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$B_s - \overline{B_s}$ mixing with a chiral light quark action

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We study the $B_s^0 - \overline{B_s^0}$ mixing amplitude in Standard Model by computing the relevant hadronic matrix element in the static limit of lattice HQET with the Neuberger light quark action. In the quenched approximation, and after matching to the $\overline{\rm MS}$ scheme in QCD, we obtain $B_{B_s}^{\overline{\rm MS}}(m_b) = 0.940(16)(22)$.

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1. Introduction

$B_s^0 - \overline{B_s^0}$ mixing is highly important in testing the Standard Model (SM) and constrains strongly its extensions. Since it is a flavor changing neutral process it occurs through loops so that the corresponding mixing amplitude is a sensitive measure of $|V_{ts}|$ and $|V_{tb}|$, as the major SM loop contribution comes from t-quark. The mixing of weak interaction eigenstates B_s^0 and $\overline{B_s^0}$ induces a mass gap ΔM_s between the mass eigenstates B_{sH} and B_{sL}. Experimentally, only a lower bound to ΔM_s is currently known, namely $\Delta M_s > 14.4\ \mathrm{ps}^{-1}$ [1], and the hope is that experimenters will soon provide us with an accurate measurement.

Theoretically the $B_s^0 - \overline{B_s^0}$ mixing is described by means of an Operator Product Expansion, *i.e.* the Standard Model Lagrangian $\mathscr{L}_{SM}$ is reduced to an effective Hamiltonian $\mathscr{H}_{eff}^{\Delta B=2}$, up to negligible terms of $\mathscr{O}(1/M_W^2)$:

$$\mathscr{H}_{eff}^{\Delta B=2} = \frac{G_F^2}{16\pi^2} M_W^2 (V_{tb}^* V_{ts})^2 \eta_B S_0(x_t) C(\mu_b) Q_{LL}^{\Delta B=2}(\mu_b), \quad Q_{LL}^{\Delta B=2} = \bar{b}\gamma_{\mu L} s\, \bar{b}\gamma_{\mu L} s, \quad \mu_b \sim m_b \quad (1.1)$$

where $\eta_B = 0.55 \pm 0.01$ $S_0(x_t)$ is a known Inami-Lim function of $x_t = m_t^2/M_W^2$ [2], $C(\mu_b)$ is the Wilson coefficient computed perturbatively to NLO in $\alpha_s(\mu_b)$ in the $\overline{\mathrm{MS}}$ (NDR) scheme, and $Q_{LL}^{\Delta B=2}$ is a four-fermions operator coming from the reduction of the box diagrams in $\mathscr{L}_{SM}$ to a local operator in the effective theory. The hadronic matrix element of $Q_{LL}^{\Delta B=2}$ is conventionally parameterized as

$$\langle \overline{B_s^0} | Q_{LL}^{\Delta B=2}(\mu_b) | B_s^0 \rangle \equiv \frac{8}{3} m_{B_s}^2 f_{B_s}^2 B_{B_s}(\mu_b)\ , \quad (1.2)$$

where $B_{B_s}(\mu_b)$ is the B_s meson bag parameter and f_{B_s} its decay constant.

So far $B_{B_s}(\mu_b)$ has been computed by using lattice QCD [3]-[9]. One of the major problems with those computations is in the following: the standard Wilson light quark lattice action breaks explicitely the chiral symmetry, which tremendously complicates the renormalization procedure of $Q_{LL}^{\Delta B=2}$ and its matching to the continuum. To get around that problem we compute $B_{B_s}(\mu_b)$ by using the lattice formulation of QCD in which the chiral symmetry is preserved at finite lattice spacing [10]. On the other hand, it should be stressed that our heavy quark is static, as the currently available lattices do not allow to work directly with the propagating b quark. Thus our results will suffer from $1/m_b$-corrections.

2. Computation on the lattice

In our numerical simulation we choose to work with the action $S = S_h^{\mathrm{EH}} + S_l^{\mathrm{N}}$, where

$$S_h^{\mathrm{EH}} = a^3 \sum_x \left\{ \bar{h}^+(x) \left[h^+(x) - V_0^{\mathrm{HYP}\dagger}(x-\hat{0}) h^+(x-\hat{0}) \right] - \bar{h}^-(x) \left[V_0^{\mathrm{HYP}}(x) h^-(x+\hat{0}) - h^-(x) \right] \right\}$$

is the static limit of HQET action [11] which has been modified after using the so-called HYP (hypercubic blocking) procedure [12], that is enough to substantially improve the signal/noise ratio [13] [the field $h^+(h^-)$ annihilates the static heavy quark (antiquark)]. $S_l^{\mathrm{N}} = a^3 \sum_x \bar{\psi}(x) D_N^{(m_0)} \psi(x)$ is the overlap light quark action with

$$D_N^{(m_0)} = \left(1 - \frac{1}{2\rho} a m_0\right) D_N + m_0, \quad D_N = \frac{\rho}{a}\left(1 + \frac{X}{\sqrt{X^\dagger X}}\right), \quad X = D_W - \frac{\rho}{a},$$

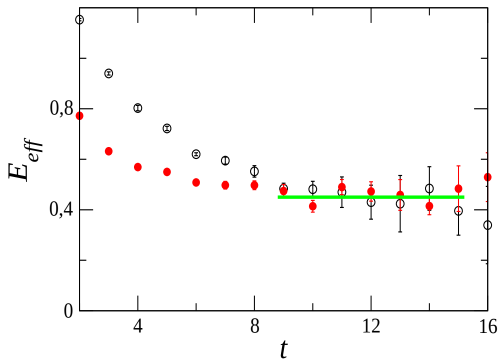

Figure 1: Effective binding energy of the 0^--state when currents are local (unfilled symbols) or smeared (filled symbols)

where D_W is the standard Wilson-Dirac operator. The overlap Dirac operator $D_N^{(m_0)}$ verifies the Ginsparg-Wilson relation $\{\gamma^5, D_N^{(m_0)}\} = \frac{a}{\rho} D_N^{(m_0)} \gamma^5 D_N^{(m_0)}$ and the overlap action is invariant under the chiral light quark transformation [14]

$$\psi \to \psi + i\varepsilon\gamma^5 \left(1 - \frac{a}{\rho} D_N^{(m_0)}\right)\psi, \quad \bar{\psi} \to \bar{\psi}(1 + i\varepsilon\gamma^5),$$

which is essential to prevent mixing of four-fermion operators of different chirality [15]. In other words, in the renormalization procedure, the subtraction of the spurious mixing with $d = 6$ operators will not be needed.

We thus compute the two- and three-point functions:

$$\tilde{C}_{AA}^{(2)\pm}(t) = \langle \sum_{\vec{x}} \tilde{A}_0^{\pm}(\vec{x},t) \tilde{A}_0^{\pm\dagger}(0) \rangle_U \xrightarrow{t \gg 0} \tilde{Z}_A e^{-\varepsilon t}, \tag{2.1}$$

$$\tilde{C}_{VV+AA}^{(3)}(t_i,t) = \langle \sum_{\vec{x},\vec{y}} \tilde{A}_0^{+}(\vec{x},t_i) \tilde{O}_1(0,0) \tilde{A}_0^{-\dagger}(\vec{y},t) \rangle_U \xrightarrow{t_i - t \gg 0} \tilde{Z}_{A\,v} \langle \overline{B_s} | \tilde{O}_1(\mu) | B_s \rangle_v \, e^{-\varepsilon(t_i - t)}, \tag{2.2}$$

$$\tilde{C}_{SS+PP}^{(3)}(t_i,t) = \langle \sum_{\vec{x},\vec{y}} \tilde{A}_0^{+}(\vec{x},t_i) \tilde{O}_2(0,0) \tilde{A}_0^{-\dagger}(\vec{y},t) \rangle_U \xrightarrow{t_i - t \gg 0} \tilde{Z}_{A\,v} \langle \overline{B_s} | \tilde{O}_2(\mu) | B_s \rangle_v \, e^{-\varepsilon(t_i - t)}, \tag{2.3}$$

$$\tilde{A}_0^{\pm} \equiv \overline{h}^{\pm} \gamma_0 \gamma^5 s, \quad \tilde{O}_1 = \overline{h}^{(+)i} \gamma_\mu (1-\gamma^5) s^i \overline{h}^{(-)j} \gamma_\mu (1-\gamma^5) s^j, \quad \tilde{O}_2 = \overline{h}^{(+)i} (1-\gamma^5) s^i \overline{h}^{(-)j} (1-\gamma^5) s^j.$$

$\sqrt{\tilde{Z}_A} = \langle 0 | \tilde{A}_0^- | B_s \rangle_v = \langle 0 | \tilde{A}_0^+ | \overline{B_s} \rangle_v$ and ε is the binding energy of the pseudoscalar heavy-light meson. In $\tilde{C}^{(2)\pm}(t_i,t)$ one current $\tilde{A}_0^{\pm}$ is local whereas the other is smeared. The role of the smearing is to isolate earlier the ground state [16], as shown in Fig. 1 [1]. We see that the same state is isolated when purely local currents are used (with those currents the signal does not exist if V_0^{HYP} is not used in the heavy quark action). The source operators in $\tilde{C}_{VV+AA}^{(3)}(t_i,t)$ and $\tilde{C}_{SS+PP}^{(3)}(t_i,t)$ are the smeared currents $\tilde{A}_0^{\pm}$, whereas the four-fermion operators $\tilde{O}_1$ and $\tilde{O}_2$ are purely local. In (2.1), (2.2) and (2.3) the subscript "v" and superscript "$\sim$" are designed to remind the reader that states and operators are defined in HQET. Note that in the computation of $\tilde{C}_{VV+AA}^{(3)}(t_i,t)$ and $\tilde{C}_{SS+PP}^{(3)}(t_i,t)$ there are two terms, coming from two different Wick contractions, namely $\sum_i B_{ii}(t) \sum_j B_{jj}(t_i)$ and $\sum_{i,j} B_{ij}(t) B_{ji}(t_i)$, where i, j are the color indices and $B_{ij}(t) = \mathrm{Tr}\left[\sum_{\vec{x}} \gamma_{\mu L} \mathscr{S}_L^{\dagger ik}(0;\vec{x},t) \gamma_0 \gamma^5 \mathscr{S}_H^{kj}(\vec{x},t;0)\right]$; $\mathscr{S}_L$ and $\mathscr{S}_H$ are the light and heavy propagators respectively and the trace is over spinor indices.

[1] Even if the time interval from which we extract the binding energy starts at $t = 9$ (green line), the overlap with radial excitations is quite reduced since $t = 6$ when currents are smeared.

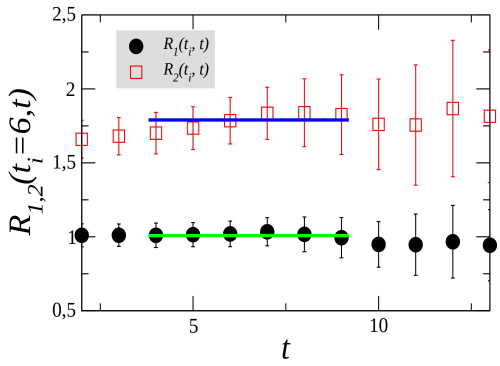

Figure 2: Signals for $R_{1,2}(t_i,t)$ defined in eq (2.4): green and blue lines indicate the time interval on which we fit the signal to a constant to extract $\tilde{B}_1(a)$ and $\tilde{B}_2(a)$ respectively

After having computed the correlation functions (2.1), (2.2) and (2.3) we build the following two ratios $R_1(t_i,t)$ and $R_2(t_i,t)$:

$$R_1(t_i,t) = \frac{\tilde{C}^{(3)}_{VV+AA}(t_i,t)}{\frac{8}{3}\tilde{Z}_A^2 \tilde{C}^{(2)+}_{AA}(t_i)\tilde{C}^{(2)-}_{AA}(t)} \xrightarrow{t_i - t \gg 0} \frac{{}_v\langle \overline{B_s}|\tilde{O}_1|B_s\rangle_v}{\frac{8}{3}|\langle 0|\tilde{A}_0^-|B_s\rangle_v|^2} \equiv \tilde{B}_1(a),$$

$$R_2(t_i,t) = \frac{C^{(3)}_{SS+PP}(t_i,t)}{-\frac{5}{3}\tilde{Z}_A^2 \tilde{C}^{(2)+}_{AA}(t_i)\tilde{C}^{(2)-}_{AA}(t)} \xrightarrow{t_i - t \gg 0} \frac{{}_v\langle \overline{B_s}|\tilde{O}_2|B_s\rangle_v}{-\frac{5}{3}|\langle 0|\tilde{A}_0^-|B_s\rangle_v|^2} \equiv \tilde{B}_2(a). \tag{2.4}$$

Those ratios are calculated either with a fixed time $t \in [-6,-8,-10,-12,-14,-16]$ and t_i free, or by fixing $t_i \in [6,8,10,12,14,16]$ while letting t free. We take the average of the two options. In Fig. 2 we show the quality of the signals for $R_{1,2}(t_i,t)$, with $t_i = 6$ fixed. The signal for $\tilde{B}_1(a)$ is quite stable as a function of t_i, whereas the signal for $\tilde{B}_2(a)$ rapidly deteriorates for larger t_i, and is completely lost for $t_i > 10$.

3. Extraction of physical B_{B_s}

Three steps are required to extract $B_{B_s} \equiv B_1$ from the lattice:
(1) $\tilde{B}_{1,2}(a)$ are matched onto the continuum $\overline{\mathrm{MS}}$(NDR) scheme at NLO in perturbation theory at the renormalization scale $\mu = 1/a$ [15],
(2) $\tilde{B}_{1,2}$ are evolved from $\mu = 1/a$ to $\mu = m_b$ by using the HQET anomalous dimension matrix, known to 2-loop accuracy in perturbation theory [7, 17],
(3) $\tilde{B}_{1,2}(\mu = m_b)$ are then matched onto their QCD counterpart, $B_{1,2}(m_b)$, in the $\overline{\mathrm{MS}}$(NDR) scheme at NLO [17].
The advantage of using a chiral light quark action for the step (1) lies in the fact that four-fermion operators can mix only with a four-fermion operator of the same chirality. In other words we have not more than 4 independent renormalization constants in the renormalization matrix, because $\tilde{O}_1$ and $\tilde{O}_2$ can mix neither with $\tilde{O}_3 \equiv \overline{h}^+\gamma_{\mu L} s\,\overline{h}^-\gamma_{\mu R} s$, nor with $\tilde{O}_4 \equiv \overline{h}^+(1-\gamma^5)s\,\overline{h}^-(1+\gamma^5)s$:

$$\begin{pmatrix} \tilde{B}_1^{\overline{\mathrm{MS}}}(\mu) \\ \tilde{B}_2^{\overline{\mathrm{MS}}}(\mu) \end{pmatrix} = \begin{pmatrix} Z_{11}(a\mu) & Z_{12}(a\mu) \\ Z_{21}(a\mu) & Z_{22}(a\mu) \end{pmatrix} \begin{pmatrix} \tilde{B}_1(a) \\ \tilde{B}_2(a) \end{pmatrix}.$$

β	V	$N_{\rm conf}$	ρ	m_0^s
6.0	$16^3 \times 32$	80	1.4	0.06 GeV
5.85	$16^3 \times 32$	30	1.6	0.09 GeV

Table 1: Parameters of our simulations: m_0^s and ρ have been chosen following [18, 19]

Actually, because of the heavy quark symmetry, those constants are not all independent. By using the heavy quark symmetry (HQS) transformations $\overline{h}^{(\pm)}(x) \overset{HQS(i)}{\longrightarrow} -\frac{1}{2}\varepsilon^{ijk}\overline{h}^{(\pm)}(x)\gamma_j\gamma_k \;\; (i = 1,2,3)$, and the equations of motion for the heavy quark $\overline{h}^{(\pm)}\gamma_0 = \pm\overline{h}^{(\pm)}$, we see that:

$$O_{SS+PP} \equiv -O_{(VV+AA)_0}, \quad O_{VV+AA} \overset{HQS(i)}{\longrightarrow} O_{VV+AA}, \quad O_{SS+PP} \overset{HQS(i)}{\longrightarrow} -O_{(VV+AA)_i}.$$

As the action is invariant under the HQS transformations, we can deduce important constraints on the renormalization matrix Z_{ij}:

$$\begin{aligned}
\langle O_{VV+AA}(\mu)\rangle &= Z_{11}\langle O_{VV+AA}(a)\rangle + Z_{12}\langle O_{SS+PP}(a)\rangle, \\
\langle O_{VV+AA}(\mu)\rangle &= Z_{11}\langle O_{VV+AA}(a)\rangle - Z_{12}\langle O_{(VV+AA)_i}(a)\rangle \quad \text{(HQS(i))},
\end{aligned}$$

which implies that $Z_{12} = 0$. Moreover

$$\begin{aligned}
\langle O_{SS+PP}(\mu)\rangle &= Z_{21}\langle O_{VV+AA}(a)\rangle + Z_{22}\langle O_{SS+PP}(a)\rangle, \\
-\langle O_{(VV+AA)_i}(\mu)\rangle &= Z_{21}\langle O_{VV+AA}(a)\rangle - Z_{22}\langle O_{(VV+AA)_i}(a)\rangle \quad \text{(HQS(i))}, \\
-\sum_{i=1,3} O_{(VV+AA)_i}(\mu) \pm O_{(VV+AA)_0}(\mu) &\equiv -\langle O_{SS+PP}(\mu)\rangle - \langle O_{VV+AA}(\mu)\rangle, \\
&= (3Z_{21} - Z_{22})\langle O_{VV+AA}(a)\rangle - Z_{22}\langle O_{SS+PP}(a)\rangle, \\
&= -(Z_{11} + Z_{21})\langle O_{VV+AA}(a)\rangle - Z_{22}\langle O_{SS+PP}(a)\rangle,
\end{aligned}$$

which leads to $Z_{21} = (Z_{22} - Z_{11})/4$.
Finally we have

$$\begin{pmatrix} \tilde{B}_1^{\overline{\rm MS}}(\mu) \\ \tilde{B}_2^{\overline{\rm MS}}(\mu) \end{pmatrix} = \begin{pmatrix} Z_{11}(a\mu) & 0 \\ \frac{1}{4}\left[Z_{22}(a\mu) - Z_{11}(a\mu)\right] & Z_{22}(a\mu) \end{pmatrix} \begin{pmatrix} \tilde{B}_1(a) \\ \tilde{B}_2(a) \end{pmatrix}$$

Therefore only two independent renormalization constants are required to match the bag parameters $\tilde{B}_{1,2}(a)$ computed on the lattice to their counterpart renormalized in $\overline{\rm MS}$ scheme.

4. Results and discussion

Our results are based on two simulations, with the parameters given in Tab. 1. We find $B_{B_s}^{\overline{\rm MS}}(m_b) = 0.940(16)(22)$, where the first error is statistical, the second is systematic and contains the error from the estimation of $\alpha_s(1/a)$ and the finite a effects. From Fig. 3 it can be seen that our value is larger than the previous static result [4]. This difference is likely due to the use of Neuberger light quark action (no subtractions), due to the use of the HYP procedure, or the combination of both. From Fig. 3 we also notice that our value is also somewhat larger than the results obtained with the propagating heavy quark, which is due to our neglect of $1/m_b$ corrections or their not so proper renormalization. JLQCD collaboration showed that the errors due to quenching are likely to be small [8, 9]. We also plan to address that issue by unquenching the $B_s^0 - \overline{B_s^0}$ mixing amplitude in the static limit and by avoiding the subtraction procedure as well. The feasibility study by means of twisted mass QCD is underway.

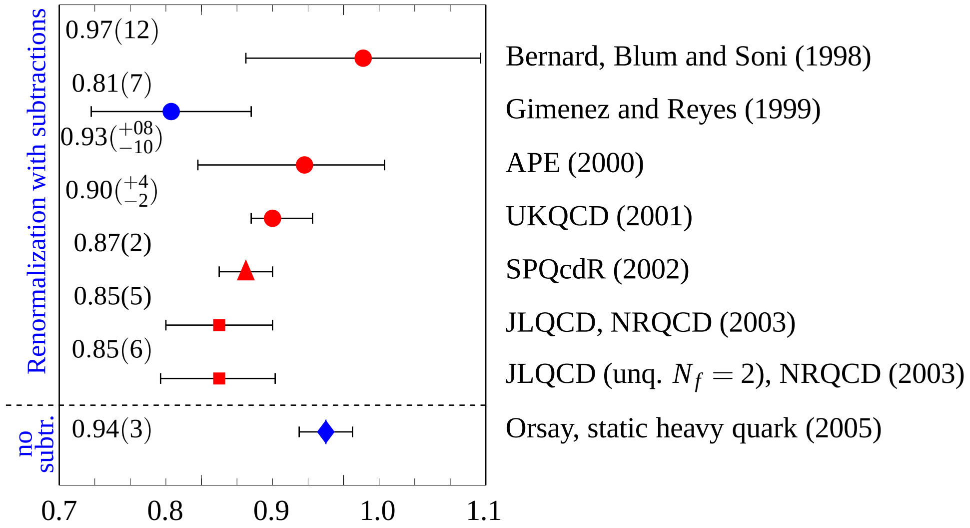

Figure 3: Various lattice values of $B_{B_s}^{\overline{MS}}(m_b)$ [3]-[9]; blue symbols correspond to a computation made with a static heavy quark

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