action. Here, we find Goldstone effects below T_c like in $aQCD$ and the $3d O(N)$ spin models, however no $O(2)/O(4)$ scaling near the chiral transition point. The result for QCD may be a consequence of the coincidence of the deconfinement transition with the chiral transition.

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1. Introduction

We study QCD with two staggered Dirac fermions both in the fundamental (QCD) and the adjoint representation ($aQCD$) near the chiral transition. The intention is to find the universality class of the transition and to verify Goldstone effects below the transition. For ordinary QCD with staggered fermions the prediction of the three-dimensional $O(2)$ universality class (the $O(4)$ class in the continuum theory) could up to now not be confirmed. We investigate in addition $aQCD$, because in that theory the deconfinement and the chiral transitions occur at different temperatures with $T_d < T_c$ (that is $\beta_d < \beta_c$) [1]. The chiral transition can therefore be studied without interference. The comparison of QCD data with the critical behaviour of $O(N)$ spin models requires the following identifications of QCD variables to $O(N)$ variables: the chiral condensate corresponds to the magnetization M and the quark mass (m_{qa}) to the magnetic field H . Instead of the temperature we use $\beta = 6/g^2$.

2. QCD with adjoint fermions ($aQCD$)

The action of $aQCD$ which we use is [1]

$$S = \beta S_G(U^{(3)}) + \sum_{x,y} \bar{\psi}_x M(U^{(8)})_{x,y} \psi_y . \quad (2.1)$$

Here, the gluon part $S_G(U^{(3)})$ is the usual Wilson one-plaquette action, but the fermions are in the 8-dimensional adjoint representation of color $SU(3)$. The standard staggered fermion matrix M depends correspondingly on $U^{(8)}$ instead of $U^{(3)}$. The links $U^{(8)}$ are *real* because

$$U_{ab}^{(8)} = \frac{1}{2} \text{tr}_3 \left[\lambda_a U^{(3)} \lambda_b U^{(3)\dagger} \right] . \quad (2.2)$$

The fermion action does not break $Z(3)$ center symmetry and the Polyakov loop L_3 is therefore order parameter for the deconfinement transition. In the continuum $aQCD$ contains an $SU(2N_f)$ chiral symmetry which breaks to $SO(2N_f)$ for $T < T_c$. This is because here N_f Dirac fermions correspond to $2N_f$ Majorana fermions. For $N_f = 2$ we have $SU(4)$ -symmetry which breaks to $SO(4)$. The corresponding continuum transition has been studied in Ref. [2] with renormalization-group methods. On the lattice an $O(2)$ -symmetry remains for staggered fermions.

Our simulations [3] were done on $N_\sigma \times 4$ lattices with $N_\sigma = 8, 12$ and 16 and a fixed length $\tau = 0.25$ of the trajectories. We used 900-2000 trajectories for measuring the chiral condensate $\langle \bar{\psi} \psi \rangle$, the Polyakov loop L_3 and the disconnected part χ_{dis} of the susceptibility.

In Ref. [1] the deconfinement transition point was found at $\beta_d = 5.236(3)$ - the same value as in QCD (!), but here it is of first order. The usual strategy to locate the chiral transition point β_c is to extrapolate the line of peak positions β_{pc} of the susceptibility to $m_{qa} = 0$ with

$$\beta_{pc} = \beta_c + c(m_{qa})^{1/\Delta} , \quad (2.3)$$

where $\Delta = \beta_m \delta$ is a product of critical exponents. On the $8^3 \times 4$ lattice we find for $m_{qa} = 0.005, 0.01, 0.02$ the values $\beta_{pc} = 5.73(8), 5.74(6), 5.75(10)$. With Δ from $O(2)$ we obtain $\beta_c = 5.7(2)$. A better method [3] is to expand the scaling ansatz $\langle \bar{\psi} \psi \rangle = m^{1/\delta} f(z)$ at small $|z|$. Here,

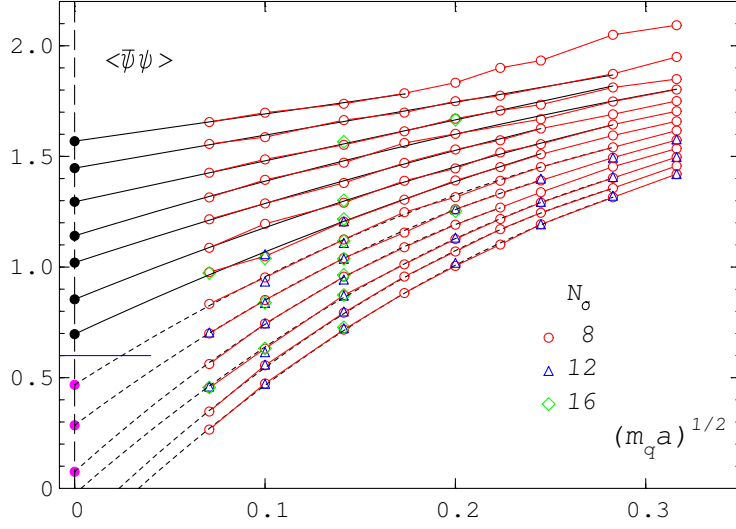


Figure 1: The chiral condensate $\langle \bar{\psi}\psi \rangle$ of $aQCD$ as a function of $(m_q a)^{1/2}$ for all β -values between 5.3 (highest values) and 5.9 (lowest values) from $N_\sigma^3 \times 4$ lattices with $N_\sigma = 8$ (circles), 12 (triangles) and 16 (diamonds). The black lines are fits with ansatz (2.5), the filled circles denote the extrapolations to $m_q a = 0$.

$m = m_q a / (m_q a)_0$, $\beta_r = (\beta - \beta_c) / \beta_0$ are the reduced field and temperature and $z = \beta_r m^{-1/\Delta}$. The result is

$$\langle \bar{\psi}\psi \rangle = (m_q a)^{1/\delta} \left\{ d_c + d_c^1 (\beta - \beta_c) (m_q a)^{-1/\Delta} + \dots \right\}. \quad (2.4)$$

Fits to only the first term at fixed β are best for $\beta \in [5.6, 5.7]$. If $O(2)$ exponents and two terms are used one finds a unique zero of the parameter $d_c^1 (\beta - \beta_c)$ as a function of β at the rather precise value $\beta_c = 5.624(2)$.

Due to the existence of massless Goldstone modes for all $T < T_c$ the susceptibility χ_L of $3d$ $O(N)$ spin models diverges for $H \rightarrow 0$ as $\chi_L \sim H^{-1/2}$. If $aQCD$ behaves effectively as such a model, a similar divergence is expected. In terms of the chiral condensate we must have then

$$\langle \bar{\psi}\psi \rangle(\beta, m_q a) = \langle \bar{\psi}\psi \rangle(\beta, 0) + c_1(\beta) (m_q a)^{1/2} + c_2(\beta) (m_q a) + \dots \quad (2.5)$$

Corresponding fits are shown in Fig. 1 for $\beta = 5.3$ to $\beta = 5.9$ with $\Delta\beta = 0.05$. The blue line separates fits above and below β_c . We see that the model expectations are met well by the data.

We have performed explicit scaling tests for the data from the $8^3 \times 4$ lattice (they coincide with the results from the $N_\sigma = 12$ and 16 lattices). In the left part of Fig. 2 the data for $\langle \bar{\psi}\psi \rangle m^{-1/\delta}$ are shown as a function of the scaling variable $z = \beta_r / m^{1/\Delta}$ using $O(2)$ exponents. The normalizations $(m_q a)_0$ and β_0 have been determined from the behaviour at β_c (2.4), and the extrapolations $\langle \bar{\psi}\psi \rangle(\beta, 0)$ obtained from the Goldstone effect [3]. In addition we show the $O(2)$ scaling function. We observe that the data scale very well with $O(2)$ exponents in the small $|z|$ -region and agree there with the $O(2)$ scaling function. For decreasing $z < 0$ outside the shown range the data for different β start to deviate and exhibit corrections-to-scaling as in the original $O(2)$ model. In the right part of Fig. 2 we show the best scaling result for the unnormalized variables obtained by varying the exponents around $\delta \approx 4.0$, $\nu \approx 1.1$, the expected continuum class values [2]. Here, $\beta_c = 5.64$. We see from Fig. 2 that the data still spread considerably, a scaling function is not known up to now.

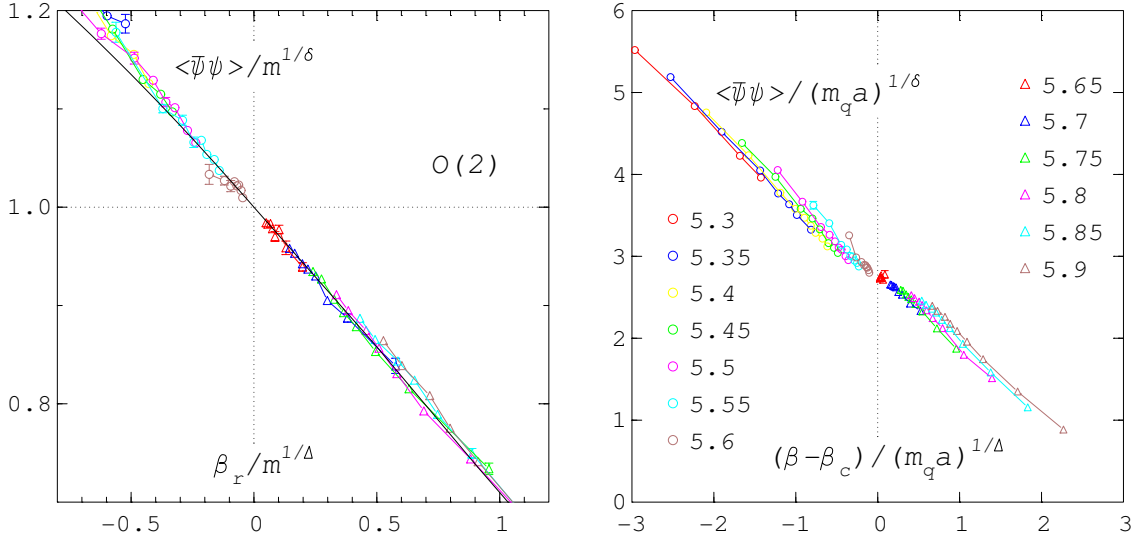


Figure 2: Scaling test in $aQCD$. Left part: $\langle \bar{\psi}\psi \rangle / m^{1/\delta}$ for $O(2)$ parameters in the vicinity of the critical point. The black line is the $O(2)$ scaling function. Right part: the unnormalized variables with $\delta = 4.4$, $\nu = 1.0$ from Ref. [2]. The data for fixed β -values are connected by straight lines to guide the eye.

3. QCD with fundamental fermions

Since the predictions for the universality class of ordinary QCD could not be confirmed with the standard staggered action we use here the $p4$ -action[4], which improves the cut-off dependence, rotational invariance and flavour symmetry. The thermodynamics of two flavour QCD has been investigated with this action in Ref. [5]. In particular, the chiral transition point was estimated to $\beta_c = 3.48(3)$ by extrapolation of the pseudocritical points $\beta_{pc}(m_q a)$ to $m_q a = 0$.

We have extended the work of Karsch et al.[5] on $N_\sigma^3 \times 4$ lattices with $N_\sigma = 8, 12, 16$ at the couplings $\beta = 2.8, 3.0, 3.2, 3.4, 3.48$ and 3.50 , that is for $\beta \leq \beta_c$. All our simulations were done with the MILC code using the R-algorithm with a mass-dependent stepsize $\delta\tau(m_q a) = \min\{0.4m_q a, 0.1\}$ and trajectory length $\tau = 1$. We produced 1000-2000 trajectories for masses $m_q a \in [0.025, 0.5]$ and somewhat less for $m_q a = 0.01$.

Like in $aQCD$ we have tested the mass dependence of the chiral condensate at fixed $\beta < \beta_c$. In Fig. 3 we show the data from $\beta = 2.8$ to $\beta = 3.48$ as a function of $(m_q a)^{1/2}$. We have fitted the data in the range $m_q a \lesssim 0.15$ to the ansatz

$$\langle \bar{\psi}\psi \rangle(\beta, m_q a) = \langle \bar{\psi}\psi \rangle(\beta, 0) + c_1(\beta)(m_q a)^{1/2}. \quad (3.1)$$

Again, the behaviour expected due to the Goldstone effect is confirmed by the data.

In Fig. 4 we investigate the scaling form $\langle \bar{\psi}\psi \rangle = d_c(m_q a)^{1/\delta}$ for the critical point in the close neighbourhood of β_c . We have plotted the data with $\delta_{O(2)} = 4.78$ in the left part of Fig. 4 at $\beta = 3.48$ (solid lines) and $\beta = 3.5$ (dotted lines). Obviously, the curves are no straight lines going through the origin. Also, the corresponding attempt to obtain a scaling function with $O(2)$ exponents fails as can be seen in the left part of Fig. 5. Here, $\beta_c = 3.51$ was used, the scales are not normalized. Other values for β_c shift the results slightly, but do not lead to scaling. The use of $O(4)$ exponents does not improve the result.

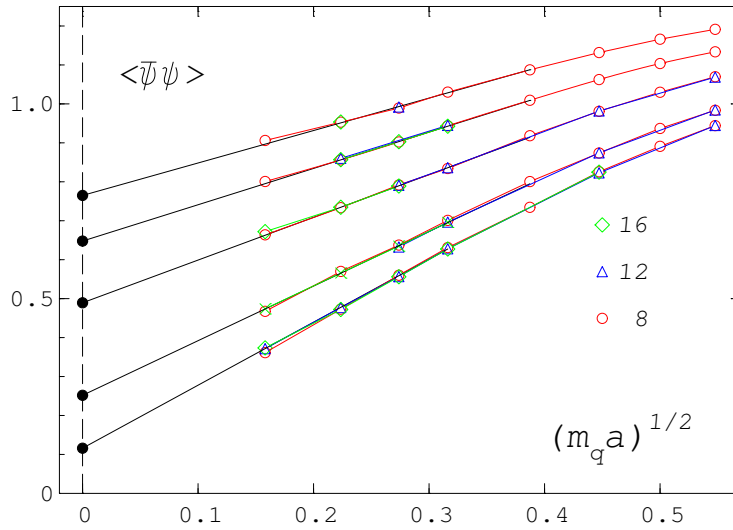


Figure 3: The chiral condensate $\langle \bar{\psi}\psi \rangle$ as a function of $(m_q a)^{1/2}$ for all β -values between 2.8 (highest values) and 3.48 (lowest values) from $N_\sigma^3 \times 4$ lattices with $N_\sigma = 8$ (circles), 12 (triangles) and 16 (diamonds). The black lines show the fit results and the filled circles the estimates for the chiral condensate at $m_q a = 0$.

In order to achieve scaling of the data we have fitted the ansatz $\langle \bar{\psi}\psi \rangle = d_c (m_q a)^{1/\delta}$ at $\beta = 3.50$ with a free amplitude d_c and a free exponent δ for $m_q a \in [0.01, 0.1]$ and obtain $\delta = 2.3$. The fit is shown in the right part of Fig. 4. Likewise, we estimated the magnetic exponent β_m to 0.6 from the values of $\langle \bar{\psi}\psi \rangle$ at $m_q a = 0$ which we gained from the Goldstone extrapolations. In the right part of Fig. 5 we show the data for these exponents. Here, $\langle \bar{\psi}\psi \rangle / (m_q a)^{1/\delta}$ is normalized to 1 at β_c . Still, scaling is not perfect, but certainly much better than for $O(N)$ exponents.

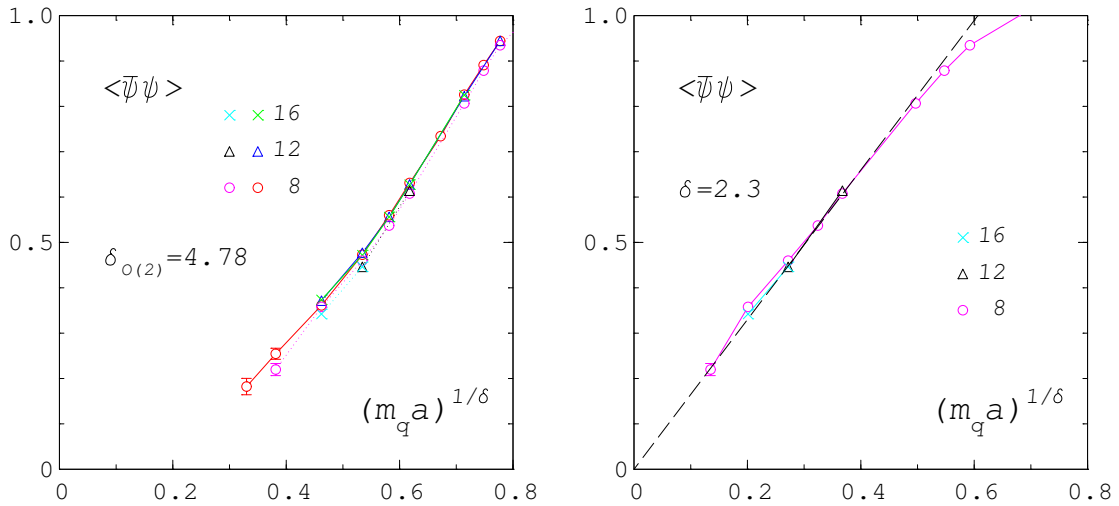


Figure 4: Test of the scaling ansatz for $\langle \bar{\psi}\psi \rangle$ at the critical point for $O(2)$ (left part) at $\beta = 3.48$ (solid lines) and 3.5 (dotted lines). In the right part we show a free fit to the scaling ansatz at $\beta = 3.5$ (dashed line). Here, the exponent is $\delta = 2.3$.

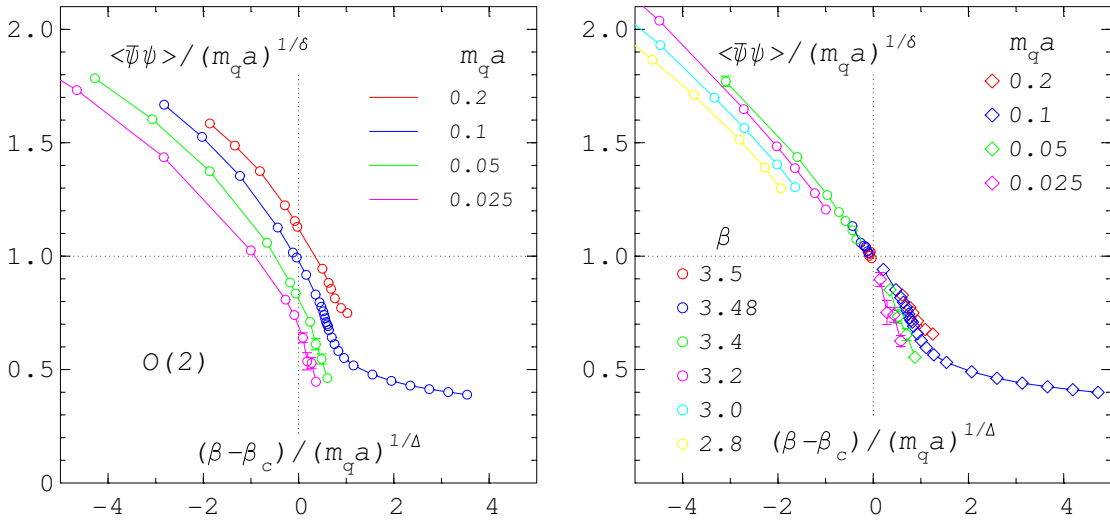


Figure 5: Test on scaling behaviour of $\langle \bar{\psi}\psi \rangle / (m_q a)^{1/\delta}$ in QCD for $O(2)$ (left) and free (right) exponents.

4. Summary

In $aQCD$ the deconfinement transition is first order and occurs below the second order chiral transition. The latter is located at $\beta_c = 5.624(2)$. The scaling behaviour of $\langle \bar{\psi}\psi \rangle$ in the vicinity of β_c is in full agreement with the $3d$ $O(2)$ universality class. The lattice data do not yet show scaling with exponents from the proposed continuum class[2]. In the region between the two phase transitions the quark mass dependence of the chiral condensate is as expected due to the existence of Goldstone modes like in $3d$ $O(N)$ spin models.

For QCD we find that the chiral condensate exhibits Goldstone effects below the chiral transition point as in $3d$ $O(N)$ spin models and like in $aQCD$. The transition is of second order or a crossover, there is no sign of a first order behaviour. In the vicinity of β_c the chiral condensate does not show the scaling behaviour of the $3d$ $O(2)$ or $O(4)$ universality classes, at best it is that of a class with exponents $\delta = 2.3$ and $\beta_m = 0.6$. This result could be a consequence of the coincidence of the deconfinement and the chiral transitions.

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