Study of 1/m corrections in HQET

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We report our exploratory study on the matching condition of HQET with QCD including $1/m$ corrections. We introduce a new observable from the dependence of the heavy-light effective energy on the twisted boundary condition parameter θ, which could be used to match the kinetic term $\vec{D}^2/2m$. Carrying out quenched QCD simulations for fixed lattice spacing in small volumes with O(a)-improved Wilson fermions, we study the $1/m$ dependence of this observable, from which the static limit and $1/m$ coefficient can be extracted. We also compare our preliminary result with HQET.

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1. Introduction

One of the primary goals in particle physics is to determine fundamental parameters of the standard model such as the quark masses and the Cabibbo-Kobayashi-Maskawa matrix elements (e.g. V_{td}, V_{ts}, V_{tb}) in order to test the standard model and find a clue to the physics beyond the standard model. However for this determination in addition to the experimental inputs from CLEO-c [1], BaBar [2] and BELLE [3], we need non-perturbative evaluation of hadronic observables from the first principles of QCD. Above all, simulation of lattice QCD is one of the most effective approaches.

In lattice QCD, the typical lattice cutoff accessible in the present computers is $a^{-1} \approx 1 \sim 3$GeV. But in the case of heavy quarks such as the bottom quark whose mass is $m \approx 4$GeV the quark mass in lattice unit is so large $ma \approx 1 \sim 4$ that the conventional lattice fermion action has no control over the discretization errors. In order to avoid this problem which has been raised of the heavy-quark calculation, we must take the following approaches:

(a) conventional fermions with much finer lattice spacing,

(b) non-relativistic QCD and their variations,

(c) anisotropic lattices [4, 5],

(d) heavy-quark effective theory(HQET) [6, 7, 8].

However (a) is still not practical in high precision computation due to computational cost and (b) has theoretical uncertainty owing to the perturbative matching, while (c) and (d) have potential of providing formalism for high precision computation. In this report, we will give our exploratory study on (d) HQET.

HQET is a theory that reproduces the low energy mode of a heavy quark in the static limit [9, 10], where the action is a systematic expansion in inverse powers of the mass

$$S_{\text{HQET}} = a^4 \sum_x \left\{ \mathscr{L}_{\text{stat}}(x) + \sum_{\nu=1}^{n} \mathscr{L}^{(\nu)}(x) \right\}, \tag{1.1}$$

$$\mathscr{L}_{\text{stat}}(x) = \bar{\psi}_h \left(\nabla_0^* + \delta m \right) \psi_h(x), \ \mathscr{L}^{(\nu)}(x) = \sum_i \omega_i^{(\nu)} \mathscr{L}_i^{(\nu)}(x), \tag{1.2}$$

$$\delta m = \mathscr{O}\left(\frac{1}{m}\right), \ \omega_1^{(1)} = \frac{1}{m} + \mathscr{O}\left(\frac{1}{m^2}\right), \ \omega_2^{(1)} = \frac{1}{m} + \mathscr{O}\left(\frac{1}{m^2}\right), \tag{1.3}$$

$$\mathscr{L}_1^{(1)}(x) = \bar{\psi}_h \left(-\frac{1}{2}\sigma \cdot \mathbf{B} \right) \psi_h, \ \mathscr{L}_2^{(1)}(x) = \bar{\psi}_h \left(-\frac{1}{2}\mathbf{D}^2 \right) \psi_h. \tag{1.4}$$

If one treats in $1/m$ corrections perturbatively to a fixed order, the theory is a renormalizable theory with well-defined continuum limit: We presume it can accept up until next-to-leading order terms of $1/m$ and can have $1 \sim 2\%$ accuracy. However HQET is an effective theory with unknown coefficients for $1/m$ corrections terms and the ultraviolet behavior is essentially different from QCD. Therefore, one has to match HQET with QCD by matching conditions defined by a set of physical observables

$$\langle \mathscr{O}_{\text{QCD}}(1/z) \rangle = \langle \mathscr{O}_{\text{HQET}}(1/z) \rangle, \ z \equiv ML \gg 1 \ (\mathscr{O}; \text{an observable}), \tag{1.5}$$

where M is a renormalization group invariant mass [11]. This can practically be done in a small volume $L \approx 0.2$fm where we take the small lattice spacing within reasonable numerical cost: $m_b a \ll 1$. Then by computing the step-scaling function [10], we can evolve the lattice HQET into coarser lattice where physical observables can be computed in large volume. In fact these two steps have been carried out for δm_q and the axial current $A_\mu^{\text{QCD}} = A_\mu^{\text{HQET}}$ in the static limit, from which the following results are obtained

$$F_{B_s}^{\text{stat}} = 253 \pm 45 \text{MeV} \; [12], \; \bar{m}_b = 4.12 \pm 0.07 \pm 0.04 \text{MeV} \; [10]. \tag{1.6}$$

For a high precision calculation, non-perturbative calculation of the $1/m$ correction is required. For this purpose it is necessary to match the coefficients of the $1/m$ corrections in the action or operators to begin with.

The matching of HQET with QCD is composed of three steps:

(e) to execute the matching condition (1.5),

(f) to match coefficients of $1/m$-correction terms from Eq. (1.5),

(g) to evaluate the step scaling functions.

In this report we give study on (e) as the first step for (f) and also computing the static limit; especially on the search for new observables which are efficient for the determination of $1/m$ term.

2. Observables

In Ref. [13], Heitger *et al.* propose using two-point correlation functions in small finite volume as the matching condition (1.5). Thus we can evaluate some HQET parameters by combining observables in varied kinematical conditions and execute heavy quark lattice simulations of both QCD and HQET in $L_0 \approx 0.2$fm.

Two-point correlation functions are defined by

$$f_A(x_0) = -\frac{1}{2} \int d^3\mathbf{y} d^3\mathbf{z} \langle A_0 \bar{\zeta}_b(\mathbf{y}) \gamma_5 \zeta_l(\mathbf{z}) \rangle, \; k_V(x_0) = -\frac{1}{6} \sum_k \int d^3\mathbf{y} d^3\mathbf{z} \langle V_k \bar{\zeta}_b(\mathbf{y}) \gamma_k \zeta_l(\mathbf{z}) \rangle, \tag{2.1}$$

where $A_\mu(x) = \bar{\psi}_l(x) \gamma_\mu \gamma_5 \psi_b(x)$, $V_\mu(x) = \bar{\psi}_l(x) \gamma_\mu \psi_b(x)$, and $\bar{\zeta}$, ζ are the boundary fields at $x_0 = 0$. Defining effective energies as

$$E_{\text{PS}}^{\text{eff}}(x_0, \theta) = -\frac{d}{dx_0} \ln[f_A(x_0)], \; E_V^{\text{eff}}(x_0, \theta) = -\frac{d}{dx_0} \ln[k_V(x_0)], \tag{2.2}$$

we can determine various observables in kinematical conditions such as

$$\Gamma_{\text{av}} = \frac{1}{4} \left(E_{\text{PS}}^{\text{eff}} \left(\frac{T}{2} \right) + 3 E_V^{\text{eff}} \left(\frac{T}{2} \right) \right), \; \Delta\Gamma = E_V^{\text{eff}} \left(\frac{T}{2} \right) - E_{\text{PS}}^{\text{eff}} \left(\frac{T}{2} \right), \tag{2.3}$$

$$\Xi = \frac{1}{4} \left(E_{\text{PS}}^{\text{eff}} \left(\frac{T}{4} \right) + 3 E_V^{\text{eff}} \left(\frac{T}{4} \right) \right) - \frac{1}{4} \left(E_{\text{PS}}^{\text{eff}} \left(\frac{T}{2} \right) + 3 E_V^{\text{eff}} \left(\frac{T}{2} \right) \right) \tag{2.4}$$

to assess δm, $\omega_1^{(1)}$, and $\omega_2^{(1)}$ respectively, defined in Eq. (1.3).

It is known that the kinetic term of $1/m$ correction $\omega_2^{(1)}$ is difficult to evaluate it with good accuracy, because of the cancellation between power divergences of effective energies at $T/2$ and

$T/4$, which causes large systematic errors. Therefore it is fruitful to look for alternative observables for consistency checks and pursuit of more efficiency. In this work we propose an alternative observable:

$$\Xi^{\text{new}} = \Gamma_{\text{av}}(1/z)|_{\theta=1.0} - \Gamma_{\text{av}}(1/z)|_{\theta=0.5}, \tag{2.5}$$

where θ is defined by twisted boundary conditions

$$\psi(x+\hat{k}L) = e^{i\theta}\psi(x), \ \bar{\psi}(x+\hat{k}L) = \bar{\psi}(x)e^{-i\theta} \quad (k=1,2,3, \ T=L, \ \theta = 0.5, 1.0). \tag{2.6}$$

3. Simulation methods

Simulations are carried out both for QCD and HQET on a quenched 12^4 lattice with the standard plaquette gauge action at $\beta = 6/g_0^2 = 7.4802$. We take the Schrödinger functional boundaries [14, 15] with $C = C' = 0$. The SF-boundary coefficients are $c_{\text{t}}^{2\text{-loop}} = 1 - 0.089g_0^2 - 0.030g_0^4$ [16]. 256 configurations are accumulated by the pseudo-heat-bath algorithm each separated by 200 Monte Carlo sweeps. Errors are estimated by the standard jackknife method.

In QCD, we use $O(a)$-improved Wilson fermion with the nonperturbative clover coefficient c_{SW} [17] and the SF boundary conditions $P_+\psi(x)|_{x_0=0} = \rho$, $P_-\psi(x)|_{x_0=T} = 0$ [18]. Six values of hopping parameters are taken for the heavy quark which correspond to $z \equiv ML = 3.0$, 3.8, 5.15, 6.0, 6.6, 9.0 whereas the hopping parameter for the light quark is set to the critical value ($\kappa_{\text{c}} = 0.133961$) following Ref. [19].

In HQET, we used the static-limit action $\mathscr{L}_{\text{stat}}(x)$ (1.2) with the boundaries for the heavy quark. In order to improve the numerical precision, we adopted gauge fields in ∇_0^* of $\mathscr{L}_{\text{stat}}(x)$ are smeared by the HYP links [20] with $(\alpha_1, \alpha_2, \alpha_3) = (1.0, 1.0, 0.5)$ [21, 22]. The light quark is the same as in QCD.

4. Results

Fig. 1 shows the average effective energies Γ_{av} (2.3) at $\theta = 0.5$, 1.0 with matching condition

$$L\Gamma_{\text{av}} \overset{M\to\infty}{\sim} C_{\text{mass}}(M/\Lambda_{\overline{\text{MS}}})z + \mathscr{O}((1/z)^0) \ [13], \tag{4.1}$$

where C_{mass} is the matching function between QCD and HQET [13] and $M/\Lambda_{\overline{\text{MS}}}$ is given in Ref. [11]. We reproduced consistent result with $\theta = 0.5$ at $\beta = 7.4802$ in Ref. [19]. As is obvious from Fig. 1, the data for $\theta = 0.5$ at $\beta = 7.4802$ deviate only by $2 \sim 3\%$ for $z = 3.0$, 3.8, 5.15, whereas for $z \equiv ML = 9.0$ the deviation is of 10% order due to the discretization error. Although there is no scaling study, we find the $1/z$ dependence of the result for $\theta = 1.0$ also shows qualitatively similar behavior. In fact, our data at $z = 3.0$, 3.8, 5.15 seem to approach the static limit value predicted as in Eq. (4.1). On the other hand, the data for $z = 9.0$ seem to deviate from the $1/m$ scaling significantly. There is need of a stringent test in further studies, since this deviation is probably due to discretization errors.

Fig. 2 shows the $1/z$ dependence of the observable Ξ^{new}. Owing to the reparameterization invariance [23], the renormalization factor is not needed for this observable. Assuming that the
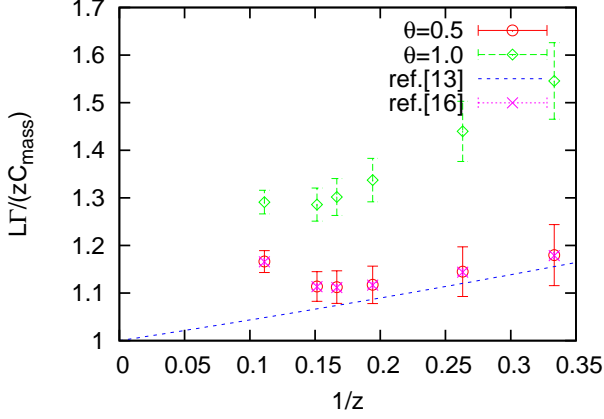
Figure 1: The averages of the effective energy at $\theta = 0.5$ (circle symbol) and $\theta = 1.0$ (diamond). The cross symbols and the dashed line denote the data for $\theta = 0.5$ at $\beta = 7.4802$ [19] and in the continuum limit [13] respectively.
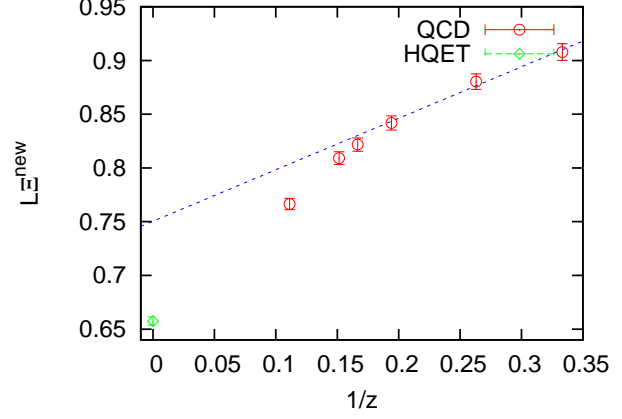
Figure 2: $1/z$ dependence of the new observable $L\Xi^{\text{new}}$. The circle symbols denote our data for QCD and the diamond denotes the HQET result. The dashed line is the linear fit using three lighter points.

data for $z = 3.0$, 3.8, 5.15 are very close to the continuum limit, we made a linear fit in $1/z$ as $L\Xi^{\text{new}} = a_0 + a_1/z$. Our preliminary results are $a_0 = 0.750(12)$ and $a_1 = 0.479(47)$, where the errors are statistical only. The data for $z = 9.0$ again deviate from the $1/z$ scaling, which is probably due to the discretization error. The extrapolation of QCD results does approach to the HQET result towards the static limit, although it gives higher values than the HQET result at $\beta = 7.4802$. Further study is needed to see if this discrepancy vanishes in the continuum limit. Nevertheless, it is encouraging that this observable has clear $1/z$ dependence so that it has sensitivity to determine the coefficient of $1/m$-correction terms $\omega_2^{(1)}$ in HQET.

5. Conclusions

We have proposed the new observable Ξ^{new} defined as the difference of the effective energy at $\theta = 0.5$, 1.0 for extracting the coefficients of the kinetic term in HQET. Our simulation at $\beta = 7.4802$ suggests that it has qualitatively correct HQET scaling and also good sensitivity to $1/m$-correction term.

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