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The screening length in hot QCD

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We discuss the temperature dependence of the screening lengths in quenched and full QCD using the non-perturbative lattice approach. We analyze the temperature dependence of distances which are defined as moments of quark antiquark free energies, *i.e.* we introduce

$$\langle r^n \rangle \equiv \frac{1}{\mathscr{Z}(\mathscr{T})} \int d^3r \, r^n F(r,T) \,, \qquad (n = 1,2,3,\ldots)$$

where $\mathscr{Z}(T)$ is a suitably chosen normalization constant and $F(r,T)$ denotes the change in free energy due to the presence of heavy quarks. These distance scales are supposed to describe the geometric size of partonic clouds which screen static charges in a medium and characterize distances beyond which the quark antiquark free energy is to large extent dominated by medium effects. At asymptotic high temperatures these moments can be related to the inverse Debye mass. In our numeric analysis we find that these moments drop rapidly in the vicinity of the phase transition and indicate distances which are about twice as large as the inverse Debye mass found in earlier studies. Our analysis supports recent findings that indicate that J/ψ will show significant medium modifications only at temperatures well above T_c.

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1. Introduction

In a hot and dense medium the screening radius rapidly decreases with increasing temperature and/or density and the occurrence of the phase transition in QCD towards a deconfined medium is intimately connected to a decreasing screening length. Due to unambiguities in perturbation theory at temperatures only moderately above phase transition most of todays quantitative discussions of the screening mass and length, and corresponding medium effects on heavy quark bound states [1, 2], are to large extent based on the non-perturbative lattice approach. Here, the Debye screening mass, $m_D(T)$, has been extracted from either the infrared limit of the gluon propagator, which should give a proper definition of the Debye mass, or from the screened Coulomb behavior of quark antiquark free energies at large distances. In such lattice studies large values for the screening mass were obtained [3, 4, 5, 6, 7, 8] which indicate a quite small screening length,

$$r_D(T) \equiv \frac{1}{m_D(T)} \,. \tag{1.1}$$

We will introduce here a screening length by characterizing the temperature dependence of the geometric size of gluonic clouds that are induced due to the presence of static color charges in the medium. For this purpose we introduce and discuss moments of distance dependent quark antiquark (free) energies at finite temperatures.

2. Quark antiquark interactions and screening

In a heat bath at fixed temperature the parton density distribution can be considered as being homogenous and the mean free path of partons is determined only by temperature and (parton) density. The presence of static charges in the medium, however, will polarize the medium and the parton density distribution will change compared to the density distribution of the heat bath without static charges. In particular, the parton density is expected to increase in the vicinity of static test charges and will depend also on the distance between them. This property is illustrated in Fig. 1 for the case of a static quark antiquark ($Q\bar{Q}$) pair: The region at which one expects a significant higher parton density might be well located in the vicinity of the test charges and will be considered by us in the following as *clouds* which screen and neutralize the additional color charges. At sufficiently large separation of the $Q\bar{Q}$ pair ($r \to \infty$) the interaction is screened through these clouds and both clouds will be well separated (see Fig. 1 (a)). In this case the energy (and entropy) which is needed to neutralize the charges will not depend on distance. When going to smaller distances, however, the clouds begin to overlap (b) and the geometric structure of the density distribution is supposed to depend also on distance (c). In this case the interaction energy of the quark antiquark will show also significant r-dependence.

The distance below which the interaction is significantly modified by distance could thus be used to characterize the sizes of the separated screening clouds. To get a more quantitative understanding of the screening length and the size of the clouds that surround the static quarks, we consider momenta, $\langle r^n \rangle$, which are supposed to characterize the parton densities induced by the

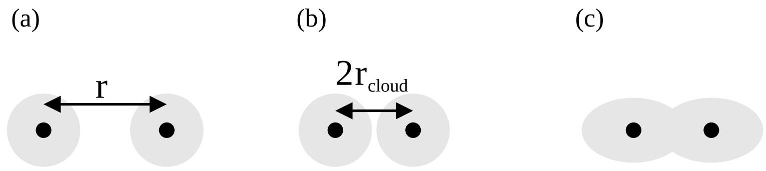

Figure 1: Illustration of screening through the polarization of gluons in the medium: (a) At large distances ($r \to \infty$) the gluon clouds which surround the static test charges are well separated. At smaller distances, $r < 2r_{cloud}$, the clouds begin to overlap (b) and the geometric structure of the clouds will depend on the separation between the test charges (c).

presence of strong interactions, *i.e.* we consider

$$\langle r^n \rangle_{1,8,av} = \frac{4\pi}{\mathscr{Z}_{1,8,av}(T)} \int_0^\infty dr\, r^{n+2}\, F_{1,8,av}(r,T) \; , \tag{2.1}$$

where the temperature dependence of the normalization constants is given by

$$\mathscr{Z}_{1,8,av}(T) \equiv 4\pi \int_0^\infty dr r^2\, F_{1,8,av}(r,T) \; . \tag{2.2}$$

In both relations we already have assumed that the free energies are normalized such that they approach zero at large distances, *i.e.* $F_{1,8,av}(r\to\infty,T) = 0$. In particular, when assuming a color screened Coulomb behavior for $F_{1,8}(r,T)$ at high temperature and large distances, *i.e.*

$$F_{1,8}(r,T) = -C_{1,8}\alpha(T) exp(-m_D(T)r)/r \; , \tag{2.3}$$

(with $C_1 = 4/3$ and $C_8 = -1/6$ for three colors, $N = 3$) we obtain

$$\langle r^n \rangle_{1,8} = (n+1)! \left(\frac{1}{m_D(T)} \right)^n \; . \tag{2.4}$$

To lowest order in g, thus, the size of the screening cloud is not expected to depend on the specific color representation of the $Q\bar{Q}$ state (singlet, octet) and the lowest moments, $\langle r \rangle_{1,8}$, are expected to be about twice as large as indicated by the inverse Debye mass[1],

$$\langle r \rangle_{1,8} = 2\frac{1}{m_D} = 2r_D \; . \tag{2.5}$$

Of course, similar relations can be obtained also for the manifest gauge invariant observables $\langle r^n \rangle_{av}$ using the high temperature perturbative relation $F_{av} \simeq -(F_1/T)^2/16$. In this case smaller distances are obtained, *i.e.* $\langle r^n \rangle_{av} = n!/(2m_D)^n$. In the limit of high temperatures the perturbative leading order relation between the coupling and the Debye mass indicate only a slowly decreasing screening length with increasing temperatures, *i.e.* $\langle r \rangle_{1,8,av} \sim 1/(gT)$.

On the other hand, at temperatures close but above T_c remnants of the confinement forces contribute to the non-perturbative properties of the quark antiquark free energies [7, 8] and deviations

[1] Of course, at small distances the coupling will be promoted to a coupling which runs also with distance, $\alpha(r,T)$, and thus the relations deduced here (and in what follows) give only the leading approximation for the relations between the Debye mass, m_D, and the given momenta at high temperature

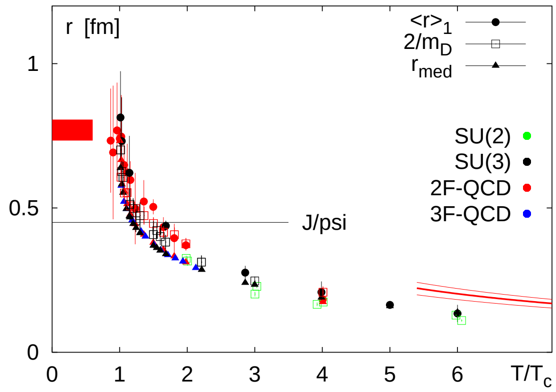

Figure 2: Lattice results for $\langle r\rangle_1$ as function of T/T_c in quenched ($N_f = 0$) and full QCD ($N_f = 2$) and compared to the size of J/ψ at $T = 0$. We also compare here $\langle r\rangle_1$ to different length scales obtained in terms of $2/m_D$ in $SU(2)$ [3], $SU(3)$ [7] and 2-flavor QCD [8] as well as to distances obtained for $SU(3)$ [10] and 2- and 3-flavor QCD [8] in terms of r_{med}. Further details on this figure, in particular on the different lines indicated here, are given in the text (see Sec. 3).

from a color screened Coulomb behavior become thus important already at intermediate distances. In particular, assuming a string-type potential in QCD at $T = 0$,

$$V(r) \simeq \begin{cases} -\frac{\pi}{12r} + \sigma(r - r_{\rm break}) + \frac{\pi}{12 r_{\rm break}} & : \quad r < r_{\rm break} \\ 0 & : \quad r > r_{\rm break} \end{cases}, \tag{2.6}$$

where $\sigma \simeq 4.5\ \mathrm{fm}^{-2}$ denotes the string tension and $r_{\rm break} \simeq (1.2 - 1.4)$ fm the distance at which the string breaks at $T = 0$, yields $\langle r\rangle_{T=0} \simeq (0.71 - 0.83)$ fm at $T = 0$.

3. Numeric results for screening mass and length

Our numeric studies of the length scale $\langle r^n\rangle_{1,8,av}$ in QCD are to large extent based on lattice calculations of quark antiquark free energies given in Refs. [9, 8] for QCD and calculations of free energies in quenched QCD [10, 7]. We also compare our results to the recent non-perturbative analysis of the Debye mass given in Refs. [7, 8] for $SU(3)$ and 2,3-flavor QCD and for $SU(2)$ [3]. To convert these masses to physical units we used $T_c \simeq 270$ MeV for quenched and $T_c \simeq 203$ MeV in 2- and $T_c \simeq 195$ MeV for 3-flavor QCD, which are the relevant transition temperatures at the quark mass values used in these calculations. We fixed the transition temperatures for $SU(2)$ being 290 MeV.

In Fig. 2 we summarize our lattice results for $\langle r\rangle_1$ obtained in quenched and full QCD (filled red circles: 2-flavor QCD; filled black circles: quenched QCD) in physical units as function of T/T_c. It can be seen that $\langle r\rangle_1 \simeq 0.7$ fm at temperatures in the close vicinity of the transition while $\langle r\rangle_1$ drops rapidly to values below 0.4 fm already at temperatures above $2T_c$. At higher temperatures $\langle r\rangle_1$ continues to decrease rather slowly with increasing temperatures. In particular, in the high temperature limit $\langle r\rangle_1$ is expected to decrease proportional to $(gT)^{-1}$. This behavior is also indicated for 2-flavor QCD in Fig. 2 by the red lines using Eq. 2.5 and the temperature dependence of the perturbative 2-loop coupling in the $\overline{MS}$-scheme (using renormalization scales $\mu \in [\pi, 4\pi]$). At the highest temperatures analyzed by us this estimate still leads to larger distances. It is, however, quite interesting to note here that at high temperatures similar distances to those indicated through $\langle r \rangle_1$ are also indicated through twice the inverse Debye mass, $2/m_D$. These distances are also shown in Fig. 2 by triangles.

Following Ref. [1] the decreasing behavior of (twice) the screening length above T_c may imply the onset of medium modifications on heavy quark bound states. To estimate the relevant temperatures at which medium effects on J/ψ may become important we also indicate in Fig. 2 the size of this state at $T = 0$ by a horizontal line. At appears quite reasonable that beyond temperatures at which twice the screening length becomes smaller than the size of the particular bound state screening effects may strongly influence the binding properties. From Fig. 2 it may thus appear reasonable that J/ψ survives the transition and may show significant medium modifications at temperatures only well above T_c. This can also be seen when comparing the data for $\langle r \rangle_1$ and $2/m_D$ in Fig. 2 with the scale r_{med} defined in [10], which is also shown in Fig. 2 by triangles. In fact, r_{med} has been introduced to indicate distances at which medium effects become important in the singlet free energy and both scales, $\langle r \rangle_1$ and r_{med}, indicate similar distances above T_c in the entire temperature range shown in Fig. 2. In particular, when comparing the different scales discussed above for quenched and 2-flavor QCD with todays attainable scales for $SU(2)$ and 3-flavor QCD no or only little differences appear at temperatures only moderately above the transition. Of course, the screening length in $SU(2)$ will diverge when going to temperatures in the close vicinity of T_c.

Finally we also compare in Fig. 2 the values for $\langle r \rangle_1$ obtained in quenched and full QCD at low temperatures to the value one may expect at zero temperature, $\langle r \rangle_{T=0}$, using the parameterization for $V(r)$ discussed in Sec. 2. This value is indicated by a thick line (left-hand side in Fig. 2). It can be seen that the values obtained for $\langle r \rangle_1$ become indeed quite similar to $\langle r \rangle_{T=0} \simeq 0.74 - 0.85$ fm already at temperatures close but above T_c. Again this may demonstrate the presence of remnants of the confinement forces above the transition and indicates that the singlet free energy deviates at intermediate and small distances from the leading order color screened Coulomb behavior expected at high temperatures. However, the rather small differences observed between $2/m_D$ and $\langle r \rangle_1$ at low temperatures presumably implies that the scale $\langle r \rangle_1$ is not very sensitive to the small distance properties of $F_1(r,T)$.

4. Conclusions

We have discussed the screening length in quenched and full QCD using properties of quark antiquark free energies calculated on the lattice. For this purpose we analyzed moments of the free energies, *i.e.* $\langle r \rangle_{1,8,av}$ defined in Eq. (2.1), which are supposed to characterize distances at which medium modification become important in quark antiquark free energies. We find that $\langle r \rangle_1$ drops rapidly at temperatures close above the phase transition while at high temperatures $\langle r \rangle_1$ is almost twice as large as the scale one obtains from the inverse Debye mass. A comparison of $\langle r \rangle_1$ with the size of J/ψ at $T = 0$ suggests that above temperatures about $1.3T_c$ the binding properties of J/ψ could be strongly influenced by screening. In particular, the length scales obtained for $SU(3)$ as well as for QCD ($N_f = 2,3$) show no or only little differences at temperatures which are moderately above the transition. Our analysis of the screening length in QCD thus supports recent

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lattice studies of quarkonium spectral functions above T_c in quenched [11, 12] and full QCD [13] which indicate that J/ψ may survive the transition and will dissolve at higher temperatures. At low temperatures, however, a more detailed analysis of $\langle r \rangle_{1,8,av}$ including also higher moments, will be interesting in future. A similar analysis of the screening length with this method may also become of considerable interest in QCD at finite temperatures and densities [14] using mesonic or baryonic [15] heavy quark free and/or internal energies [16].

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References

[1] T. Matsui and H. Satz, Phys. Lett. **B178** (1986) 416.

[2] For a recent review see F. Karsch, hep-lat/0502014.

[3] U. Heller, F. Karsch and J. Rank, Phys. Rev. **D57** (1998) 1438, hep-lat/9710033.

[4] O. Kaczmarek, F. Karsch, E. Laermann and M. Lutgemeier, Phys. Rev. **D62** (2000) 034021.

[5] S. Digal, S. Fortunato and P. Petreczky, Phys. Rev. **D68** (2003) 034008.

[6] A. Nakamura and T. Saito, Prog. Theor. Phys. **111** (2004) 733.

[7] O. Kaczmarek, F. Karsch, P. Petreczky and F. Zantow, Phys. Rev. **D70** (2004) 074505.

[8] O. Kaczmarek and F. Zantow, Phys. Rev. **D71** (2005) 114510.

[9] O. Kaczmarek *et al.*, hep-lat/0312015.

[10] O. Kaczmarek, F. Karsch, P. Petreczky and F. Zantow, Phys. Lett. **B543** (2002) 41.

[11] M. Asakawa and T. Hatsuda, Phys. Rev. Lett. **92** (2004) 012001.

[12] S. Datta, F. Karsch, P. Petreczky and I. Wetzorke, Phys. Rev. **D69** (2004) 094507.

[13] J.I. Skullerud *et al.*, PoS(LAT2005)176.

[14] M. Doring *et al.*, hep-lat/0509001 and PoS(LAT2005)193.

[15] K. Hubner, O. Kaczmarek, F. Karsch and O. Vogt, hep-lat/0408031 and K. Hubner, O. Kaczmarek and O. Vogt, PoS(LAT2005)194.

[16] O. Kaczmarek and F. Zantow, hep-lat/0506019 and O. Kaczmarek *et al.*, hep-lat/0309121.