Critical temperature in QCD with two flavors of dynamical quarks[*]

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We present results obtained in QCD with two flavors of non-perturbatively improved Wilson fermions at finite temperature on $16^3 \times 8$ and $24^3 \times 10$ lattices. We determine the transition temperature in the range of quark masses $0.6 < m_\pi/m_\rho < 0.8$ at lattice spacing a≈0.1 fm and extrapolate the transition temperature to the continuum and to the chiral limits. We also discuss the order of phase transition.

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1. Introduction

Knowledge of the QCD phase structure is important in order to understand the physics of the early Universe, structure of neutron stars, processes in heavy ion collisions, *etc*. In order to obtain predictions for the real world from lattice QCD, we have to extrapolate the lattice data to the continuum and to the chiral limit. Determination of the transition temperature T_c was studied by many groups. T_c from staggered fermions with 2 and 2+1 flavors of dynamical quarks [1, 2, 3, 4], and from Wilson fermions with 2 flavors [5] was found to be consistent in the chiral limit. Edwards and Heller [6] determined T_c for $L_t = 4$ and 6 using nonperturbatively improved Wilson fermions. We compute T_c on finer lattices with $L_t = 8$ and 10 and extrapolate our high statistics data to the continuum limit. Our results for $L_t = 8$ at $\beta = 5.2$ were previously reported in Ref. [7].
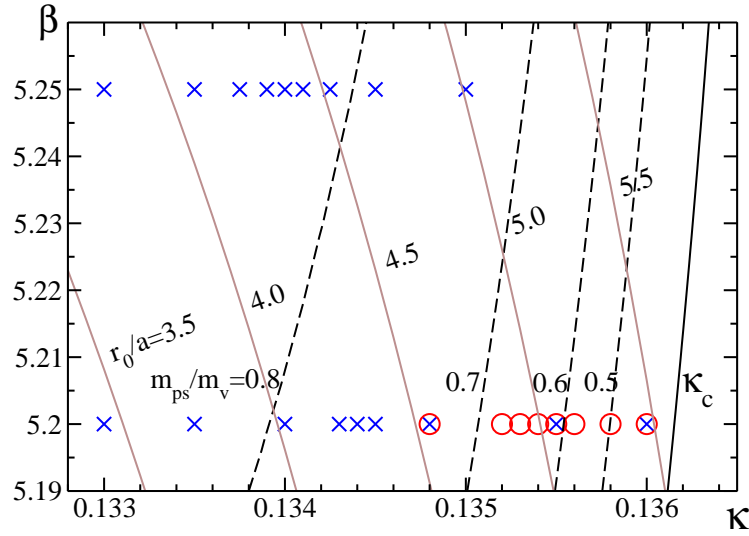
2. Simulation

We use the Wilson gauge action and the non-perturbatively improved Wilson fermions with c_{sw} given in Ref. [8]. Configurations are generated on $16^3 \times 8$ ($\beta = 5.2$ and 5.25) and $24^3 \times 10$ ($\beta = 5.2$) lattices at various κ. The values of κ and the corresponding number of configurations for $24^3 \times 10$ and $16^3 \times 8$ lattices can be found in Table 1. We use T=0 results obtained by QCDSF-UKQCD to fix the scale. The contour plot of lines of constant r_0/a and m_π/m_ρ is shown in Fig. 1.

| $24^3 \times 10$		$16^3 \times 8$			
$\beta = 5.2$		$\beta = 5.2$		$\beta = 5.25$	
κ	# of conf.	κ	# of traj.	κ	# of traj.
0.1348	678	0.1330	7,120	0.1330	1,540
0.1352	2058	0.1335	4,500	0.1335	7,439
0.1353	2725	0.1340	3,000	0.13375	9,225
0.1354	3934	0.1343	6,615	0.1339	12,470
0.1355	3816	0.1344	11,275	0.1340	19,479
0.1356	1730	0.1345	9,077	0.1341	13,750
0.1358	1210	0.1348	5,813	0.13425	5,155
0.1360	617	0.1355	5,650	0.1345	2,650
		0.1360	3,699	0.1350	1,780

Table 1: Simulation statistics.

3. Critical temperature

We use the Polyakov loop susceptibility to determine the transition point. We identify the critical value of κ, κ_t, as a point, where the Polyakov loop susceptibility reaches its maximum. Applying a Gaussian fit in the vicinity of the maximum (as shown in Figs. 2, 3 and 4) we find $\kappa_t = 0.13542(6)$ at $\beta = 5.2$ on $24^3 \times 10$, $\kappa_t = 0.13444(6)$ at $\beta = 5.2$ on $16^3 \times 8$, and $\kappa_t = 0.13406(6)$ at $\beta = 5.25$ on $16^3 \times 8$. Using the Sommer scale r_0=0.5 fm and interpolating r_0/a to the critical point,

Figure 1: Lines of constant r_0/a (solid lines) and constant m_π/m_ρ (dotted lines) at $T = 0$. Crosses and circles indicate points where simulations are done.

we obtain for the critical temperatures $T_c = 197(2)$MeV (at $m_\pi/m_\rho = 0.63(2)$), $T_c = 212(2)$MeV (at $m_\pi/m_\rho = 0.76(1)$), and $T_c = 217(2)$MeV (at $m_\pi/m_\rho = 0.81(1)$), respectively.
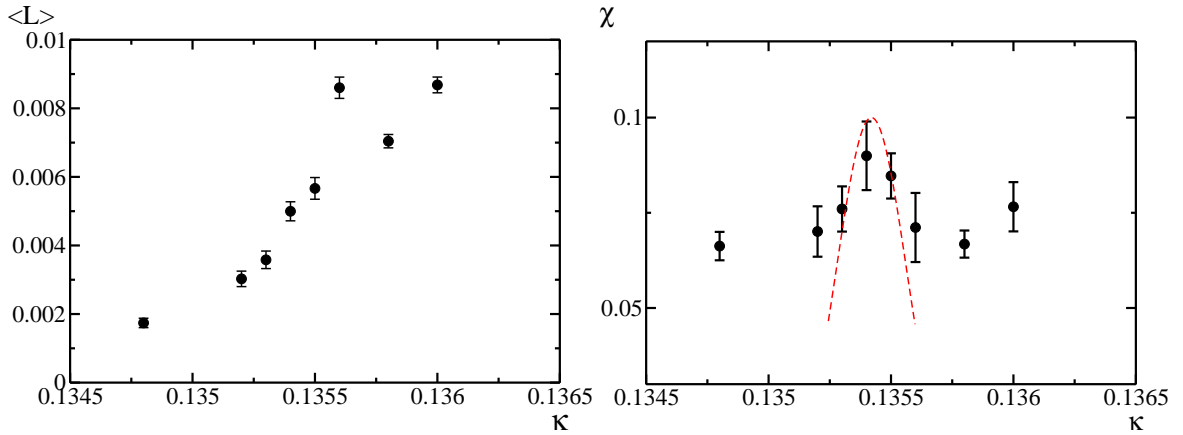
Figure 2: Polyakov loop (left) and its susceptibility (right) at $\beta = 5.2$ on $24^3 \times 10$ lattice.

In Fig. 5 we compare our result for the critical temperature with those of Refs. [1], [5] and [6], where we have assumed $\sqrt{\sigma} = 425$ MeV, and m_v and m_{ps} are vector meson mass and pseudoscalar mass obtained on the zero-temperature lattice. Our result is in quantitative agreement with the results of the Bielefeld group and CP-PACS. This is reassuring, as [1], [5] and [6] work at larger lattice spacing.

4. Continuum and chiral limits

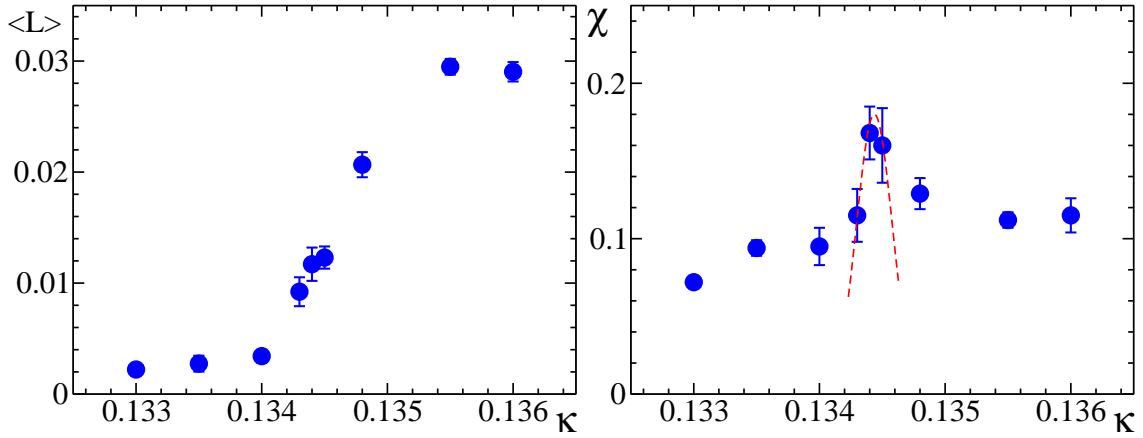In order to estimate T_c in the continuum and the chiral limits we apply the extrapolation for-

Figure 3: Polyakov loop (left) and its susceptibility (right) at $\beta = 5.2$ on $16^3 \times 8$ lattice.
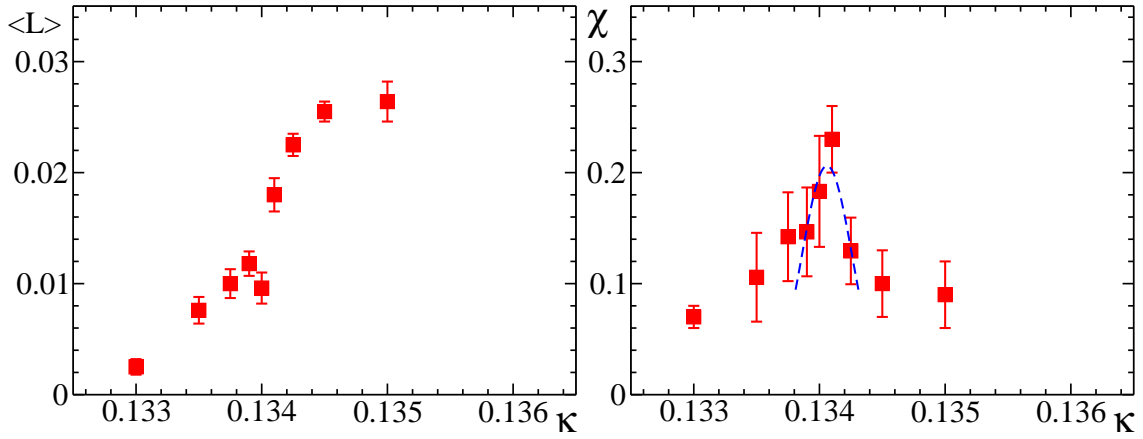
Figure 4: Polyakov loop (left) and its susceptibility (right) at $\beta = 5.25$ on $16^3 \times 8$ lattice.
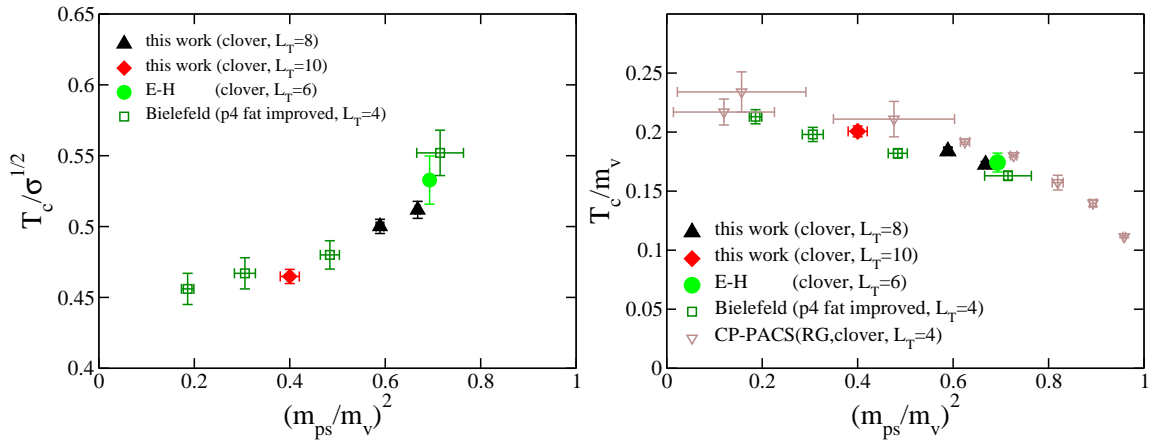

Figure 5: (left) The transition temperature T_c in units of the string tension $\sigma(T = 0)$ as a function of m_{ps}/m_v and (right) T_c in units of m_v as a function of $(m_{ps}/m_v)^2$ (right). The data is from this work (the up triangles and the diamond), Ref. [1] (the squares), Ref. [6] (the circle), and Ref. [5] (the down triangles).

mula, which takes into account the lowest order corrections coming from the finite lattice spacing and quark masses. We use two extrapolation functions: the first one is of the bare quark mass type (TYPE 1) and the second one is of the pseudoscalar mass type (TYPE 2).

$$\textbf{TYPE 1}: \qquad T_c r_0 = T_c^{m_q, a \to 0} r_0 + C_a \left(\frac{a}{r_0}\right)^2 + C_q \left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)^{\frac{1}{\beta\delta}}, \qquad (4.1)$$

$$\textbf{TYPE 2}: \qquad T_c r_0 = (T_c r_0)^{m_q, a \to 0} + C_a \left(\frac{a}{r_0}\right)^2 + C_q (m_{ps})^{\frac{2}{\beta\delta}}. \qquad (4.2)$$

where $T_c^{m_q, a \to 0}$ corresponds to the extrapolated value of the critical temperature, and β and δ are the critical indices. Since $O(a)$ improved Wilson fermions are employed in this studies, we expect corrections of $O(a^2)$ only.

We make an attempt to fit four values for $T_c r_0$ (see Table 2), obtained at rather large quark masses, in order to estimate the extrapolation parameters used in Eqs. (4.1,4.2). If the transition in two-flavor QCD is of the second order, then the transition is expected to belong to the universality class of the 3D $O(4)$ spin model with $1/\beta\delta \approx 0.54$. Current lattice calculations suggest that transition in QCD for physical quark masses is not a true phase transition but a crossover [1, 2, 3, 4, 5, 10]. Since results obtained by CP-PACS indicated a good $O(4)$ scaling [5], we extrapolate the value of the critical temperature using the value of 0.54 as $1/\beta\delta$. We rely on the assumption that the finite temperature phase transition in two-flavor QCD is second order. The best fit parameters are presented in Table 3.

$T_c r_0$	a/r_0	$1/\kappa - 1/\kappa_c$	m_{ps}	L_t	β
0.501(5)	0.200(2)	0.040(4)	1.58(6)	10(DIK)	5.2
0.539(5)	0.232(3)	0.094(4)	2.25(4)	8(DIK)	5.2
0.551(5)	0.227(2)	0.123(4)	2.61(4)	8(DIK)	5.25
0.57(2)	0.290(9)	0.174(12)	2.74(5)	6(E-H)	5.2

Table 2: The available data for the critical temperature $T_c r_0$.

$1/\beta\delta$	$(T_c r_0)^{m_q, a \to 0}$	C_a	C_q	χ^2/dof	TYPE
0.54	0.439(7)	0.1(2)	0.33(3)	0.09	1
0.54	0.420(7)	0.5(1)	0.036(2)	0.04	2

Table 3: The best fit parameters for the case if the transition is of second order.

We get the critical temperature in the continuum and in the chiral limits,

TYPE1 fitting: $T_c = 173(3)\text{MeV}$,   TYPE2 fitting: $T_c = 166(3)\text{MeV}$.

The value of T_c extrapolated by using the TYPE 1 fitting agrees with the values obtained by other groups, but the value from the TYPE 2 fitting is slightly smaller. Both fits should give the same result. The difference gives an indication of the systematic error.

Finally, we mention the results of Ref. [9] which support the first order of the transition. If the transition is indeed of the first order, then $1/\beta\delta = 1$. In order to test the sensitivity of T_c to the order of the transition, we also performed a fit with $1/\beta\delta = 1$.

$1/\beta\delta$	$(T_c r_0)^{m_q, a \to 0}$	C_a	C_q	χ^2/dof	TYPE
1	0.486(17)	0.0(4)	0.5(1)	0.11	1
1	0.460(11)	0.5(2)	0.09(2)	0.08	2

Table 4: The best fit parameters for the case if the transition is of first order.

TYPE1 fitting: $T_c = 192(7)\text{MeV}$,   TYPE2 fitting: $T_c = 182(4)\text{MeV}$.

These values are rather large compared to the value obtained from $N_f = 2 + 1$ staggered fermions in Ref. [4]. The results of our fits do not discriminate between the first and the second order phase transitions because of rather large errors in the $T_c r_0$ values. We are continuing simulations on $24^3 \times 10$ lattice in order to get a better precision of the T_c value for this lattice.

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