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$B \to D^* l \nu$ with staggered chiral perturbation theory

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An unquenched calculation of the form factor for $B \to D^* l \nu$ is needed to improve the determination of $|V_{cb}|$. The MILC lattices, computed with a 2+1 improved staggered action for the light quarks, are well suited to this purpose. The relevant staggered chiral perturbation theory (SChPT) must be known in order to correctly account for the "taste" breaking discretization effects associated with the staggered quarks to NLO in $1/m_{D^*}$. This SChPT calculation is presented.

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1. Introduction

The CKM element V_{cb} is important for the phenomenology of flavor physics in determining the apex of the unitarity triangle in the complex plane. $|V_{cb}|$ can be determined from inclusive and exclusive semileptonic B decays, and they are both limited by theoretical uncertainties. The inclusive method makes use of the heavy quark expansion [1, 2], but is limited by the breakdown of local quark-hadron duality, the errors of which are difficult to estimate. The exclusive method requires reducing the uncertainty of the form factor $\mathscr{F}_{B\to D^*}$, which has been calculated using lattice QCD in the quenched approximation [3]. In an effort to eliminate the errors due to quenching, as well as to reach lighter quark masses, we anticipate using lattices with 2+1 flavors of light staggered quarks generated by the MILC collaboration [4] to calculate $\mathscr{F}_{B\to D^*}$. It is therefore important to understand the taste violations in this heavy-light system, and for this we use the staggered heavy-light chiral perturbation theory of Aubin and Bernard [5].

We make use of the result from Aubin and Bernard that at $O(a^2)$ all taste-violations in the Symanzik action are those of the light quark sector [6, 7]. The discretization effects due to the heavy quarks are not taken into account explicitly in the ChPT, but can be absorbed into the definitions of the lattice heavy quarks. The heavy-light mesons are combined into a single field

$$H_a = \frac{1+\not{v}}{2}[\gamma^\mu B^*_{\mu a} - \gamma_5 B_a], \tag{1.1}$$

with the conjugate, $\overline{H}_a \equiv \gamma_0 H_a^\dagger \gamma_0$. The light mesons appear in the form $\Sigma = \sigma^2 = \exp[2i\Phi/f]$, where Φ is a 12×12 matrix that contains the pions

$$\Phi = \begin{pmatrix} U & \pi^+ & K^+ \\ \pi^- & D & K^0 \\ K^- & \overline{K}^0 & S \end{pmatrix}, \tag{1.2}$$

where $U = \sum_{a=1}^{16} U_a T_a$, *etc*, and $T_a = \{\xi_5, i\xi_{\mu 5}, i\xi_{\mu\nu}, \xi_\mu, \xi_I\}$.

The calculation makes use of an expansion in m_q (the light quark mass), a^2, and the heavy-light residual momentum. The Lagrangian is

$$\mathscr{L}^{(2)} = i\mathrm{tr}_D[\overline{H}_a v^\mu(\delta_{ab}\partial_\mu + iV_\mu^{ba})H_b] + g_\pi \mathrm{tr}_D[\overline{H}_a H_b \gamma^\nu \gamma_5 A_\nu^{ba}] + \mathscr{L}_{S\chi PT}, \tag{1.3}$$

where $V_\mu \equiv (i/2)[\sigma^\dagger\partial\sigma + \sigma\partial_\mu\sigma^\dagger]$, and $A_\mu \equiv (i/2)[\sigma^\dagger\partial\sigma - \sigma\partial_\mu\sigma^\dagger]$. The leading correction to this Lagrangian at $1/m_c$ relevant for this calculation is

$$\mathscr{L}_M^{(2)} = \frac{\lambda_2}{m_c}\mathrm{tr}[\overline{H}_a\sigma^{\mu\nu}H_a\sigma_{\mu\nu}], \tag{1.4}$$

and gives rise to the splitting between the D and D^* masses, $\lambda_2 = -\frac{m_c}{2}\Delta^{(c)} = -\frac{m_c}{2}(m_{D^*} - m_D)$. The staggered light Lagrangian is

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$$\mathscr{L}_S^{(2)} = \frac{f^2}{8}\mathrm{tr}[\partial_\mu \Sigma \partial^\mu \Sigma] + \frac{f^2 B_0}{4}\mathrm{tr}[\chi^\dagger \Sigma + \Sigma^\dagger \chi] + \frac{2m_0^2}{3}(U_I + D_I + S_I + \ldots)^2 + a^2 \mathscr{V}, \tag{1.5}$$

where

$$\mathscr{V} = \sum_k C_k \mathscr{O}_k + \sum_{k'} C_{k'} \mathscr{O}_{k'}, \tag{1.6}$$

are taste breaking operators that can be found in [7]. The primed operators are taste non-diagonal, while the unprimed operators are taste-diagonal. The terms in Eq. (1.3) give rise to the NLO chiral logarithms. There are additional terms not shown in this equation coming from a new taste-breaking potential that involves both heavy and light mesons, and although this potential is of higher order than the terms in Eq. (1.3), they can contribute analytic terms at NLO.

2. Obtaining $|V_{cb}|$

The differential rate for the semileptonic decay $\overline{B} \to D^* l \overline{\nu}_l$ is

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2 - 1}\,\mathscr{G}(w) |V_{cb}|^2 |\mathscr{F}_{B\to D^*}(w)|^2 \tag{2.1}$$

where $w = v' \cdot v$ is the velocity transfer from the initial state to the final state, $\mathscr{G}(w)$ is a kinematic factor and $\mathscr{F}_{B\to D^*}$ is a matrix element which must be calculated nonperturbatively. This matrix element is a combination of several form factors, but at zero recoil it simplifies to $\mathscr{F}_{B\to D^*}(1) = h_{A_1}(1)$. Heavy quark symmetry plays an important role in constraining $h_{A_1}(1)$, leading to the heavy quark expansion [8, 9]

$$h_{A_1}(1) = \eta_A \left[1 - \frac{l_V}{(2m_c)^2} + \frac{2 l_A}{2m_c 2m_b} - \frac{l_P}{(2m_b)^2}\right], \tag{2.2}$$

up to order $1/m_Q^2$. The above works were generalized to lattice gauge theory in [10]. The l's are long-distance matrix elements of the heavy quark effective theory (HQET).

It was realized that these l's could be computed precisely by making use of the double ratios of various matrix elements at zero recoil [3]:

$$\mathscr{R}_+ = \frac{\langle D|\bar{c}\gamma_4 b|\overline{B}\rangle \langle \overline{B}|\bar{b}\gamma_4 c|D\rangle}{\langle D|\bar{c}\gamma_4 c|D\rangle \langle \overline{B}|\bar{b}\gamma_4 b|\overline{B}\rangle} = |h_+(1)|^2, \tag{2.3}$$

$$\mathscr{R}_1 = \frac{\langle D^*|\bar{c}\gamma_4 b|\overline{B}^*\rangle \langle \overline{B}^*|\bar{b}\gamma_4 c|D^*\rangle}{\langle D^*|\bar{c}\gamma_4 c|D^*\rangle \langle \overline{B}^*|\bar{b}\gamma_4 b|\overline{B}^*\rangle} = |h_1(1)|^2, \tag{2.4}$$

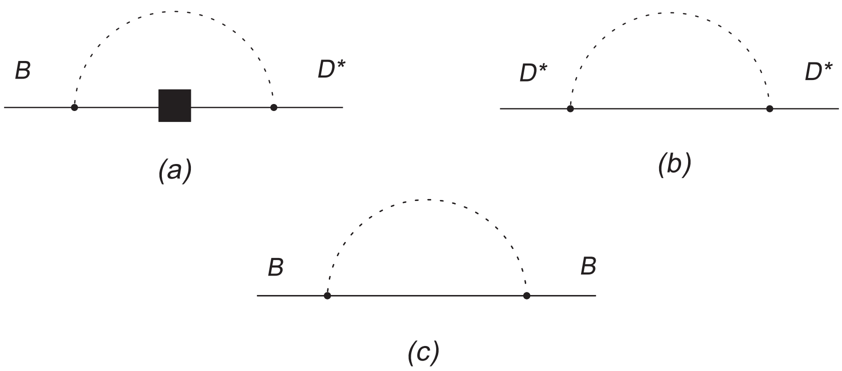

Figure 1: One-loop diagrams that contribute to $B \to D^*$. The solid line represents a meson containing a heavy quark, and the dashed line represents light mesons. The small solid circles are strong vertices and contribute a factor of g_π. The large solid square is a weak interaction vertex. Diagram (a) is a vertex correction, and (b) and (c) correspond to wavefunction renormalization.

$$\mathscr{R}_{A_1} = \frac{\langle D^*|\bar{c}\gamma_j\gamma_5 b|\overline{B}\rangle \langle \overline{B}^*|\bar{b}\gamma_j\gamma_5 c|D\rangle}{\langle D^*|\bar{c}\gamma_j\gamma_5 c|D\rangle \langle \overline{B}^*|\bar{b}\gamma_j\gamma_5 b|\overline{B}\rangle} = \left|\check{h}_{A_1}(1)\right|^2. \tag{2.5}$$

Statistical fluctuations in the numerator and denominator are highly correlated and therefore cancel in the ratio. The normalization uncertainty in the lattice currents also largely cancels in the ratio. Thus, all uncertainties scale as $\mathscr{R}-1$ rather than as $\mathscr{R}$. Making use of the heavy quark expansions of the above double ratios, we can obtain the three l's needed to construct h_{A_1} to order $1/m_Q^2$ (Eq 2.2), one from each ratio.

$$h_+(1) \ = \ \eta_V \left[1 - l_P\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right], \tag{2.6}$$

$$h_1(1) \ = \ \eta_V \left[1 - l_V\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right], \tag{2.7}$$

$$\check{h}_{A_1}(1) \ = \ \check{\eta}_A \left[1 - l_A\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right], \tag{2.8}$$

where η_V and $\check{\eta}_A$ are short-distance coefficients of HQET.

3. Chiral corrections to $B \to D^*$ at zero recoil

The one loop diagrams that contribute to $B \to D^*$ are shown in Fig. 1. In general, the light mesons represented by the dotted lines in Fig. 1 include one or more insertions of the hairpin diagrams (see [5]) for the singlet, axial and vector taste flavor-neutral mesons. The original continuum ChPT result was obtained by Randall and Wise [11], and was generalized to the partially quenched case by Savage [12]. The result for the 2+1 ($m_u = m_d \neq m_s$) full lattice QCD case including taste-breaking terms is

$$
\begin{aligned}
h_{A_1}^{2+1}(1) \;=\; & 1+\frac{X_A}{m_c^2}+\frac{g_\pi^2}{48\pi^2 f^2}\left[\frac{3}{2}\overline{F}_{\pi_I}+\overline{F}_{K_I}+\frac{1}{6}\overline{F}_{\eta_I}+a^2\delta_V'\left(\frac{m_{S_V}^2-m_{\pi_V}^2}{(m_{\eta_V}^2-m_{\pi_V}^2)(m_{\pi_V}^2-m_{\eta_V'}^2)}\overline{F}_{\pi_V}\right.\right. \\
& \left.\left.+\frac{m_{\eta_V}^2-m_{S_V}^2}{(m_{\eta_V}^2-m_{\eta_V'}^2)(m_{\eta_V}^2-m_{\pi_V}^2)}\overline{F}_{\eta_V}+\frac{m_{S_V}^2-m_{\eta_V'}^2}{(m_{\eta_V}^2-m_{\eta_V'}^2)(m_{\eta_V'}^2-m_{\pi_V}^2)}\overline{F}_{\eta_V'}\right)+(V\to A)\right],
\end{aligned}
\tag{3.1}
$$

where $\overline{F}_j \equiv F\left(m_j, -\Delta^{(c)}/m_j\right)$, and

$$
\begin{aligned}
F\left(m_j, x\right) \;=\; & \frac{m_j^2}{x}\left\{x^3\ln\frac{m_j^2}{\Lambda^2}-\frac{2}{3}x^3-4x+2\pi\right. \\
& \left.-\sqrt{x^2-1}(x^2+2)\left(\ln\left[1-2x(x-\sqrt{x^2-1})\right]-i\pi\right)\right\} \\
& \longrightarrow -(\Delta^{(c)})^2\ln\left(\frac{m_j^2}{\Lambda^2}\right)+\mathscr{O}[(\Delta^{(c)})^3],
\end{aligned}
\tag{3.2}
$$

with g_π the $D^*-D-\pi$ coupling, $\Delta^{(c)}=m_{D^*}-m_D=142$ MeV and X_A is a term that is independent of the light quark masses that must exactly cancel the scale dependence of the logarithms. In principle, this term also contains taste-breaking contributions which vanish as the lattice spacing goes to zero. From the discussion after Eq (1.4) we see that the $D-D^*$ splitting begins at $1/m_c$, so that the one-loop chiral corrections to the above formula begin at $1/m_c^2$. We do not account for the smaller deviations of the b-quark mass from the heavy quark limit in this calculation.

Fig. 2 is a plot of $h_{A_1}(1)$ vs m_π^2, illustrating the importance of accounting for staggered effects in the chiral limit. The part of the graph that asymptotes to a straight line is a guess (based on the earlier quenched result [3]) as to what a linear fit to data points might look like for the MILC lattices for $h_{A_1}(1)$. The curves add to the linear behavior the contribution from the chiral logs with $g_\pi=0.60$. The curve with the large cusp is the continuum extrapolated curve; the one without the cusp also includes the staggered effects with values determined from the MILC coarse lattices ($a=0.125$ fm). The way the procedure for the extrapolation would work in practice is one would fit lattice data to Eq. (3.1), and then the taste-breaking effects would be eliminated by setting the terms proportional to a^2 in Eq. (3.1) to zero. The staggered data are expected to be linear, even when the continuum result is not; this is a characteristic effect of taste-breaking terms due to the mass-splittings of the different taste pions. However, simulations would not likely be sensitive to the cusp anytime soon even if staggering did not smooth it out, given that the cusp only occurs at values very close to the physical pion mass. In this case one is especially dependent on the ChPT in order to extrapolate to the physical light quark masses.

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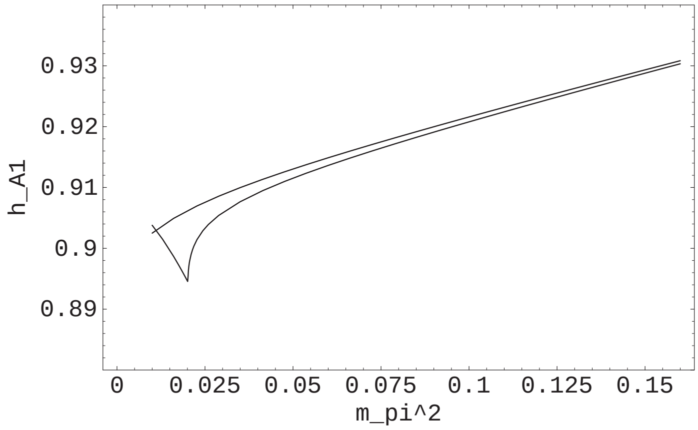

Figure 2: These curves add to linear behavior the contribution from chiral logs with $g_\pi = 0.60$. The curve with the large cusp is the continuum extrapolation; the one without the cusp includes also the staggered effects.

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