PROCEEDINGS OF SCIENCE

B and D meson semileptonic decays in three-flavor lattice QCD

P.B. Mackenzie[*,a], C. Aubin[c,d], C. Bernard[d], C. DeTar[e], M. Di Pierro[f], Steven Gottlieb[h], E. Gregory[j], U.M. Heller[k], J.E. Hetrick[l], A.X. El-Khadra[m], A.S. Kronfeld[a], L. Levkova[h], F. Maresca[e], D. Menscher[m], M. Nobes[n,o], M. Okamoto[a], M.B. Oktay[q], J. Osborn[e], D. Renner[j], J.N. Simone[a], R. Sugar[p], D. Toussaint[j], H.D. Trottier[o]

E-mail: mackenzie@fnal.gov

[a]*Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA*
[c]*Physics Department, Columbia University, New York, New York, USA*
[d]*Department of Physics, Washington University, St. Louis, Missouri 63130, USA*
[e]*Physics Department, University of Utah, Salt Lake City, Utah 84112, USA*
[f]*School of Computer Science, Telecommunications and Information Systems, DePaul University, Chicago, Illinois 60604, USA*
[h]*Department of Physics, Indiana University, Bloomington, Indiana 47405, USA*
[j]*Department of Physics, University of Arizona, Tucson, Arizona 85721, USA*
[k]*American Physical Society, Ridge, New York 11961, USA*
[l]*Physics Department, University of the Pacific, Stockton, California 95211, USA*
[m]*Physics Department, University of Illinois, Urbana, Illinois 61801, USA*
[n]*Laboratory of Elementary-Particle Physics, Cornell University, Ithaca, New York 14853, USA*
[o]*Physics Department, Simon Fraser University, Burnaby, British Columbia V5A 1S6, Canada*
[p]*Department of Physics, University of California, Santa Barbara, California 93106, USA*
[q]*School of Mathematics, Trinity College, Dublin, Ireland*

Fermilab Lattice, MILC, and HPQCD Collaborations

We have calculated the semileptonic form factors of B and D mesons using unquenched, improved staggered light quarks, and improved clover heavy quarks. We discuss the use of unitarity constraints to bound the form factors in the high recoil momentum region.

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1. Introduction

Semileptonic decays of B and D mesons have a central place in lattice phenomenology of the Standard Model. They are processes with single, stable mesons, and so are among the most tractable of current lattice calculations. They are of crucial importance in determining Standard Model parameters: they can pin down four elements of the CKM matrix. D semileptonic decays are of particular interest now because CLEO is measuring them more accurately than ever before (a few per cent). B semileptonic decays are of interest because they feed directly into the analysis of the unitarity triangle. We are making a study of these decays with unquenched improved staggered fermions, following the general approach begun in Ref. [1]. Results for D mesons were reported in Ref. [2]. Earlier results were reported in Ref. [3]. Note also this year's plenary review at Lattice 2005 by Okamoto.

2. Methods

We use $\mathcal{O}(a)$ improved clover heavy quarks with the Fermilab interpretation [4]. We use $\mathcal{O}(a^2)$ improved, "asqtad" light quarks [5] and $\mathcal{O}(a^2)$ improved glue. Our main results employ the asqtad "coarse" data set, with lattice spacing $a = 0.121$ fm., volume=$20^3 \times 64$, and light quark masses m_l=0.12-0.3 m_s. The data sets for each light quark mass contained around 500 configurations each. We used four separate time sources on each gauge configuration. No correlations were visible between the time sources. Staggered chiral perturbation theory [6] was used in the chiral extrapolations. One-loop lattice perturbation theory [7] was used, with a nonperturbative estimate of the current renormalization following Ref. [8].

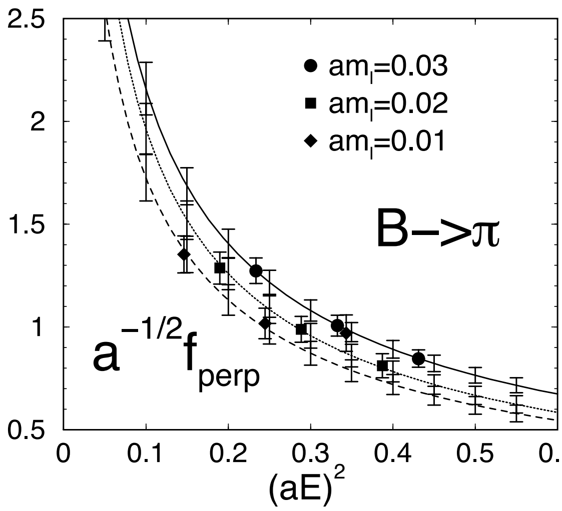

Figure 1: The form factor f_{perp} for B semileptonic decay. The Bećerivić-Kaidalov function was used to interpolate to fiducial values of the energy. The analogous graphs for D decay are given in Refs. [2] and [3].

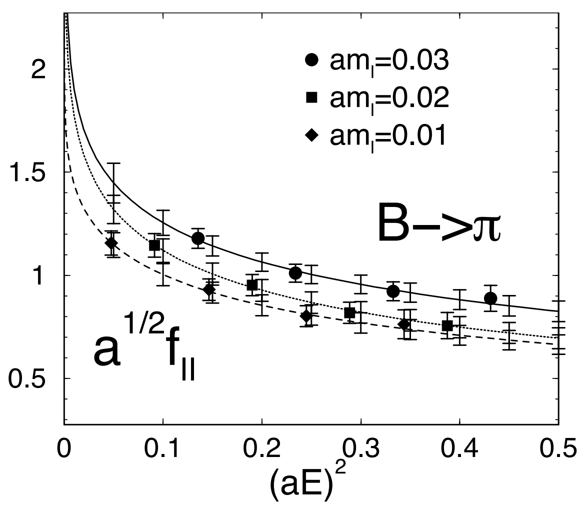

Figure 2: The same as Fig. 1, but for $f_{\|}$.

In the main analysis, for a given light quark mass, the Bećirević-Kaidalov parameterization of the form factors [9] was used to interpolate the form factors to fiducial values of the energy, as illustrated in Figs. 1 and 2. These interpolated form factors were then extrapolated to the physical quark mass, as shown in Figs. 3 and 4.

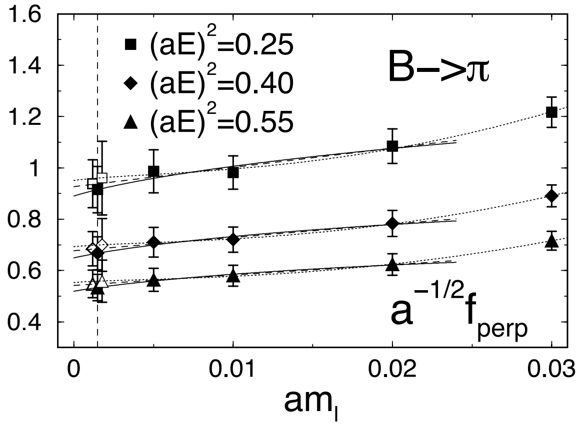

Figure 3: Chiral extrapolation of $f_{\rm perp}$.

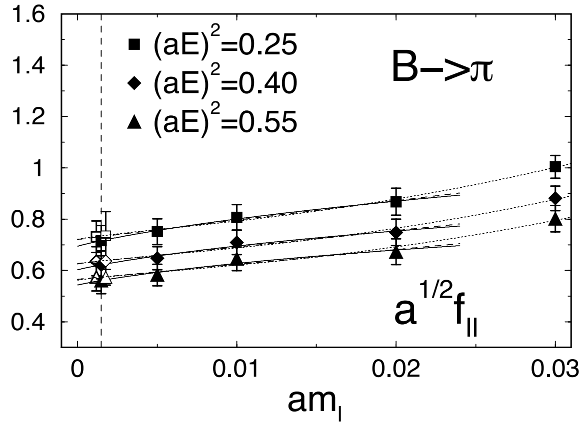

Figure 4: Chiral extrapolation of $f_{\|}$.

To check whether the use of the BK parameterization introduced any model-dependence into the results, we redid some fits, replacing the two-stage fit (energy interpolation followed by chiral extrapolation) with a two-dimensional fit in energy and quark mass simultaneously. Results were consistent, but with larger statistical errors in the two-dimensional fit, especially in the high pion momentum region, as shown in Figs. 5 and 6. The difference is used as an estimate of uncertainty

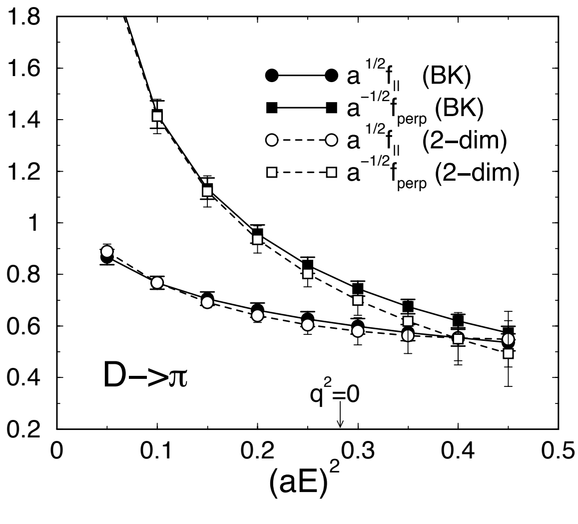

Figure 5: The form factors for D semileptonic decay obtained with a two-dimensional chiral and E^2 fit compared with those obtained with Bećirević-Kaidalov based energy interpolation, followed by chiral extrapolation.

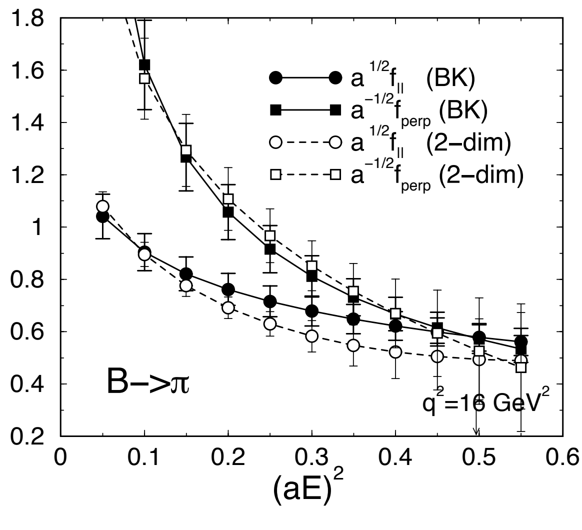

Figure 6: The same as Fig. 5, but for B decay.

due to the BK parameterization. The BK-based fitting produces reduced statistical errors in the high-momentum region, at the cost of introducing possible model dependence.

As a preliminary check for discretization errors, the form factors for decay into relatively heavy u and d quarks have been compared on the asqtad coarse and fine ($a = 0.086$ fm) lattices. Results agreed within statistical errors. Discretization uncertainties were estimated with HQET power counting, and are our largest current uncertainty. Analyzing form factor data on a larger range of lattice spacings is likely to give the greatest future improvement in our uncertainties.

3. Preliminary results

Preliminary estimates for the uncertainties in CKM matrix element determinations are shown in Table 1. Comparing predictions with experiment, we obtain for the CKM matrix elements:

$$|V_{ub}| \times 10^3 = 3.48(29)(38)(47), \tag{3.1}$$
$$|V_{cd}| = 0.239(10)(24)(20), \tag{3.2}$$
$$|V_{cs}| = 0.969(39)(94)(24), \tag{3.3}$$

where the errors are statistical, systematic, and experimental.

If we instead use the accepted values of the CKM matrix elements as inputs, we obtain for the D meson decay rates

$$\begin{aligned} \Gamma(D^0 \to \pi^- l^+ \nu) &= (7.7 \pm 0.6 \pm 1.5 \pm 0.8) \times 10^{-3} \text{ps}^{-1}, \\ \Gamma(D^0 \to K^- l^+ \nu) &= (9.2 \pm 0.7 \pm 1.8 \pm 0.2) \times 10^{-2} \text{ps}^{-1}, \\ \frac{\Gamma(D^0 \to \pi^- l^+ \nu)}{\Gamma(D^0 \to K^- l^+ \nu)} &= 0.084 \pm 0.007 \pm 0.017 \pm 0.009, \end{aligned} \tag{3.4}$$

where the errors are statistical, systematic, and from CKM matrix element.

After the publication of our results for the q^2 dependence of the $D \to Kl\nu$ form factor, a high-statistics measurement of the shape appeared from the FOCUS Collaboration [10]. As can be seen in Fig. 7, the results agree well.

Table 1: Systematic errors for CKM matrix elements from the semileptonic decays. Errors for V_{ub} decay are obtained from the integration with $q^2_{\text{min}} = 16\ \text{GeV}^2$.

decay	$D \to \pi(K)l\nu$	$B \to \pi l\nu$
CKM matrix element	$\lvert V_{cd(s)}\rvert$	$\lvert V_{ub}\rvert$
discretization effect	9%	9%
fitting 3- and 2-point functions	3%	3%
chiral extrapolation	3%(2%)	4%
q^2 dependence (BK parameterization)	2%	4%
current renormalization	0%	1%
a uncertainty	1%	1%
total systematic	10%	11%

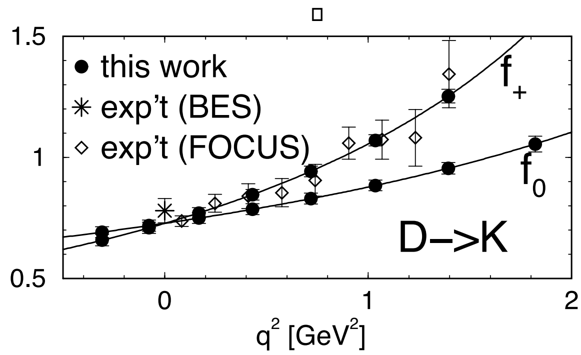

Figure 7: The form factors in $D \to Kl\nu$ decay, compared with experimental data for f_+.

4. Unitarity constraints

Unitarity constraints can be used without introducing model dependence to constrain the shape of form factors in the high recoil momentum region (where our statistical uncertainties range from poor to infinite). Arnesen, Grinstein, Rothstein, and Stewart [11] have recently given a lucid explanation. The function

$$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \tag{4.1}$$

maps $q^2 = t > t_+$ onto $|z| = 1$, and $t < t_+$ onto $[-1,1]$ in the complex plane. Here, $t = (p_H - p_L)^2$, $t_+ = (m_H + m_L)^2$, and $t_- = (m_H - m_L)^2$. t_0, taken as 0.65 t_- here, is a free parameter adjusted to center the physical region on $z \sim 0$.

For the various semileptonic decays, the physical region, $0 < t < t_-$, is mapped into

$B \to \pi l\nu$: $-0.34 < z < 0.22$,
$D \to \pi l\nu$: $-0.17 < z < 0.16$,
$D \to Kl\nu$: $-0.04 < z < 0.06$,
$B \to Dl\nu$: $-0.02 < z < 0.04$.

When unitarity is taken into account, the smallness of the physical range of z will constitute a small parameter that limits the number of parameters required to describe the form factors to a given accuracy.

In this parameterization, the form factors have the form:

$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0}^{\infty} a_k(t_0) z(t,t_0)^k. \tag{4.2}$$

$P(t)$ and $\phi(t,t_0)$ contain most of the complexity of the form factors. Unitarity requires simply that $\sum a_k^2 < 1$. In B semileptonic decay, just five terms in the series are necessary for 1% accuracy. Fitting three points of our raw data for f_+ at $m_l = 0.02$ (red squares in Fig.8), we obtain from the fit (shown without error bars as a green line) good constraints on the form factor beyond the region where we have data. (Two sample points are shown with error bars as blue circles. The leftmost blue point corresponds to a recoil momentum of around 1.7 GeV.) (Our data end at recoil momenta of around 1 GeV.) We are currently examining a simultaneous fit of f_+ and f_0, and the use of unitarity-based interpolation parameters to supplement or replace.the BK-based fitting.

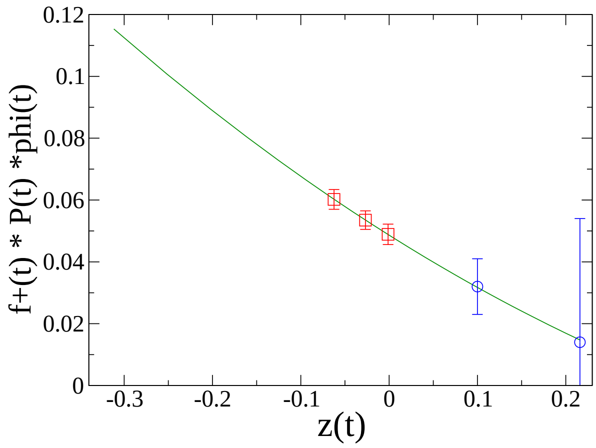

Figure 8: A fit (green line) of our data (red squares) for f_+ normalized by $P(t)$ and $\phi(t,t_0)$, as a function of $z(t,t_0)$. Unitarity-based extrapolation of our data into the high recoil-momentum region (blue circles) gives useful constraints beyond the range of the lattice data.

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