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Radiative Transitions in Charmonium

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The form factors for the radiative transitions between charmonium mesons are investigated. We employ an anisotropic lattice using a Wilson gauge action, and domain-wall fermion action. We extrapolate the form factors to $Q^2 = 0$, corresponding to a real photon, using quark-model-inspired functions. Finally, comparison is made with photocouplings extracted from the measured radiative widths, where known. Our preliminary results find photocouplings commensurate with these experimentally extracted values.

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1. Introduction

The GlueX program at Jefferson Laboratory will produce hybrid mesons through photoproduction, in contrast to the pion production mechanism generally employed in other searches. This proposal is supported by flux-tube model calculations, which show no suppression for conventional-hybrid photocouplings compared with those for conventional-conventional transitions[1, 2, 3]:

$$\Gamma(\pi_{1H}^{+} \rightarrow a_2^{+}\gamma) \sim \mathscr{O}(100)\ \text{keV}, \tag{1.1}$$

$$\Gamma(b_{JH}^{+} \rightarrow \rho^{+}\gamma) \sim \mathscr{O}(1000)\ \text{keV}, \tag{1.2}$$

$$\Gamma(b_1^{+} \rightarrow \pi^{+}\gamma) = 230 \pm 60\ \text{keV}. \tag{1.3}$$

As a first examination of the validity of these calculations, we have embarked on a program to examine transition form factors in the charmonium sector. Here we expect both a flux-tube model and lattice calculations to provide reasonable descriptions of the physics. In this contribution, we focus on transitions between conventional charmonium mesons as a precursor to future studies of hybrid mesons. Many charmonium states below the $D\bar{D}$ threshold have measured radiative transtions into ligher charmonium states, and these transtions are reasonably well described in quark potential models[4].

2. Formalism

We write the transition matrix element between an initial state i and a final state f as

$$\langle f(\vec{p}_f, \lambda_f) \mid V_\mu(0) \mid i(\vec{p}_i, \lambda_i)\rangle$$

where $\vec{p}_i$ and $\vec{p}_f$ are the initial and final momenta respectively, λ_i and λ_f label helicities, and V_μ is the electromagnetic vector current. For comparison both with experiment and with quark potential models, we express the transition form factors in a multipole expansion. For the transitions we consider in this paper, the expansion is as follows:

2.1 $\eta_c(0^-)$ form factor

Whilst charge conjugation requires that this vanish, we can nevertheless examine the "form factor" by coupling the vector current only to the quark. This is the paradigm process, analogous to the pion form factor; continuum current conservation implies there is only a single form factor:

$$\langle \eta_c(\vec{p}_f) \mid V^\mu(0) \mid \eta_c(\vec{p}_i)\rangle = f(Q^2)\left[p_f^\mu + p_i^\mu\right], \tag{2.1}$$

where $-Q^2 = q^2 = (E_f - E_i)^2 - \vec{q}^2$, with $\vec{q} = \vec{p}_f - \vec{p}_i$ the three-momentum carried by the photon.

2.2 $J/\psi(1^-) \longrightarrow \gamma\eta_c(0^-)$ transition

This is an M_1-transition, and likewise described by only a single form factor

$$\langle \eta_c(\vec{p}_{\rm PS}) \mid V^\mu(0) \mid \psi(\vec{p}_{\rm V}, \lambda)\rangle = \frac{2V(Q^2)}{m_{\eta_c} + m_\psi}\varepsilon^{\mu\alpha\beta\gamma} p_{\rm PS}^\alpha p_{\rm V}^\beta \varepsilon^\gamma(\vec{p}_{\rm V}, \lambda), \tag{2.2}$$

where λ is the helicity label of the J/ψ.

	η_c	J/ψ	χ_{c0}	χ_{c1}	h_c
Lattice (MeV)	2819(7)	2917(7)	3288(15)	3401(29)	3351(19)
PDG (MeV)	2980(1)	3097	3415	3511	3526

Table 1: Determination of the spectrum, together with the corresponding PDG values[3].

2.3 $\chi_{c0}(0^+) \longrightarrow \gamma J/\psi(1^-)$ transition

This transition involves an electric dipole $E_1(Q^2)$ coupling to the transverse component of the photon, and a further form factor $C_1(Q^2)$ coupling to the longitudinal component:

$$\begin{aligned}
&\langle \chi_{c0}(\vec{p}_{\rm S})|V^\mu(0)|J/\psi(\vec{p}_{\rm V},\lambda)\rangle = \\
&\Omega^{-1}(Q^2)\Bigg(E_1(q^2)\left[\Omega(Q^2)\varepsilon^\mu(\vec{p}_{\rm V},\lambda) - \varepsilon(\vec{p}_{\rm V},\lambda).p_{\rm S}(p_{\rm V}^\mu p_{\rm V}.p_{\rm S} - m_{\rm V}^2 p_{\rm S}^\mu)\right] + \\
&\frac{C_1(Q^2)}{\sqrt{q^2}} m_V \varepsilon(\vec{p}_{\rm V},\lambda).p_{\rm S}\left[p_{\rm V}.p_{\rm S}(p_{\rm V}+p_{\rm S})^\mu - m_{\rm S}^2 p_{\rm V}^\mu - m_{\rm V}^2 p_{\rm S}^\mu\right]\Bigg)
\end{aligned} \tag{2.3}$$

For real photons, only E_1 is of interest, but in our calculation we will extract both form factors.

3. Computational details

The computations are performed in the quenched approximation to QCD on 300 configurations of a $12^3 \times 48$ lattice. We employ an anisotropic Wilson gauge action [5], with a renormalized anisotropy $\xi \equiv a_s/a_t = 3$. The temporal lattice spacing obtained from the static quark-antiquark potential is $a_t^{-1} = 6.05(1)$ GeV.

The quark propagators are computed using an anisotropic domain-wall fermion (DWF) action, with a domain-wall height $M = 1.7$ and $L_5 = 16$. We employ standard Gaussian smearing for the interpolating fields, using sources of various widths. For this exploratory study, we do not attempt a precise tuning of the charm quark mass to yield the physical m_{η_c}. The charmonium spectrum obtained on our lattices is listed in Table 1; in general, the whole spectrum is too light, reflecting the imprecision in our tuning.

The three-point functions are computed using the usual sequential-source method, following the strategy outlined in ref. [6]. Sequential-source propagators are computed for both a final scalar χ_{c0} and a pseudoscalar η_c, placed at the mid-point of the lattice $t_f = 24$, for both $\vec{p}_f = (0,0,0)$ and $\vec{p}_f = (1,0,0)$ where the momentum is expressed in the appropriate lattice units. The form factors are computed using the local, non-conserved vector current. The form factors are obtained from the three-point functions with the remaining amplitudes and masses extracted from fits to the two-point functions. The matching factor to the continuum, Z_V, is determined by imposing charge conservation on the pseudoscalar form factor $f(Q^2 = 0) = 1$. We find a discrepancy of around 11% between the matching factor obtained for a pion at rest, and that for a pion at the lowest non-zero value of the lattice momentum $\vec{p} = (1,0,0)$, representing an important systematic uncertainty on our calculation.

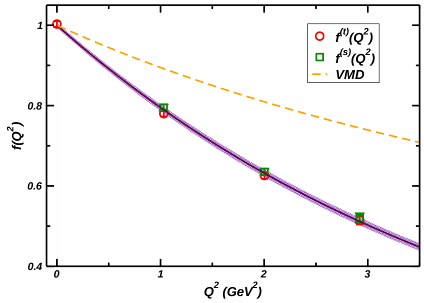

Figure 1: The green and red data points show the lattice determination of the η_c "form factor" obtained using the spatial and temporal components of the electromagnetic current respectively. The band corresponds to a quark-model-inspired fit to the data, while the dashed line is the VMD expectation, as described in the text.

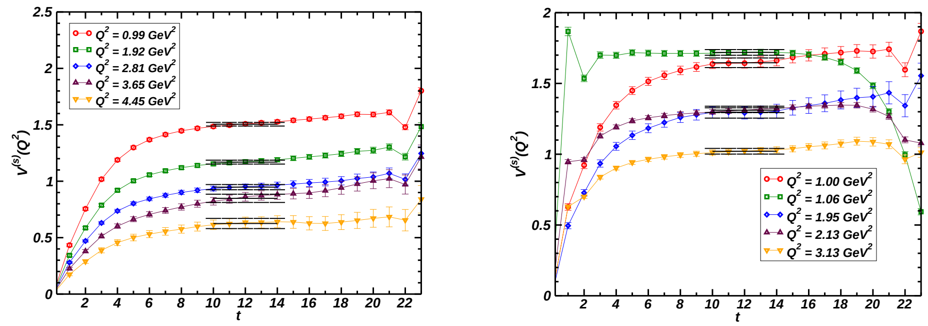

Figure 2: The left- and right-hand figures show plateaux in the determination of $V(Q^2)$ at several Q^2 for $\vec{p}_f = (0,0,0)$ and $\vec{p}_f = (1,0,0)$, respectively.

4. Results

The η_c "form factor" is shown in figure 1, for the case $\vec{p}_f = (0,0,0)$. In order to describe the Q^2-dependence of the data, we appeal to a non-relativistic quark model using harmonic-oscillator wave functions, characterized by a single parameter β:

$$f(Q^2) = e^{-Q^2/16\beta^2}. \tag{4.1}$$

A single-parameter fit to the data yields $\beta = 0.522(5)$ GeV, a value not uncharacteristic of that typically employed in quark-model charmonium wavefunctions; the fit is shown as the band in the figure. Also shown as the dashed line is the VMD form, using the lattice value of the J/ψ mass; VMD provides a notably poor description of the data, in contrast to the case of the pion[6].

Proceeding now to the transition form factors, we consider first the transition $J/\psi \longrightarrow \gamma\eta_c$. The quality of the extraction of the form factor is illustrated in Figure 2 where we show the plateaux at various Q^2 using the spatial components of the vector current.

The Q^2 dependence of the transition form factor is shown as the left-hand plot in Figure 3. The systematic difference arising from the different determinations of Z_V is manifest in the discrepancy between the values obtained with $\vec{p}_f = (0,0,0)$ and those with $\vec{p}_f = (1,0,0)$; the right-hand plot shows the data with those for $\vec{p}_f = (1,0,0)$ rescaled by this 11%, greatly reducing this discrepancy.

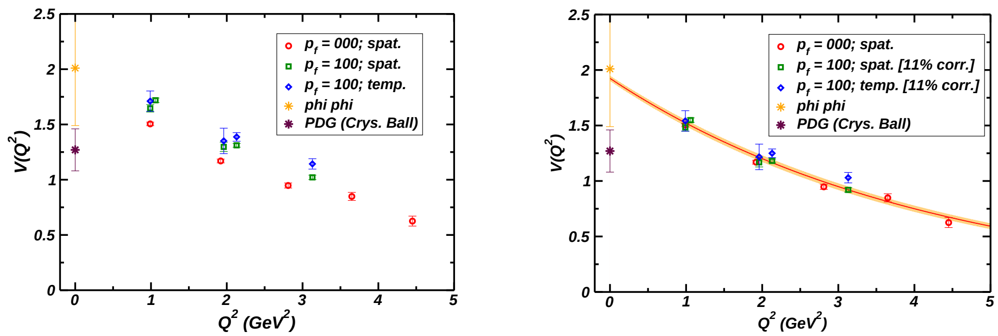

Figure 3: The left-hand plot shows the transition form factor $J/\psi \longrightarrow \gamma\eta_c$, for $p_f = (0,0,0)$ and $p_f = (1,0,0)$. The purple and yellow bursts are phenomenological values obtained from the PDG[3], and using the transition rate to $\phi\phi$ respectively. The right-hand plot shows the same data with those corresponding to $\vec{p}_f = (1,0,0)$ scaled by the 11% discrepancy in Z_V; the band is a quark-model-inspired fit to the lattice data, as described in the text.

In order to extrapolate to $Q^2 = 0$, we once again employ a non-relativistic quark-model-inspired form

$$V(Q^2) = V(0)e^{-Q^2/16\beta^2}. \tag{4.2}$$

A combined fit to the $\vec{p}_f = (0,0,0)$ and rescaled $\vec{p}_f = (1,0,0)$ data yields $\beta = 0.52(1)$ GeV and $V(0) = 1.92(2)$.

The radiative width is related to the photocoupling $V(Q^2 = 0)$ by

$$\Gamma(J/\psi \to \eta_c\gamma) = \alpha \frac{|\vec{q}|^3}{(m_\psi + m_{\eta_c})^2} \frac{64}{27} |V(Q^2 = 0)|^2, \tag{4.3}$$

whence we can obtain an "experimental" determination of the photocoupling using the measured radiative width. This is shown for the PDG value as the purple burst[3]; an alternative value obtained from the ratio of the product branching fraction of $\psi \longrightarrow \gamma\eta_c \longrightarrow \gamma\phi\phi$ and the $\eta_c \longrightarrow \phi\phi$ branching fraction is shown in yellow.

Finally, we turn to the $\chi_{c0} \longrightarrow \gamma J/\psi$ transtion. The two form factors $E_1(Q^2)$ and $C_1(Q^2)$ are shown as the left- and right-hand plots in Figure 4. Using a quark-model form for the extrapolation to $Q^2 = 0$, we have for the electric-dipole form factor

$$E_1(Q^2) = c \mid \vec{q}(Q^2) \mid e^{-Q^2/16\beta^2} \tag{4.4}$$

where $\vec{q}$ is the three-momentum of the photon in the rest frame of the decaying χ_{c0},

$$\mid \vec{q}(Q^2) \mid^2 = \frac{(m_\psi^2 - m_\chi^2)^2 + 2Q^2(m_\psi^2 + m_\chi^2) + Q^4}{4m_\chi^2}, \tag{4.5}$$

with an analogous form for the longitudinal multipole $C_1(Q^2)$. The extrapolations are shown as the bands on the figure. Once again, we can use the measurement of the radiative width to obtain an "experimental" value for the physical multipole $E_1(Q^2 = 0)$. This is shown as the purple burst using the PDG width[3], and as the yellow burst using a recent CLEO determination[7]. The

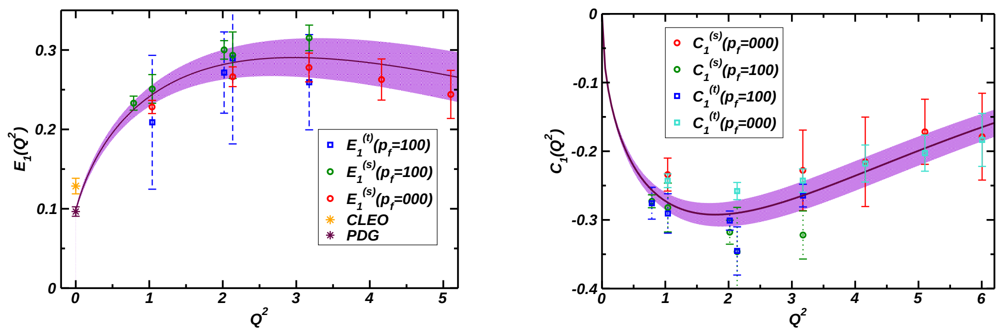

Figure 4: The left- and right-hand plots show the form factors $E_1(Q^2)$ and $C_1(Q^2)$ respectively for the radiative transition $\chi_{c0} \longrightarrow \gamma J/\psi$. The bands correspond to independent, quark-model-inspired fits to the lattice data at $\vec{p}_f = (0,0,0)$ and $\vec{p}_f = (1,0,0)$, as described in the text. The purple and yellow bursts are phenomenological values obtained from the PDG[3], and from a recent CLEO determination[7]

corresponding values for the parameter β are $0.60(3)$ GeV and $0.47(1)$ GeV for the E_1 and C_1 multipoles respectively.

In this work, we present a first study of radiative transitions in charmonium. While the results are preliminary, we find quark-model extrapolations yield values for the photocouplings in reasonable proximity to experimental expectations, with broadly consistent values for the wave-function parameter β. Computations of further form factors are in progress. A future goal is to extend this study to the light-quark sector relevant for JLab, and to transitions involving hybrid mesons.

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