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Finite-temperature chiral transition in QCD with quarks in the fundamental and adjoint representation.

Francesco Basile
Dipartimento di Fisica dell'Università di Pisa and INFN,
Via Buonarroti 2, 56127 Pisa, Italy. E-mail: basile@sns.it

Andrea Pelissetto
Dipartimento di Fisica dell'Università di Roma "La Sapienza" and INFN,
Piazzale Moro 2, 00185 Roma, Italy. E-mail: Andrea.Pelissetto@roma1.infn.it

Ettore Vicari*
Dipartimento di Fisica dell'Università di Pisa and INFN,
Via Buonarroti 2, 56127 Pisa, Italy. E-mail: vicari@df.unipi.it

We study the nature of the finite-temperature chiral transition in QCD with N_f light quarks in the fundamental and adjoint representation. Universality and renormalization-group (RG) arguments show that the possibility of having a continuous transition is related to the existence of a stable fixed point (FP) in the RG flow of a 3D Landau-Ginzburg-Wilson Φ^4 theory with the same chiral symmetry-breaking pattern. The RG flow of these theories is studied by field-theoretical approaches, computing and analyzing high-order perturbative series, up to six loops. According to this RG analysis, the transition in QCD can be continuous only for $N_f = 2$. In this case it belongs to the 3D O(4) universality class. We also find a stable FP corresponding to a 3D universality class with symmetry breaking $U(2)_L \otimes U(2)_R \rightarrow U(2)_V$, which implies that the transition can be continuous also if the axial-anomaly effects are suppressed at T_c. In the case of quarks in the adjoint representation, we can have a continuous transition for $N_f = 1, 2$. For $N_f = 1$ it belongs to the O(3) universality class. For $N_f = 2$ it belongs to a new 3D universality class characterized by the symmetry breaking SU(4)→SO(4).

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The thermodynamics of Quantum Chromodynamics (QCD) is characterized by a transition from a low-T hadronic phase, in which chiral symmetry is broken, to a high-T phase with deconfined quarks and gluons (quark-gluon plasma), in which chiral symmetry is restored [1]. Our understanding of the finite-T phase transition is essentially based on the relevant symmetry and symmetry-breaking pattern (SBP). In the presence of N_f light quarks the relevant symmetry is the chiral symmetry $\mathrm{U}(1)_V \otimes \mathrm{SU}(N_f)_L \otimes \mathrm{SU}(N_f)_R$. At $T = 0$ this symmetry is spontaneously broken to $\mathrm{U}(1)_V \otimes \mathrm{SU}(N_f)_V$ with a nonzero quark condensate $\langle \bar{\psi}\psi \rangle$. The finite-$T$ transition is related to the restoring of the chiral symmetry. It is therefore characterized by an $N_f \times N_f$ complex-matrix order parameter Φ_{ij}, related to the bilinear quark operator $\bar{\psi}_{Li}\psi_{Rj}$, and the SBP

$$\mathrm{SU}(N_f)_L \otimes \mathrm{SU}(N_f)_R \rightarrow \mathrm{SU}(N_f)_V. \tag{1}$$

If the axial $\mathrm{U}(1)_A$ symmetry is effectively restored at T_c, the expected SBP becomes

$$\mathrm{U}(N_f)_L \otimes \mathrm{U}(N_f)_R \rightarrow \mathrm{U}(N_f)_V. \tag{2}$$

Lattice Monte Carlo (MC) simulations suggest that these is not the case in three-color QCD. However, since the anomaly gets suppressed in the large-N_c limit ($\partial_\mu J_5^\mu \propto \frac{1}{N_c} Q$), SBP (2) may be relevant in the large-N_c limit.

Although deconfinement and chiral symmetry restoration are apparently related to different mechanisms, they seem to be somehow coupled in QCD, since lattice computations show that the Polyakov loop has a sharp increase at T_c where the chiral condensate vanishes. However, the interplay between the two effects is not clear yet. Insight into this question may be gained by investigating related models, such as $\mathrm{SU}(N_c)$ gauge theories with N_f Dirac fermions in the adjoint representation (aQCD).[1] Unlike QCD, aQCD is also invariant under global $\mathbb{Z}_{N_c}$ transformations related to the center of the gauge group $\mathrm{SU}(N_c)$, as in pure $\mathrm{SU}(N_c)$ gauge theories. There are two well-defined order parameters in the light-quark regime, related to the confining and chiral modes, i.e. the Polyakov loop and the quark condensate. One generally expects two transitions: a deconfinement transition at T_d associated with the breaking of the $\mathbb{Z}_{N_c}$ symmetry, and a chiral transition at T_c in which chiral symmetry is restored. In aQCD with N_f massless flavors the chiral-symmetry group extends to $\mathrm{SU}(2N_f)$. At $T = 0$ this symmetry is expected to spontaneously break to $\mathrm{SO}(2N_f)$, due to quark condensation. Therefore the SBP at the finite-T chiral transition is expected to be

$$\mathrm{SU}(2N_f) \rightarrow \mathrm{SO}(2N_f) \tag{3}$$

with a symmetric $2N_f \times 2N_f$ complex matrix as order parameter related to the bilinear quark condensate. If the axial $\mathrm{U}(1)_A$ symmetry is restored at T_c, the SBP is

$$\mathrm{U}(2N_f) \rightarrow \mathrm{O}(2N_f). \tag{4}$$

MC simulations for $N_c = 3$ and $N_f = 2$ [2, 3] show that the deconfinement transition at T_d is first order. Results at the chiral transition appear consistent with a continuous transition. Interestingly, the ratio between the two critical temperatures turns out to be quite large, $T_c/T_d \approx 8$, suggesting a rather weak interplay between the corresponding underlying mechanisms.

[1] AQCD is asymptotically free only for $N_f < 11/4$, thus only the cases $N_f = 1, 2$ are interesting.

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In order to study the nature of the finite-T chiral transition in QCD and aQCD, we exploit universality and renormalization-group (RG) arguments, as originally applied by Pisarski and Wilczek in Ref. [4]. They can be summarized as follows.

(i) Let us first assume that the phase transition at T_c is continuous for vanishing quark masses. In this case the length scale of the critical modes diverges approaching T_c, becoming eventually much larger than $1/T_c$, which is the size of the euclidean "temporal" dimension at T_c. Therefore, the asymptotic critical behavior must be associated with a 3D universality class with the same SBP.
(ii) The existence of such a 3D universality class can be investigated by considering the most general Landau-Ginzburg-Wilson (LGW) Φ^4 theory compatible with the given SBP, which describes the critical modes at T_c. Neglecting the $U(1)_A$ anomaly, it is given by

$$\mathscr{L}_{\mathrm{U}(N)} = \mathrm{Tr}(\partial_\mu\Phi^\dagger)(\partial_\mu\Phi) + r\mathrm{Tr}\Phi^\dagger\Phi + \frac{u_0}{4}\left(\mathrm{Tr}\Phi^\dagger\Phi\right)^2 + \frac{v_0}{4}\mathrm{Tr}\left(\Phi^\dagger\Phi\right)^2. \tag{5}$$

If Φ_{ij} is a generic $N\times N$ complex matrix, the symmetry is $\mathrm{U}(N)_L\otimes\mathrm{U}(N)_R$, which breaks to $\mathrm{U}(N)_V$ if $v_0>0$, thus providing the LGW theory relevant for QCD with $N_f=N$. If Φ_{ij} is also symmetric, the global symmetry is $\mathrm{U}(N)$, which breaks to $\mathrm{O}(N)$ if $v_0>0$, which is the case relevant for aQCD with $N_f=N/2$. The reduction of the symmetry to $\mathrm{SU}(N_f)_L\otimes\mathrm{SU}(N_f)_R$ for QCD [$\mathrm{SU}(2N_f)$ for aQCD], due to the axial anomaly, is achieved by adding determinant terms, such as

$$\mathscr{L}_{\mathrm{SU}(N)} = \mathscr{L}_{\mathrm{U}(N)} + w_0\left(\det\Phi^\dagger + \det\Phi\right). \tag{6}$$

Nonvanishing quark masses can be accounted for by adding an external-field term $\mathrm{Tr}\left(H\Phi + \mathrm{h.c.}\right)$.
(iii) The critical behavior at a continuous transition is determined by the fixed points (FPs) of the RG flow: the absence of a stable FP generally implies first-order transitions. Therefore, a necessary condition of consistency with the initial hypothesis (i) of a continuous transition is the existence of stable FP in the corresponding LGW Φ^4 theory. If no stable FP exists, the finite-T chiral transition of QCD (aQCD) is predicted to be first order. If a stable FP exists, the transition can be continuous, and its universal critical behavior is determined by the FP; but this does not exclude a first-order transition if the system is outside the attraction domain of the stable FP.

The above arguments show that the nature of the finite-T transition in QCD and aQCD can be investigated by studying the RG flow of the corresponding 3D LGW Φ^4 theories. For this purpose we consider two different field-theoretical (FT) perturbative approaches. One is defined within the massive (disordered) phase using a zero-momentum renormalization (MZM) scheme. The other one is defined within the massless (critical) theory using a minimal subtraction ($\overline{\mathrm{MS}}$) scheme (we consider a 3D-$\overline{\mathrm{MS}}$ scheme without ε expansion). The RG flow is determined by the FPs, which are given by the common zeroes of the β-functions associated with the quartic couplings. A FP is stable if all eigenvalues of its stability matrix have positive real part. Using symbolic manipulations programs, we computed the expansion up to six loops in the MZM scheme (which requires the calculation of approximately 1500 Feynman diagrams) and up to five loops in the $\overline{\mathrm{MS}}$ scheme. Since perturbative FT expansions are asymptotic, it is necessary to resum the series. This is done by exploiting Borel summability and knowledge of the large-order behavior, which is inferred by semiclassical calculations of instanton solutions. We first resum the β-functions, and then search for their common zeroes. This computation is essentially nonperturbative, because the resummation uses nonperturbative information on their large-order behavior. The comparison of the analyses

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of the MZM and 3D-$\overline{\rm MS}$ expansions provides nontrivial crosschecks of the results. Details of these calculations can be found in Refs. [5, 6] (see also [7]). In the following we summarize the main results.

For $N = 1$, the case relevant to $N_f = 1$ QCD, $\mathscr{L}_{\rm U(1)}$ reduces to the O(2) symmetric Φ^4 theory, corresponding to the XY universality class of superfluid transition in ^{4}He, see, e.g., [8]. The determinant term related to the axial anomaly, cf. Eq. (6), plays the role of an external field, thus no continuous transition is expected, but a crossover.

The case $N = 2$ without anomaly, cf. Eq. (5), relevant for $N_f = 2$ QCD, was originally analyzed by Pisarski and Wilczek [4] within the $\varepsilon \equiv 4 - D$ expansion to one-loop order, which means close to 4D. No stable FP is found close to 4D, as also in the case of symmetric matrix field relevant for $N_f = 1$ aQCD. Thus a naive extrapolation to 3D indicates first-order transitions. However, in some physically interesting cases the extrapolation of ε-expansion calculations to $\varepsilon = 1$ fails to provide the correct physical picture in 3D: for example, in the Ginzburg-Landau model of superconductors [10], and in O(2)⊗O(N) theories of some frustrated spin models [11]. Actually, this happens also in the case of the LGW Φ^4 theory (5) with $N = 2$. Indeed, the analysis of the high-order series shows the presence of stable FPs in both MZM and 3D-$\overline{\rm MS}$ schemes, contradicting earlier analyses based on the ε expansion around 4D.[2] These results imply the existence of 3D universality classes with SBP $\mathrm{U(2)}_L\otimes\mathrm{U(2)}_R\rightarrow\mathrm{U(2)}_V$ and U(2)→O(2),[3] corresponding respectively to $N_f = 2$ QCD and $N_f = 1$ aQCD in the case of suppressed $\mathrm{U(1)}_A$ anomaly.

In two-flavor QCD, taking into account the $\mathrm{U(1)}_A$ anomaly, SBP (1) becomes equivalent to the one of the O(4) vector universality class, i.e. SO(4)→SO(3). SBP (3) of $N_f = 1$ aQCD is instead equivalent to the one of the O(3) vector universality class. This means that, if the transition is continuous, it must show the O(4) scaling behavior in $N_f = 2$ QCD and the O(3) one in $N_f = 1$ aQCD. See, e.g., Ref. [9] for a recent review on the O(N) universality classes. Actually, the LGW Φ^4 theory corresponding to these cases is

$$\mathscr{L}_{\rm SU(2)} = \mathrm{Tr}(\partial_\mu\Phi^\dagger)(\partial_\mu\Phi) + r\mathrm{Tr}\Phi^\dagger\Phi + \frac{u_0}{4}\left(\mathrm{Tr}\Phi^\dagger\Phi\right)^2 + \frac{v_0}{4}\mathrm{Tr}\left(\Phi^\dagger\Phi\right)^2 \tag{7}$$
$$+w_0\left(\det\Phi^\dagger + \det\Phi\right) + \frac{x_0}{4}\left(\mathrm{Tr}\Phi^\dagger\Phi\right)\left(\det\Phi^\dagger + \det\Phi\right) + \frac{y_0}{4}\left[(\det\Phi^\dagger)^2 + (\det\Phi)^2\right],$$

where $w_0, x_0, y_0 \sim g$ and g parametrizes the effective breaking of the $\mathrm{U(1)}_A$ symmetry. If the anomaly is suppressed ($g = 0$), then $w_0 = x_0 = y_0 = 0$. $\mathscr{L}_{\rm SU(2)}$ contains two quadratic (mass) terms, therefore it describes several transition lines in the T-g plane, which meet at a multicritical point for $g = 0$. In the case of QCD the multicritical behavior is controlled by the $\mathrm{U(2)}_L\otimes\mathrm{U(2)}_R$ symmetric theory. Possible phase diagrams in the T-g plane are shown in Fig. 1. When $g \neq 0$ the transition may be first order or continuous in the O(4) universality class. Actually, we may also have a mean-field behavior (apart from logarithms) for particular values of g, see Fig. 1. A similar scenario applies also to $N_f = 1$ aQCD.

No stable FPs are found for $N > 2$ in the LGW theory (5). Thus, neglecting the anomaly, transitions are always first order when $N_f > 2$ for QCD and $N_f > 1$ for aQCD. In most cases this result does not change if we take into account the axial anomaly, cf. Eq. (6). The only exception is

[2] This result was overlooked in Ref. [6]. Some details can be found in Ref. [5].

[3] The U(2)/O(2) universality class is also the one of the normal-to-planar superfluid transition in ^{3}He [12, 5].

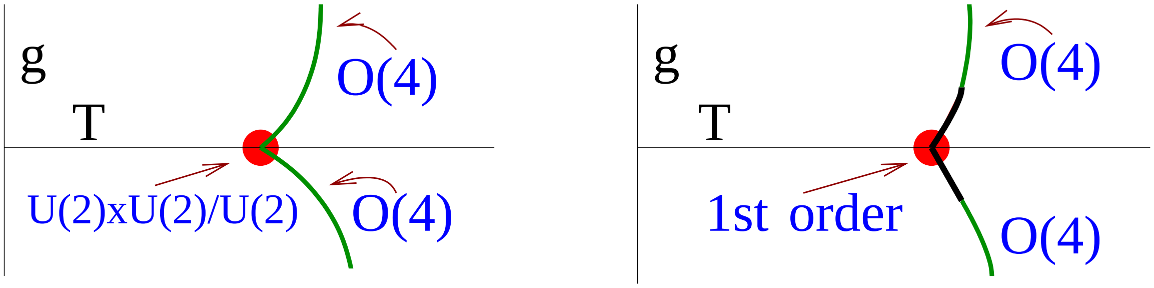

Figure 1: Possible phase diagrams in the T-g plane for the LGW theory (7) describing the transition of $N_f = 2$ QCD, in the case the transition at the multicritical point, i.e. for $g = 0$, is continuous (left) or first order (right). Thick black lines indicate first-order transitions. At their end points, thus for particular values of g, the transition should be of mean-field type (apart from logarithms).

	$U(1)_A$ anomaly	suppressed anomaly at T_c
QCD	$SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V$	$U(N_f)_L \otimes U(N_f)_R \to U(N_f)_V$
$N_f = 1$	crossover or first order	O(2) or first order
$N_f = 2$	O(4) or first order	$U(2)_L \otimes U(2)_R / U(2)_V$ or first order
$N_f \geq 3$	first order	first order
aQCD	$SU(2N_f) \to SO(2N_f)$	$U(2N_f) \to O(2N_f)$
$N_f = 1$	O(3) or first order	U(2)/O(2) or first order
$N_f = 2$	SU(4)/SO(4) or first order	first order

Table 1: Summary of the RG predictions. For each case we report the possible types of transition, indicating the universality class when the transition can also be continuous.

the case related to the $N_f = 2$ aQCD, where we find a stable FP corresponding to a 3D SU(4)/SO(4) universality class, with critical exponents $\nu \approx 1.1$ and $\eta \approx 0.2$. Note that, although SU(4)$\simeq$O(6), the SBP of $N_f = 2$ aQCD, i.e. SU(4)$\to$SO(4), differs from that of the O(6) vector model, i.e. SO(6)$\to$ SO(5). Thus, according to the standard paradigm that relates the universality class to the SBP, the corresponding universality classes and critical behaviors must be different.

The predictions of our RG analysis for the finite-T chiral transitions in QCD and aQCD are summarized in Table 1. These transitions have also been investigated by lattice MC simulations. Overall, MC results for two-flavor QCD seem to favor a continuous transition in the continuum limit. However, a satisfactory check of the O(4) scaling behavior has not been achieved yet. Results obtained using Wilson fermions appear consistent with a continuous transition in the O(4) universality class [13]; MC simulations using staggered fermions appear more problematic [14, 15, 16, 17, 3]. Unlike $N_f = 2$ QCD, the transition scenario appears settled for $N_f \geq 3$: MC simulations [14, 18] show first-order transitions, in agreement with the RG predictions. Finally, in the case of $N_f = 2$ aQCD the available MC results [2, 3] favor a continuous transitions. But they are not yet sufficiently precise to check the critical behavior of the 3D SU(4)/SO(4) universality class.

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