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Lattice QCD with chemical potential: the properties of hadrons at the extreme conditions

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A new technique to investigate small chemical potential effects on hadron quantities using lattice QCD simulations is developed. *A Taylor series expansion* of hadron screening mass in μ/T is used. It is successfully applied to study the properties of nucleon, pseudo-scalar and vector meson screening masses around the deconfinement phase transition at finite baryonic density by evaluating the Taylor coefficients with respect to the iso-scalar ($\mu_S = \mu_u = \mu_d$) and iso-vector ($\mu_V = \mu_u = -\mu_d$) chemical potentials, where μ_u and μ_d are u and d quark chemical potentials, respectively. We simulate 2-flavor lattice QCD with staggered fermions on a $12^2 \times 24 \times 6$ lattice with $ma = 0.025$, 0.05 and 0.10. The meson screening mass after chiral extrapolation is presented.

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1. Introduction

To study the effects of nuclear matter on particles in lattice QCD, it is necessary to overcome the sing-problem or to use an alternative method to avoid it. Although direct simulations at finite chemical potential μ are very difficult, we can investigate the effect of small chemical potential on an observable by evaluating the coefficients of its Taylor expansion with respect to μ in a simulation at $\mu = 0$. This *Taylor expansion method* was developed in the calculation of the meson screening masses [1]. Recently, it has been extensively applied to the equation of state of QCD [4], as well as two alternative approaches, Multiparameter reweighting [2] and Analytic continuation from imaginary μ [3]. The theories with positive fermionic determinant have been also studied, which include $SU(2)$ color QCD with finite μ_B [5] and $SU(3)$ color with finite isospin chemical potential μ_I [6]. In this work, we extend the work in Ref. [1] to baryons [8] and to more systematic study including chiral extrapolation.

2. Taylor expansion method

For hadron screening masses, the Taylor expansion method reads

$$\begin{aligned}\frac{M(\mu)}{T} &= \left.\frac{M}{T}\right|_{\mu=0} + \left(\frac{M}{T}\right)\left.\frac{\partial M}{\partial\mu}\right|_{\mu=0} + \left(\frac{M}{T}\right)^2\left.\frac{\partial^2 M}{\partial\mu^2}\right|_{\mu=0} + \left[\left(\frac{\mu}{T}\right)^3\right] \\ &= C_0 + C_1\left(\frac{\mu}{T}\right) + C_2\left(\frac{\mu}{T}\right)^2 + \mathcal{O}\left[\left(\frac{\mu}{T}\right)^3\right].\end{aligned}$$

We have chosen to include temperature T in the denominators, in order to make our observables dimensionless. The region of interest for present experiments (RHIC, LHC) is that of high temperatures and small chemical potential, with $\mu/T \sim 0.1$.

To extract the hadron masses and their derivatives with respect to μ, we assume that the hadron correlators are represented in the following forms:

$$\begin{aligned}C_\pi(z) &= C_1\left(e^{-\hat{m}_1\hat{z}} + e^{-\hat{m}_1(N_z-\hat{z})}\right) + C_2\left(e^{-\hat{m}_2\hat{z}} + e^{-\hat{m}_2(N_z-\hat{z})}\right),\\ C_\rho(z) &= C_1'\left(e^{-\hat{m}_1'\hat{z}} + e^{-\hat{m}_1'(N_z-\hat{z})}\right) + C_2'(-1)^z\left(e^{-\hat{m}_2'\hat{z}} + e^{-\hat{m}_2'(N_z-\hat{z})}\right),\\ C_N(z) &= C_1''\left(e^{-\hat{m}_1''\hat{z}} + (-1)^z e^{-\hat{m}_1''(N_z-\hat{z})}\right) + C_2''\left((-1)^z e^{-\hat{m}_2''\hat{z}} + e^{-\hat{m}_2''(N_z-\hat{z})}\right)\end{aligned}$$

Then we can proceed from the quantities which are the easiest to measure by means of the numerical simulations ($C(z)$, $dC(z)/d\mu$, $d^2C(z)/d\mu^2$) to the others (M, $dM/d\mu$, $d^2M/d\mu^2$).

We examine the responses of meson screening mass to the isospin ($\mu_I = \mu_u$ = -μ_d) and baryon ($\mu_B = \mu_u = \mu_d$) chemical potentials in dynamical simulations with two flavors of Kogut-Susskind fermions and plaquette gauge action. The simulation was performed in the range of coupling $5.30 \leq \beta \leq 5.65$ and the quark masses $ma = 0.025$, 0.05 and 0.10.

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3. Numerical results

The $\mu = 0$ screening masses divided by T are shown in Fig.1. In the confined phase, the pseudo-scalar meson retains the nature of the Nambu-Goldstone boson. It will be shown later more clearly by performing the chiral extrapolation. As expected, the pseudo-scalar and vector mesons become degenerate above T_c because of chiral symmetry restoration. With increasing temperature, the screening masses of mesons (baryon) approach $2T$ ($3T$), the limit which corresponds to the free-quark contribution at high temperature with the lowest Matsubara frequency [7]. The vector meson screening mass approaches this asymptotic value at a surprisingly low temperature. In contrast, the pseudo-scalar and nucleon masses approach those limits slowly.

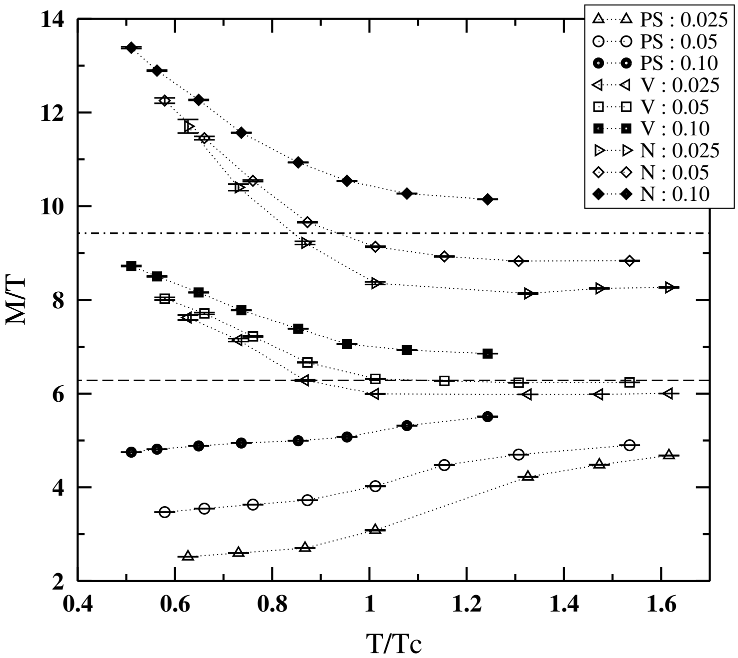

Figure 1: The screening masses of pseudo-scalar (PS), vector (V) mesons and nucleon (N), divided by the temperature T. The dashed (dot-dashed) line represents the free-quark limit 2π (3π).

3.1 $\mu_B \neq 0$, $\mu_I = 0$

Figure 2 shows the second order response of the meson screening masses with respect to μ. The first derivatives vanish because of a symmetry under $\mu_B \to -\mu_B$. In the confinement phase (the hadronic phase) the chiral symmetry is broken. The second order term in the Taylor series expansion of the meson screening mass is consistent with zero. Across the phase transition, the quarks and gluons are deconfined and the spontaneously broken symmetry restored. In this quark-gluon plasma phase QCD is believed to be asymptotically free and dominated by partons. There are at least two possible phase transitions in hadronic matter as temperature increases: deconfinement of quarks and gluons and restoration of the broken chiral symmetry. The response here increases with T and therefore pseudo-scalar meson becomes heavier when μ_B is finite. According to the present understanding of the phase diagram of full QCD with degenerate quark masses, the phase transition line is shifted to the region of high values of temperature when the quark mass increases. Such behavior influences the hadron properties and causes a non-analyticity in the meson mass.

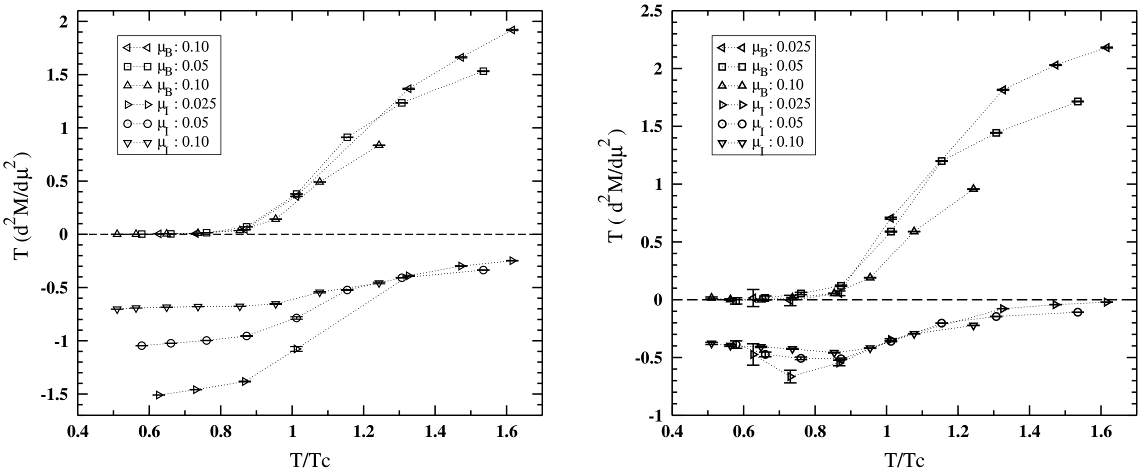

Figure 2: The second response of the screening masses of pseudo-scalar (PS) and vector (V) mesons.

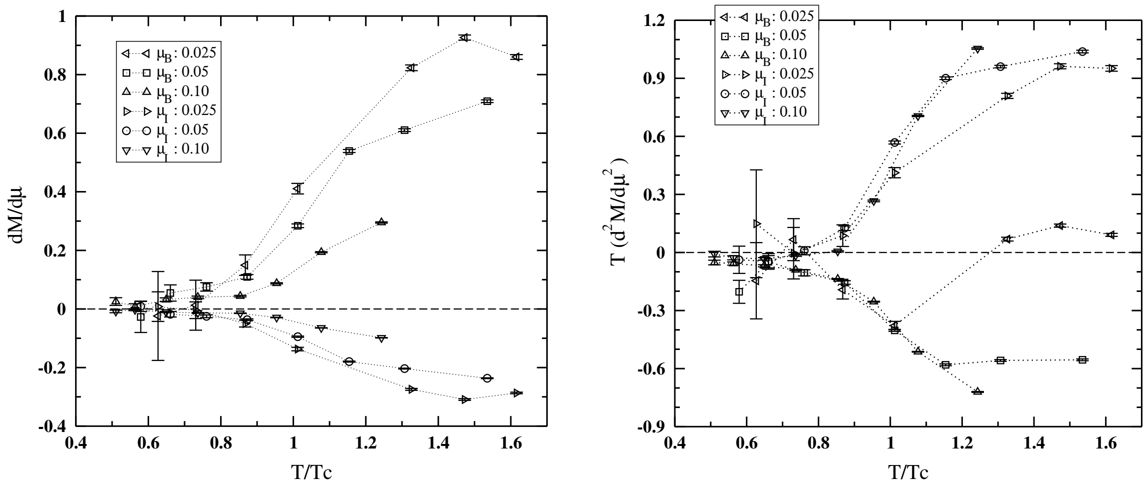

Figure 3: The 1st and 2nd responses of the nucleon screening masse.

Unlike the case of mesons, the first order response of the nucleon does not vanish for either μ_B and μ_I. Figure 3 displays the first and the second responses of baryon screening mass. Both The first and the second responses of baryon mass is close to zero in the confined phase. In the deconfined phase, the first and the second responses have opposite sign (except for the data of lightest quark mass above $1.2T_c$). Consequently, the baryon mass is less sensitive to chemical potential than meson masses.

3.2 $\mu_B = 0$, $\mu_I \neq 0$

In the confinement phase the second order response of mesons to μ_I is negative. At $\mu_I = m_\pi$ and $T = 0$, there is a second order phase transition to a pion condensation phase where π becomes massless. According to the chiral perturbation theory, the free energy of a pion drops to zero. Consequently, the second order response is negative, the more so as m_π decreases. With decreasing quark mass, the response increases in the case of pseudo-scalar meson. The vector meson does not show such strong dependence on the quark mass. At high temperature ($T > T_c$), it approaches zero quickly.

It is also possible to study the effects of μ_I on nucleon mass 3. The first comes from the isospin of the baryon, which effectively reduces the neutron mass by $\frac{1}{2}|\mu_I|$ (see the paper by Son and Stephanov SU3). Another effect which comes into play long before that is the repulsion of the nucleon by π^-'s is the condensate. It means that the baryon mass never drops to zero.

3.3 Chiral extrapolation

The question of how the chiral extrapolation should be done is much more subtle. It is fair to say that no unique satisfactory answer has been obtained. In order to guide the extrapolation of the hadron masses calculated in lattice QCD with dynamical fermions, we consider the following possible schemes: extrapolation with fixed (1) lattice spacing a, (2) coupling constant β, (3) ratio T/T_c. Here, we present results for the first scheme. The last two cases will be published elsewhere.

The result of chiral extrapolation of π and ρ-meson screening masses is shown in Fig.4. The behavior of the pion mass m_π at zero chemical potential is basically described by the Gell-Mann-Oakes-Renner relation. The π mass appears to be going to zero with the square root of the quark mass. The vector meson screening mass linearly depends on quark mass.

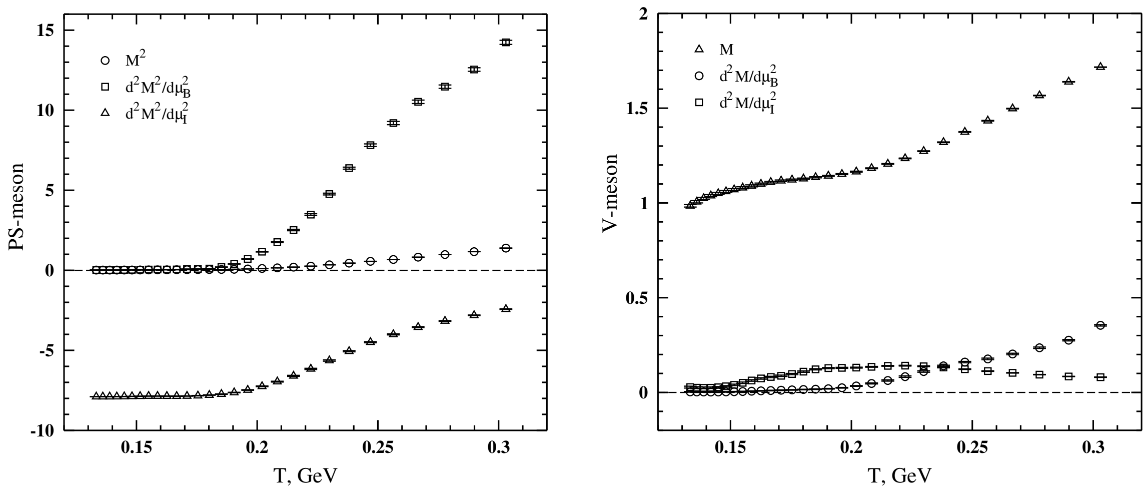

Figure 4: Chiral extrapolation of π and ρ-meson screening masses and their responses in physical units (GeV). The lattice spacing is fixed.

In the confined phase, the pion is protected from acquiring a large mass because chiral symmetry is broken. When the effects of density are switched on, the pion is getting heavier in the baryon rich environment. The case of vector meson is somewhat different. Both responses to μ_B and μ_I are positive with increasing (μ_B) and decreasing (μ_I) dependence on temperature in the region of high T.

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