

## The deconfining phase transition for SU(N) theories in 2+1 dimension

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**Jack Liddle\***

*University of Oxford*

*E-mail: j.liddle1@physics.ox.ac.uk*

**Mike Teper**

*University Of Oxford*

*E-mail: teper@thphys.ox.ac.uk*

The critical temperatures for the 2 + 1 dimensional  $SU(N_c)$  gauge theories are calculated, for  $N_c = 4, 5, 6$ . The transition is shown to be first order for  $N_c \geq 5$ . The critical temperature and latent heat are extrapolated to  $N_c = \infty$ .

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\*Speaker.

## 1. Introduction

We wish to learn more about the large  $N_c$  limit for  $SU(N_c)$  gauge theories. Quantities of interest to this study include the order of the phase transition, the critical temperature and the strength of the transition (measured by the latent heat). In this paper I will discuss the progress made so far in investigating the limit in 2 + 1 dimensions.

### 1.1 Why 2 + 1 dimensional pure gauge theories?

The deconfining phase transition for  $SU(N_c)$  theories in 3 + 1 dimensions has been looked at extensively (see [1, 2, 3]). The pure gauge theory is of interest, not only for the relative computational ease (compared to theories containing quarks), but for the rapidity with which the  $N_c = \infty$  limit can be approached, pure gauge theories having an  $O(\frac{1}{N_c^2})$  leading order correction and quark containing theories having an  $O(\frac{1}{N_c})$  correction [4].

The 2 + 1 dimensional theory differs from the 3 + 1 dimensional theory, in that it possesses a dimensionful coupling constant. But it shares many important similarities with the 3 + 1 dimensional theory, possessing a confining phase, weak coupling in the ultra-violet and strong in the infra-red.

## 2. Deconfinement and choice of order parameter

To determine the critical coupling an order parameter which can distinguish the two phases is needed. The Polyakov loop,

$$l_p = \text{Tr} \prod_{t=1}^{L_t} U_{(x,t)\hat{t}} \quad (2.1)$$

does not possess the global  $Z_{N_c}$  symmetry, as such we can use it to identify the whether the field is in a confining or deconfining phase. We define the loop susceptibility,

$$\frac{\chi}{V}(\beta) = \langle |\bar{l}_p|^2 \rangle - \langle |\bar{l}_p| \rangle^2 \quad (2.2)$$

which measures the size of the fluctuations of the loops, peaking when the Polyakov loop is equally likely to be found in either phase. A more physical quantity which we can think of is the specific heat, defined as

$$\frac{1}{\beta^2} C(\beta) = N_p \langle \bar{U}_p^2 \rangle - N_p \langle \bar{U}_p \rangle^2 \quad (2.3)$$

where  $\bar{U}_p$  is the average plaquette over a given configuration. This is related to latent heat of the transition, in the infinite volume limit,

$$\lim_{V \rightarrow \infty} \frac{1}{\beta_c^2 N_p} C(\beta_c) = \frac{1}{4} L_h^2 \quad (2.4)$$

Where  $N_p$  is the number of plaquettes. Pseudo critical couplings,  $\beta_c$ , are defined where the loop susceptibility and specific heat peak.

### 3. Reweighting

In order to locate these peaks in the loop susceptibility and specific heat we require the observables as a continuous function of the coupling. This is accomplished using a density of states reweighting technique [5]. Data from several Monte Carlo runs is used to estimate that density of states  $D(S)$ . This estimate for the density of states can be used, together with a Boltzmann factor, to estimate the expected action distribution  $P(S)$ .

$$P(S, \beta) = \frac{1}{Z(\beta)} D(S) e^{-\beta S} \quad (3.1)$$

where  $Z(\beta)$  is included as a normalisation factor. As a matter of computational practicality we bin the data into histograms  $P(S_i)$ . Knowing this estimate for  $P(S_i, \beta)$  and an estimate for an observable  $O$  given a particular  $S_i$ ,  $O(S_i)$ , we can construct the expectation value of the observable as a continuous function of  $\beta$ ,

$$\langle O(\beta) \rangle = \sum_{S_i} P(S_i, \beta) O(S_i) \quad (3.2)$$

When investigating a first order transition we must make sure we see enough transitions to ensure that our estimate for the density of states is not biased significantly in favour of either phase. Errors on the location and height of these peaks are obtained through a jackknife procedure.

### 4. Determining the order of the transition

The critical couplings receive finite volume corrections, it is through these corrections that the order of the transition is determined [6]. In physical units

$$\frac{T_c(\infty) - T_c(V)}{T_c(\infty)} = \begin{cases} \frac{h}{VT_c(\infty)^2} & \text{1st order} \\ \frac{h}{(VT_c(\infty)^2)^{\frac{1}{d\nu}}} & \text{2nd order} \end{cases} \quad (4.1)$$

The critical exponents  $\gamma, \nu$  parameterise the behaviour of the temperature divergence for a second order transition, with the correlation length diverging as  $|T - T_c|^{-\nu}$  and the loop susceptibility diverging as  $|T - T_c|^{-\gamma}$ . Writing eqn(4.1) in lattice units we have

$$\beta_c(V) = \begin{cases} \beta_c(\infty) \left(1 - h \left(\frac{N_t}{N_s}\right)^2\right) & \text{1st order} \\ \beta_c(\infty) \left(1 - h \left(\frac{N_t}{N_s}\right)^{\frac{2}{d\nu}}\right) & \text{2nd order.} \end{cases} \quad (4.2)$$

With the loop susceptibility behaving as

$$\chi \propto \begin{cases} V & \text{1st order} \\ V^{\frac{\gamma}{d\nu}} & \text{2nd order.} \end{cases} \quad (4.3)$$

## 5. Results

For  $SU(4)$  the study of the transition is challenging as it is believed that large, fine lattices are needed before it reveals itself to be a second order transition [7]. At  $L_t = 3$  it is weakly first order, when compared to  $SU(5)$  and  $SU(6)$ . This is shown in figures 1,2,3 with the  $SU(4)$  transition showing relatively frequent tunnelling between the phases, compared to  $SU(5)$  and  $SU(6)$  on the same volume. For  $SU(5)$  and  $SU(6)$  the transition is clearly first order, without any particular surprises as we approach the continuum limit. For  $SU(6)$  the presence of clear separation between the phases admits the possibility of calculating the domain wall tension between the phases.

### 5.1 $\frac{T_c}{\sqrt{\sigma}}$

The critical temperature in the infinite volume limit calculated on  $L_t = 3$  lattices (see figure 4) can be extrapolated with a conventional  $O(\frac{1}{N_c^2})$  correction to the  $N_c = \infty$  limit.

### 5.2 Latent Heat

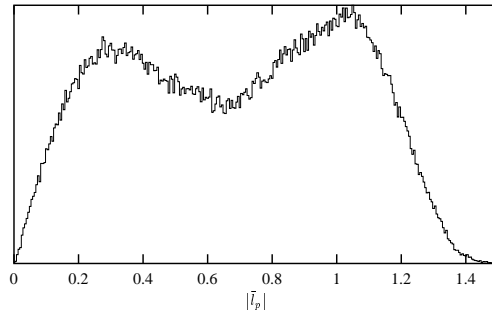
The latent heat calculated on  $L_t = 3$  lattice is shown in see figure 5 where it is extrapolated with an  $O(\frac{1}{N_c^2})$  correction. The latent heat is zero by the time  $N_c = 3$ . The latent heat of the  $SU(4)$  transition is finite but small, but may well be zero in the continuum limit.

## 6. Conclusion

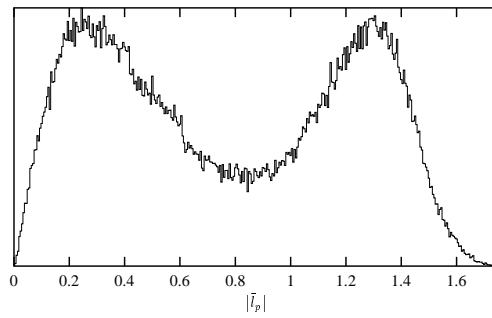
The transition for  $SU(5)$  and  $SU(6)$  are certainly first order. The large  $N_c$  limit is approached in a manner consistent with the expected  $O(\frac{1}{N_c^2})$  corrections. The strength of the transition appears to be growing with  $N_c$ . Work is currently underway to determine the critical temperatures in the continuum limit and find the appropriate  $N_c \rightarrow \infty$  limit.

## References

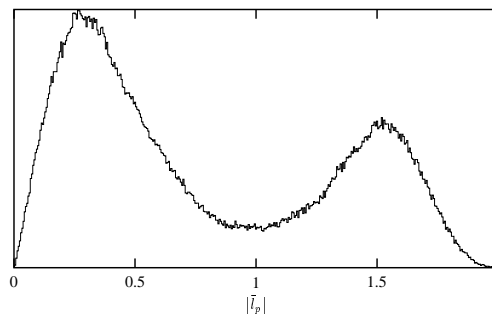
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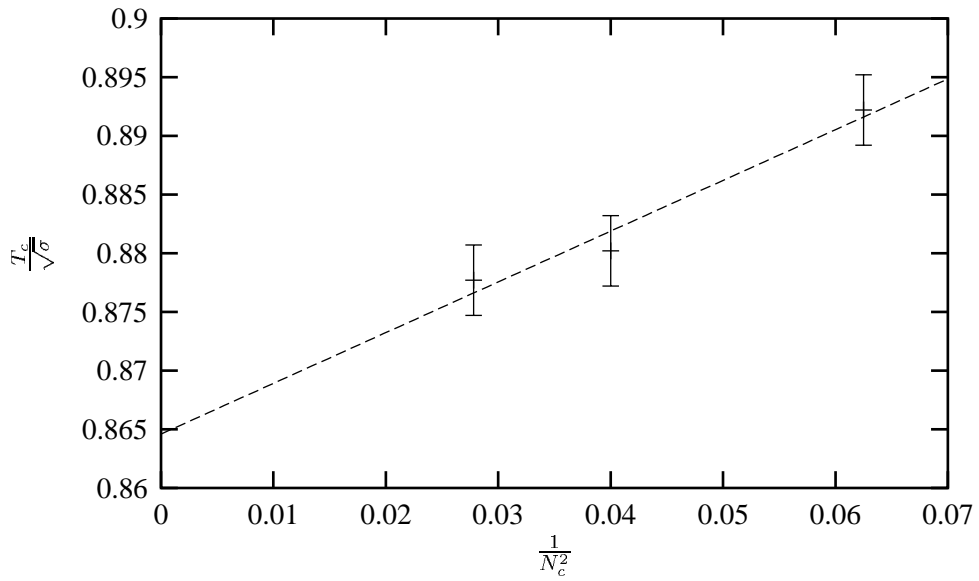
**Figure 1:**  $SU(4)$ : Distribution of  $|\bar{l}_p|$  on a  $25^2 \times 3$  lattice at  $\beta = 20.2$  close to the critical coupling



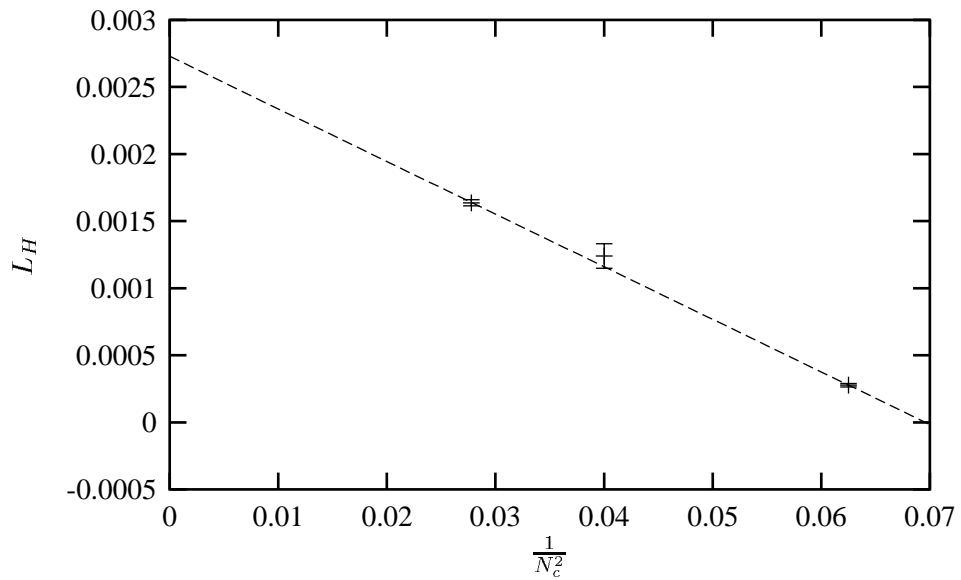
**Figure 2:**  $SU(5)$ : Distribution of  $|\bar{l}_p|$  on a  $25^2 \times 3$  lattice at  $\beta = 31.95$  close to the critical coupling



**Figure 3:**  $SU(6)$ : Distribution of  $|\bar{l}_p|$  on a  $25^2 \times 3$  lattice at  $\beta = 46.2$  close to the critical coupling



**Figure 4:** Extrapolation of  $\frac{T_c}{\sqrt{\sigma}}$  at  $L_t = 3$  to  $N_c = \infty$  with a  $\frac{1}{N_c^2}$  correction



**Figure 5:** Extrapolation of  $L_H$  at  $L_t = 3$  to  $N_c = \infty$  with a  $\frac{1}{N_c^2}$  correction