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Onium Masses with Three Flavors of Dynamical Quarks

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We have greatly extended an earlier calculation of the charmonium spectrum on three flavor dynamical quark ensembles by using more recent ensembles generated by the MILC collaboration. The heavy quarks are treated using the Fermilab formulation. The charmonium state masses are in reasonable agreement with the observed spectrum; however, some of the spin splittings may still be too small.

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1. Introduction

Calculating the spectrum of onium states is a significant challenge for lattice gauge theory. A number of levels can be studied for both charm and bottom quarks. However, dealing with heavy quarks requires special care [1, 2]. Using improved staggered sea quarks [3], it is possible to reproduce many of the most important features of the spectrum [4], which had not been done in the quenched approximation. This paper updates our work presented at Lattice 2003 [5].

2. Calculational Details

Ensembles for three lattice spacings were provided by the MILC Collaboration [6]: $a \approx 0.18$ fm ("extra-coarse"), $a \approx 0.12$ fm ("coarse"), and $a \approx 0.086$ fm ("fine"). (See Table 1.) For the extra coarse am_q / am_s = 0.6, 0.4, 0.2 and 0.1; for the coarse lattice, we also have 0.14, but we have only analyzed two values 0.4 and 0.2 for the fine lattice. From 400 to 600 configurations have been analyzed in most ensembles. The most notable exception is the coarse ensemble with am_q / $am_s = 0.1$. For each of the lattice spacings, the scale of each ensemble with different sea quark masses was kept approximately fixed using the length r_1 [7, 8] from the static quark potential. The absolute scale from the Υ $2S$–$1S$ splitting was determined by the HPQCD/UKQCD group [9, 4] on most of our ensembles implying $r_1 = 0.318(7)$ fm.

am_q / am_s	$10/g^2$	size	volume	config.	a (fm)
0.0492 / 0.082	6.503	$16^3 \times 48$	$(2.8\text{ fm})^3$	401	0.178
0.0328 / 0.082	6.485	$16^3 \times 48$	$(2.8\text{ fm})^3$	331	0.177
0.0164 / 0.082	6.467	$16^3 \times 48$	$(2.8\text{ fm})^3$	645	0.176
0.0082 / 0.082	6.458	$16^3 \times 48$	$(2.8\text{ fm})^3$	400	0.176
0.03 / 0.05	6.81	$20^3 \times 64$	$(2.4\text{ fm})^3$	559	0.120
0.02 / 0.05	6.79	$20^3 \times 64$	$(2.4\text{ fm})^3$	460	0.120
0.01 / 0.05	6.76	$20^3 \times 64$	$(2.4\text{ fm})^3$	593	0.121
0.007 / 0.05	6.76	$20^3 \times 64$	$(2.4\text{ fm})^3$	403	0.121
0.005 / 0.05	6.76	$24^3 \times 64$	$(2.9\text{ fm})^3$	136	0.120
0.0124 / 0.031	7.11	$28^3 \times 96$	$(2.4\text{ fm})^3$	261	0.0863
0.0062 / 0.031	7.09	$28^3 \times 96$	$(2.4\text{ fm})^3$	472	0.0861

Table 1: Ensembles used in this calculation.

We use the Asqtad improved staggered sea quark action that has errors of $O(\alpha_s a^2)$. The improved gluon action has errors of $O(\alpha_s^2 a^2)$. For the heavy valence quarks, we use the Sheikholeslami-Wohlert action [10] (which has $O(\alpha_s a)$ errors) with the Fermilab interpretation [2]. To compute heavy quark propagators, we use point and smeared sources and sinks. The smearing approximates 1S or 2S wavefunctions. At the sink, spatial momentum $2\pi/(La)[p_x, p_y, p_z]$ is given to the onium state. We restrict the range of p such that $\sum p_i^2 \leq 9$.

To find the onium masses, we fit two channels simultaneously for the zero momentum states. A delta function and a 1S smearing wave function are used as the source and sink. The ground state and up to three excited states are included in the fit. The minimum and maximum distance from

the source are varied, and the best fit is selected based on the confidence level and size of error in the ground state and first excited state masses. After choosing the fit range, 250 bootstrap samples are generated to provide an error estimate.

We must tune the hopping parameter κ to the charm or bottom mass. For each lattice spacing, we select a sea quark mass independent value for κ. The tuning is done on an ensemble with small sea quark mass. In fact, as this project was done in conjuction with a study of heavy-light mesons, the tuning was done for the D_s mass. The precision of that tuning was only about 8%. Because of lattice artifacts that arise for heavy states, we distinguish between the rest mass aM_1 and the kinetic mass aM_2. We use $\kappa = 0.120$, 0.119 and 0.127 on the extra coarse, coarse and fine ensembles, respectively. The imprecision of our tuning is immediately seen in Fig. 1.

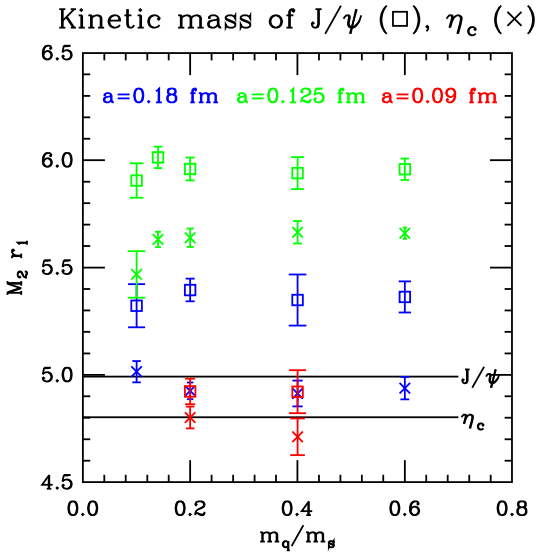


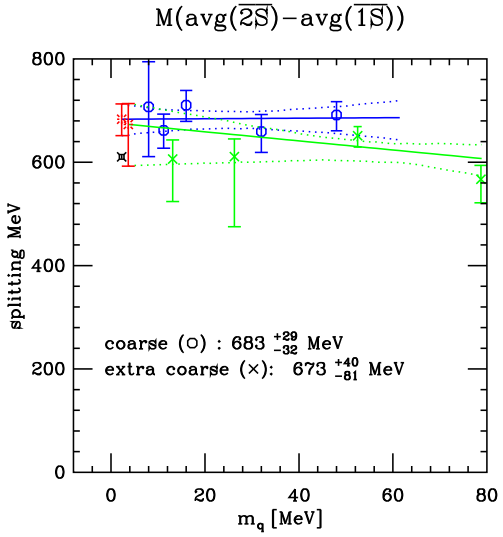
Figure 1: The kinetic masses of J/ψ and η_c on each ensemble plotted as a function m_q/m_s the light sea to strange quark mass ratio. Masses are in units of r_1. The physical masses are shown as lines.

The kinetic masses have two disadvantages: their statistical errors are large compared to those of the rest masses, and the pattern of systematic errors is more subtle [11]. However, for level splittings, a large discretization effect in the quark's rest mass drops out of the energy differences of hadron rest masses [12]. So, having tuned to approximately the right charm mass, we will now consider splittings based upon the rest masses of the various states. These states have been studied: $\eta_c(1S)$, $\eta_c(2S)$, $\psi(1S)$, $\psi(2S)$, $h_c(1P)$, $\chi_{c0}(1P)$ and $\chi_{c1}(1P)$. The $\chi_{c2}(1P)$ is also under study with a nonrelativistic P-wave source [13]. Currently, results for $\chi_{c2}(1P)$ are only available on one extra coarse ensemble. We also use the spin-averaged mass, *e.g.*, $\overline{1S} = [3M_{\psi(1S)} + M_{\eta_c(1S)}]/4$ to display some of the splittings in the spectrum.

3. Results

For each lattice spacing, we plot the splittings as a function of the mass of the light sea quarks. A linear chiral fit is made and the splitting is extrapolated to the physical value of $\hat{m} = (m_u + m_d)/2$

where the lattice spacing dependent value of $\hat{m}$ is determined from analysis of π and K meson decays constants [14]. The light meson decay constant analysis has not yet been completed on the extra coarse ensembles, so the value of $\hat{m}$ used there is only a rough estimate.



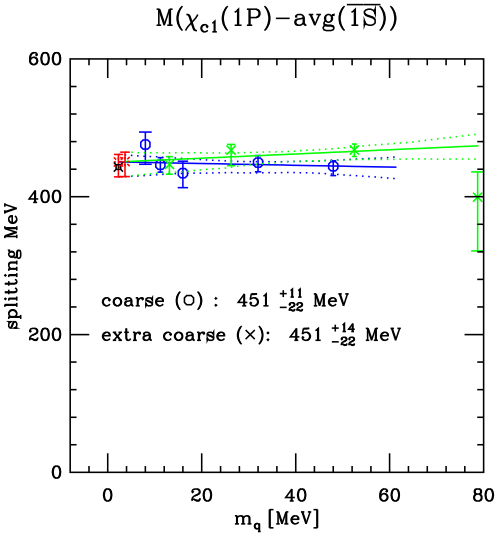


Figure 2: (left) The chiral extrapolation of the spin-averaged splitting between the 2S and 1S states on the extra coarse and coarse ensembles. The extrapolated values are shown in red, and the physical value in black.

Figure 3: (right) The splitting between the χ_{c1} ($^3P_1$) and spin-averaged 1S states on the extra coarse and coarse ensembles.

Within our current statistical uncertainties, we see reasonable agreement with the experimental value of the splittings of the spin-averaged 2S and 1S levels. The coarse value is about 2 σ high. (See Fig. 2.) As we do not yet have a full set of results for the χ_{c2}, we cannot construct the spin average of the 1P states. Instead, we use the χ_{c1} and the h_c. In nonrelativistic potential models, these two states are degenerate with each other and the spin average. The experimental splittings are well reproduced for these states. (See Figs. 3 and 4.)

As seen in Fig. 5, the spin splittings are too small. For J/ψ and η_c it amounts to about 10–22 MeV. The splitting is 19% too small for the extra coarse ensemble, 14% too small for the coarse, and 9% too small for the fine. The splitting seems to systematically improve as the lattice spacing decreases. We have not yet attempted a continuum extrapolation.

The overall agreement between this calculation with dynamical quarks and the observed spectrum is very good. The most obvious issue is the smallness of spin splittings, as seen in the J/ψ – η_c splitting, and the mass of the χ_{c0} state. There is some evidence of improvement as the lattice spacing is reduced.

4. Outlook

There are several ways to improve this calculation in the near future: We can include another fine ensemble with $m_q = 0.1m_s$. This more chiral ensemble is still being generated, but is far enough along that it would be worth starting the analysis. We also need to examine alternative fits to the ones that were selected by our automated procedure. Fermilab/MILC are almost finished

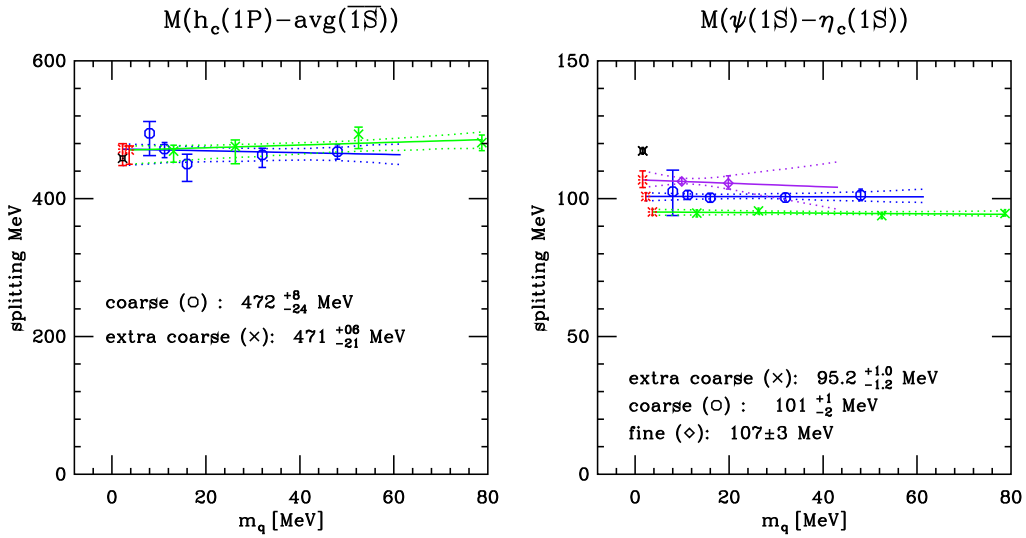


Figure 4: (left) Same as Fig. 3, except for the h_c ($^1P_1$).

Figure 5: (right) Splitting between J/ψ and η_c for all three lattice spacings. There are only two ensembles for the fine lattice spacing, shown in purple.

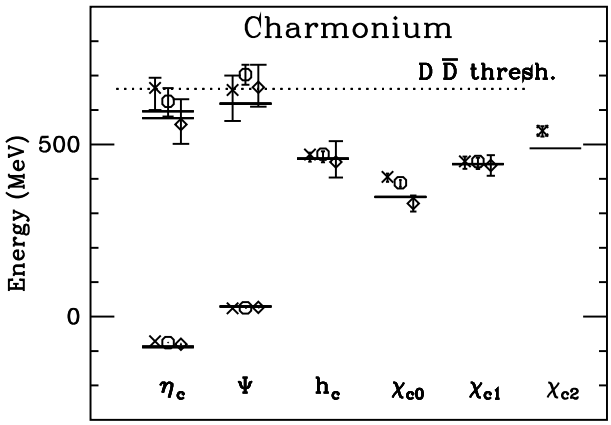


Figure 6: Charmonium spectrum for all three lattice spacings compared with experimental values. Energy is offset so that zero represents the spin-averaged 1S energy. From left to right for each state, crosses, octagons and diamonds are from the extra coarse, coarse and fine ensembles, respectively. The extra coarse χ_{c2} value without chiral extrapolation is the fancy cross.

generating a new set of ensembles at a lattice spacing between extra coarse and coarse. Production running on additional ensembles for the new *P*-wave code will be done. We also plan to use heavier quarks to study bottomonium, which has already been studied on these configurations using NRQCD [9].

In the longer term, MILC is generating new ensembles with $a \approx 0.06$ fm that should help us better understand the continuum limit. However, in the current calculation, lattice spacing dependence does not seem very large compared with statistical errors for most states.

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