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J/Ψ at high temperatures in anisotropic lattice QCD

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J/Ψ and η_c above the QCD critical temperature T_c are studied in anisotropic quenched lattice QCD, considering whether the $c\bar{c}$ systems above T_c are compact quasi-bound states or scattering states. We adopt the standard Wilson gauge action and $O(a)$-improved Wilson quark action with renormalized anisotropy $a_s/a_t = 4$ at $\beta = 6.10$ on $16^3 \times (14-26)$ lattices, which correspond to the spatial lattice volume $V \equiv L^3 \simeq (1.55\text{fm})^3$ and temperatures $T \simeq (1.11-2.07)T_c$. To clarify whether compact charmonia survive in the deconfinement phase, we investigate spatial boundary-condition dependence of the energy of the $c\bar{c}$ systems above T_c. In fact, for low-lying $c\bar{c}$ scattering states, there appears a significant energy difference $\Delta E \equiv E(\text{APBC}) - E(\text{PBC})$ between periodic and anti-periodic boundary conditions as $\Delta E \simeq 2\sqrt{m_c^2 + 3\pi^2/L^2} - 2m_c$ (m_c: charm quark mass) on the finite-volume lattice. In contrast, for compact charmonia, there is no significant energy difference between periodic and anti-periodic boundary conditions. As a lattice QCD result, we find almost no spatial boundary-condition dependence for the energy of the $c\bar{c}$ system in J/Ψ and η_c channels for $T \simeq (1.11-2.07)T_c$, which indicates that J/Ψ and η_c would survive as compact $c\bar{c}$ quasi-bound states below $2T_c$.

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1. Introduction

Since QCD is established, the quark-gluon-plasma (QGP) phase has been studied with much attention as a “new phase of matter" at high temperatures both in theoretical and experimental sides [1, 2, 3, 4]. In recent years, QGP creation experiments are actually performed at SPS [3] and RHIC [4] in high-energy heavy-ion collisions. As an important signal of the QGP creation, J/Ψ suppression [1, 2] was theoretically proposed and has been tested in the SPS/RHIC experiments. The basic assumption of J/Ψ suppression is that J/Ψ disappears above T_c due to vanishing of the confinement potential and the Debye screening effect [2].

Very recently, some lattice QCD calculations indicate an interesting possibility that J/Ψ and η_c seem to survive even above T_c [5, 6, 7], which may lead a serious modification for the J/Ψ suppression scenario in QGP physics. However, as a possible problem, the observed $c\bar{c}$ state on lattices may not be a nontrivial charmonium but a trivial $c\bar{c}$ scattering state, because it is difficult to distinguish these two states in lattice QCD.

In this paper, we aim to clarify whether the $c\bar{c}$ system above T_c is a compact quasi-bound state or a scattering state, which is spatially spread. To distinguish these two states, we investigate spatial boundary-condition dependence of the energy of the $c\bar{c}$ system by comparing results in periodic and anti-periodic boundary conditions. If the $c\bar{c}$ system is a scattering state, there appears an energy difference ΔE between the two boundary conditions as $\Delta E \simeq 2\sqrt{m_c^2+3\pi^2/L^2}-2m_c$ with the charm quark mass m_c on a finite-volume lattice with L^3. If the $c\bar{c}$ system is a compact quasi-bound state, the boundary-condition dependence is small even in finite volume. In Ref.[8], this method is actually applied for distinction between a scattering state and a compact resonance.

2. Method to distinguish a compact state from a scattering state

To begin with, we briefly explain the method to distinguish a compact state from a scattering state in term of its spatial extension. For this purpose, we investigate the $c\bar{c}$ system in the periodic boundary condition (PBC) and in the anti-periodic boundary condition (APBC), respectively, and examine spatial boundary-condition dependence for the $c\bar{c}$ system. Here, in the PBC/APBC case, we impose periodic/anti-periodic boundary condition for c and $\bar{c}$ on a finite-volume lattice.

Table 1: Periodic boundary condition (PBC) and anti-periodic boundary condition (APBC): the relation between spatial boundary condition and the minimum momentum $|\vec{p}_{\min}|$ of c, $\bar{c}$ and compact charmonia $c\bar{c}$.

PBC		
particle	spatial BC	$\lvert\vec{p}_{\min}\rvert$
c	periodic	0
$\bar{c}$	periodic	0
charmonia ($c\bar{c}$)	periodic	0

APBC		
particle	spatial BC	$\lvert\vec{p}_{\min}\rvert$
c	anti-periodic	$\sqrt{3}\pi/L$
$\bar{c}$	anti-periodic	$\sqrt{3}\pi/L$
charmonia ($c\bar{c}$)	periodic	0

For a compact $c\bar{c}$ quasi-bound state, the wave function of each quark is spatially localized and insensitive to spatial boundary conditions in lattice QCD, so that the charmonium behaves as a compact boson and its energy in APBC is almost the same as that in PBC [8]. For a $c\bar{c}$ scattering state, both c and $\bar{c}$ have non-zero relative momentum $\vec{p}_{\min} = (\pm\frac{\pi}{L}, \pm\frac{\pi}{L}, \pm\frac{\pi}{L})$, i.e., $|\vec{p}_{\min}| = \sqrt{3}\pi/L$

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in APBC, while they can take zero relative momentum $\vec{p}_{\rm min} = 0$ in PBC. In fact, if the $c\bar{c}$ system is a scattering state, there appears a significant energy difference ΔE between PBC and APBC due to the finite lattice volume of L^3, and it is estimated as $\Delta E \simeq 2\sqrt{m_c^2 + 3\pi^2/L^2} - 2m_c$. In our lattice QCD calculation, $|\vec{p}_{\rm min}|$ and ΔE for the $c\bar{c}$ scattering state are estimated as $|\vec{p}_{\rm min}| = \sqrt{3}\pi/L \simeq 0.69$GeV and $\Delta E \simeq 2\sqrt{m_c^2 + 3\pi^2/L^2} - 2m_c \simeq 0.35$GeV for $L \simeq 1.55$fm and $m_c \simeq 1.3$GeV.

3. Anisotropic lattice QCD

In this paper, we adopt anisotropic lattice QCD for the study of high-temperature QCD. In lattice QCD at temperature T, (anti)periodicity is imposed in the temporal direction with the period $1/T$, and hence it is technically difficult to measure temporal correlators at high temperatures. To overcome this problem, we use the anisotropic lattice with anisotropy $a_s/a_t = 4$. Owing to the finer temporal mesh, we can obtain detailed information for temporal correlators.

For the gauge field, we adopt the standard plaquette action on an anisotropic lattice as [8, 9]

$$S_G = \frac{\beta}{N_c}\frac{1}{\gamma_G}\sum_{s,i<j\le 3} {\rm ReTr}\{1 - P_{ij}(s)\} + \frac{\beta}{N_c}\gamma_G \sum_{s,i\le 3} {\rm ReTr}\{1 - P_{i4}(s)\}, \tag{3.1}$$

where $P_{\mu\nu}$ denotes the plaquette operator. In the simulation, we take $\beta \equiv 2N_c/g^2 = 6.10$ and the bare anisotropy $\gamma_G = 3.2103$, which lead to renormalized anisotropy as $a_s/a_t = 4.0$. The scale is set by the Sommer scale $r_0^{-1} = 395$MeV. Then, the spatial and temporal lattice spacing are estimated as $a_s^{-1} \simeq 2.03$GeV (i.e., $a_s \simeq 0.097$fm), and $a_t^{-1} \simeq 8.12$GeV (i.e., $a_t \simeq 0.024$fm), respectively. The adopted lattice size is $16^3 \times (14-26)$, which corresponds to the spatial lattice size as $L \simeq 1.55$fm and the temperature as $T = (1.11-2.07)T_c$. We use 999 gauge configurations, which are picked up every 500 sweeps after the thermalization of 20,000 sweeps.

For quarks, we use $O(a)$-improved Wilson (clover) action on the anisotropic lattice as [8, 9]

$$\begin{aligned} S_F &\equiv \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y), \\ K(x,y) \equiv \delta_{x,y} &- \kappa_t\{(1-\gamma_4)U_4(x)\delta_{x+\hat{4},y} + (1+\gamma_4)U_4^\dagger(x-\hat{4})\delta_{x-\hat{4},y}\} \\ &- \kappa_s \sum_i \{(r-\gamma_i)U_i(x)\delta_{x+\hat{i},y}\} + (r+\gamma_i)U_i^\dagger(x-\hat{i})\delta_{x-\hat{i},y}\} \\ &- \kappa_s c_E \sum_i \sigma_{i4}F_{i4}\delta_{x,y} - r\kappa_s c_B \sum_{i<j} \sigma_{ij}F_{ij}\delta_{x,y}, \end{aligned} \tag{3.2}$$

which is anisotropic version of the Fermilab action [10]. κ_s and κ_t denote the spatial and temporal hopping parameters, respectively, and r the Wilson parameter. c_E and c_B are the clover coefficients. The tadpole improvement is done by the replacement of $U_i(x) \to U_i(x)/u_s$, $U_4(x) \to U_4(x)/u_t$, where u_s and u_t are the mean-field values of the spatial and the temporal link variables, respectively. The parameters $\kappa_s, \kappa_t, r, c_E, c_B$ are to be tuned so as to keep the Lorentz symmetry up to $O(a^2)$. At the tadpole-improved tree-level, this requirement leads to $r = a_t/a_s$, $c_E = 1/(u_s u_t^2)$, $c_B = 1/u_s^3$ and the tuned fermionic anisotropy $\gamma_F \equiv (u_t\kappa_t)/(u_s\kappa_s) = a_s/a_t$. For the charm quark, we take $\kappa = 0.112$ with $1/\kappa \equiv 1/(u_s\kappa_s) - 2(\gamma_F + 3r - 4)$, which corresponds to the hopping parameter in the isotropic lattice. The bare quark mass m_0 in spatial lattice unit is expressed as $m_0 = \frac{1}{2}(\frac{1}{\kappa} - 8)$. We summarize the lattice parameters and related quantities in Table 2. In the present lattice QCD, the masses of J/Ψ and η_c are found to be $m_{J/\Psi} \simeq 3.07$GeV and $m_{\eta_c} \simeq 2.99$GeV at zero temperature.

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Table 2: Lattice parameters and related quantities in our anisotropic lattice QCD calculation.

β	lattice size	a_s^{-1}	a_t^{-1}	γ_G	u_s	u_t	γ_F	κ
6.10	$16^3 \times (14-26)$	2.03GeV	8.12GeV	3.2103	0.8059	0.9901	4.0	0.112

4. Temporal correlators of $c\bar{c}$ systems at finite temperature on anisotropic lattice

To investigate the low-lying state at high temperatures from the temporal correlator, it is practically desired to use a "good" operator with a large ground-state overlap, due to limitation of the temporal lattice size. To this end, we use a spatially-extended operator of the Gaussian type as

$$O(t,\vec{x}) \equiv N \sum_{\vec{y}} \exp\left\{-\frac{|\vec{y}|^2}{2\rho^2}\right\} \bar{c}(t,\vec{x}+\vec{y})\Gamma c(t,\vec{x}) \tag{4.1}$$

in the Coulomb gauge [8, 9]. N is a normalization. The size parameter ρ is optimally chosen in terms of the ground-state overlap. $\Gamma = \gamma_k (k = 1-3)$ and $\Gamma = \gamma_5$ correspond to $1^-(J/\Psi)$ and $0^-(\eta_c)$ channels, respectively. The energy of the low-lying state is calculated from the temporal correlator,

$$G(t) \equiv \frac{1}{V}\sum_{\vec{x}} \langle O(t,\vec{x}) O^\dagger(0,\vec{0})\rangle, \tag{4.2}$$

where the total momentum of the $c\bar{c}$ system is projected to be zero.

In accordance with the temporal periodicity at finite temperature, we define the effective mass $m_{\rm eff}(t)$ from the correlator $G(t)$ by the cosh-type function as [11]

$$\frac{G(t)}{G(t+1)} = \frac{\cosh[m_{\rm eff}(t)(t-N_t/2)]}{\cosh[m_{\rm eff}(t)(t+1-N_t/2)]} \tag{4.3}$$

with the temporal lattice size N_t. In the plateau region of $m_{\rm eff}(t)$, $m_{\rm eff}(t)$ corresponds to the energy of the low-lying $c\bar{c}$ state. To find the optimal value of ρ, we calculate the correlator $G(t)$ for $\rho = 0.2, 0.3, 0.4$ and 0.5fm at each temperature, and examine the ground-state overlap by comparing $G(t)/G(0)$ with the fit function of $g_{\rm fit}(t) = A\cosh[m(t-N_t/2)]$ [11]. As a result, the optimal size seems to be $\rho \simeq 0.2$fm for $c\bar{c}$ systems. Hereafter, we only show the numerical results for $\rho = 0.2$fm.

5. Lattice QCD results for the $c\bar{c}$ system above T_c

We investigate the $c\bar{c}$ systems above T_c both in $J/\Psi(J^P = 1^-)$ and $\eta_c(J^P = 0^-)$ channels. For each channel, we calculate the temporal correlator $G(t)$ and the effective mass $m_{\rm eff}(t)$ defined by Eq.(4.3) both in PBC and APBC, and examine their spatial boundary-condition (b.c.) dependence.

Figures 1-4 show the effective-mass plot $m_{\rm eff}(t)$ of the $c\bar{c}$ system in the J/Ψ channel for ρ=0.2fm. From the cosh-type fit for the correlator $G(t)$ in the plateau region of $m_{\rm eff}(t)$, we extract the energies, E(PBC) and E(APBC), of the low-lying $c\bar{c}$ system in PBC and APBC, respectively. Table 3 summarizes the $c\bar{c}$ system in the J/Ψ channel in PBC and APBC at each temperature.

As a remarkable fact, almost no spatial b.c. dependence is found for the low-lying energy of the $c\bar{c}$ system, i.e., $\Delta E \equiv E(\text{APBC}) - E(\text{PBC}) \simeq 0$, which is contrast to the $c\bar{c}$ scattering case of

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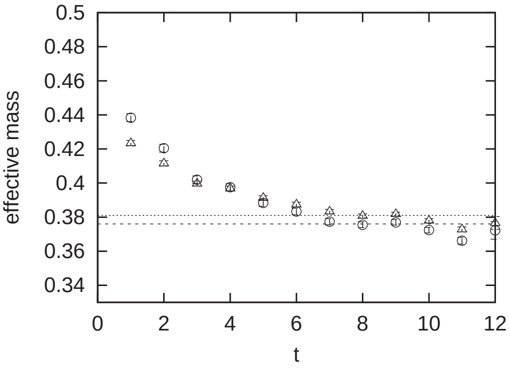

Figure 1: Effective mass of J/Ψ at $T = 1.11T_c$ in lattice unit. The circular/triangle symbols denote the results in PBC/APBC. The dashed/dotted lines denote E(PBC/APBC) in PBC/APBC.

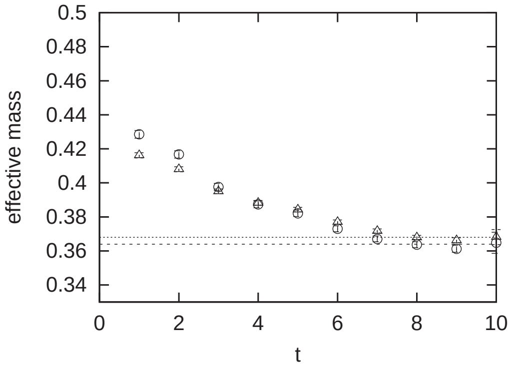

Figure 2: Effective mass of J/Ψ at $T = 1.32T_c$. The symbols, lines, units are same as Figure 1.

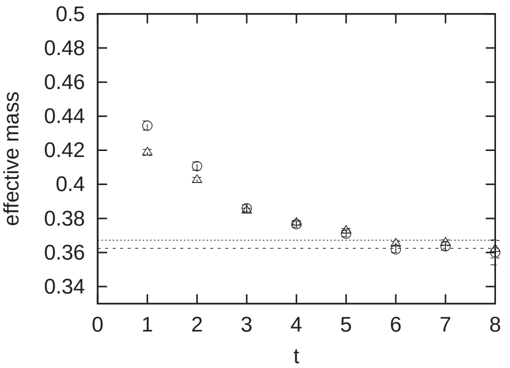

Figure 3: Effective mass of J/Ψ at $T = 1.61T_c$. The symbols, lines, units are same as Figure 1.

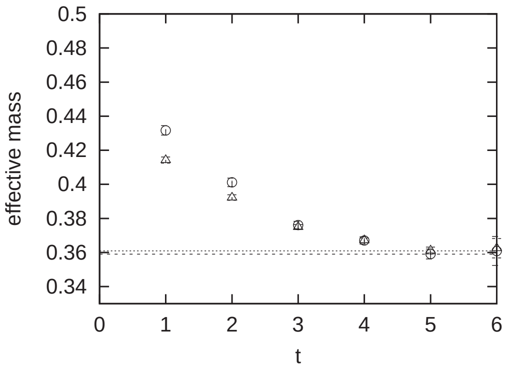

Figure 4: Effective mass of J/Ψ at $T = 2.07T_c$. The symbols, lines, units are same as Figure 1.

$\Delta E \simeq 2\sqrt{m_c^2 + 3\pi^2/L^2} - 2m_c \simeq 0.35$GeV for $L \simeq$1.55fm and $m_c \simeq 1.3$GeV as was discussed in Sect.2. This result indicates that J/Ψ survives for $T = (1.11 - 2.07)T_c$.

Table 4 summarizes the $c\bar{c}$ system in the η_c channel in PBC and APBC at each temperature. Again, almost no spatial b.c. dependence is found as $\Delta E \equiv E(\mathrm{APBC}) - E(\mathrm{PBC}) \simeq 0$, and this result indicates that η_c also survives for $T = (1.11 - 2.07)T_c$ as well as J/Ψ.

In contrast to J/Ψ and η_c, our preliminary lattice results show a large spatial b.c. dependence for the $c\bar{c}$ system in the χ_{c1} ($J^P = 1^+$) channel even near T_c, which seems consistent with Ref.[7].

6. Summary and conclusions

We have investigated J/Ψ and η_c above T_c with anisotropic quenched lattice QCD to clarify whether the $c\bar{c}$ systems above T_c are compact quasi-bound states or scattering states. We have adopted $O(a)$-improved Wilson quark action with renormalized anisotropy $a_s/a_t = 4$. Anisotropic lattice is technically important for the measurement of temporal correlators at high temperatures. We have use $\beta = 6.10$ on $16^3 \times (14 - 26)$ lattices, which correspond to $T = (1.11 - 2.07)T_c$.

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Table 3: The energy of the $c\bar{c}$ system in the J/Ψ channel ($J^P = 1^-$) in PBC and APBC at $\beta = 6.10$ and $\rho = 0.2$fm at each temperature. The statistical errors are smaller than 0.01GeV. We list also uncorrelated $\chi^2/N_{\rm DF}$ and $\Delta E \equiv E(\text{APBC}) - E(\text{PBC})$.

temperature	fit range	E(PBC) [$\chi^2/N_{\rm DF}$]	E(APBC) [$\chi^2/N_{\rm DF}$]	ΔE
$1.11T_c$	7–11	3.05GeV [0.14]	3.09GeV [0.61]	0.04GeV
$1.32T_c$	8–11	2.95GeV [0.34]	2.98GeV [0.33)]	0.03GeV
$1.61T_c$	6–9	2.94GeV [0.10]	2.98GeV [0.22]	0.04GeV
$2.07T_c$	5–7	2.91GeV [0.03]	2.93GeV [0.04]	0.02GeV

Table 4: The energy of the $c\bar{c}$ system in the η_c channel ($J^P = 0^-$) in PBC and APBC at $\beta = 6.10$ and $\rho = 0.2$fm at each temperature. The statistical errors are smaller than 0.01GeV. We list also uncorrelated $\chi^2/N_{\rm DF}$ and $\Delta E \equiv E(\text{APBC}) - E(\text{PBC})$.

temperature	fit range	E(PBC) [$\chi^2/N_{\rm DF}$]	E(APBC) [$\chi^2/N_{\rm DF}$]	ΔE
$1.11T_c$	7–11	3.03GeV [0.04]	3.02GeV [0.17]	-0.01GeV
$1.32T_c$	7–11	2.99GeV [0.78]	2.98GeV [0.82]	-0.01GeV
$1.61T_c$	6–9	3.00GeV [0.31]	2.97GeV [0.38]	-0.03GeV
$2.07T_c$	5–7	3.01GeV [0.03]	3.00GeV [0.07]	-0.01GeV

To conclude, we have found almost no spatial boundary-condition dependence of the energy of the low-lying $c\bar{c}$ system both in J/Ψ and η_c channels even on the finite-volume lattice. These results indicate that J/Ψ and η_c survive as compact $c\bar{c}$ quasi-bound states for $T = (1.11 - 2.07)T_c$.

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