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A strategy for the computation of m_b including $1/m$ terms

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We consider HQET including the first order correction in $1/m$. A strategy for the computation of the b-quark mass following the scheme

$$\begin{tikzcd}
\boxed{\text{experiment}} & & \boxed{\text{Lattice with } am_{\mathrm{q}} \ll 1} \\
m_{\mathrm{B}} = 5.4\,\mathrm{GeV} \arrow[d] & L_2 = 2L_1 & \Phi_1(L_1,M),\Phi_2(L_1,M) \arrow[d] \\
\Phi_1^{\mathrm{HQET}}(L_2),\Phi_2^{\mathrm{HQET}}(L_2) & & \Phi_1^{\mathrm{HQET}}(L_1),\Phi_2^{\mathrm{HQET}}(L_1) \arrow[ll, "\sigma_{\mathrm{m}}(u_1)", "\sigma_1^{\mathrm{kin}}(u_1),\sigma_2^{\mathrm{kin}}(u_1)"']
\end{tikzcd}$$

is discussed. Only two quantities $\Phi_{1/2}$ have to be considered in order to match QCD and HQET, since the spin-dependent interaction is easily eliminated due to the spin symmetry of the static theory. Quite simple formulae relate the renormalization group invariant b-quark mass (M_{b}) to the B-meson mass. All entries in these formulae are non-perturbatively defined and can be computed in the continuum limit of the lattice regularized theory. For the numerically most critical part, we illustrate the cancellation of power divergences by a numerical example.
Numerical results for the $1/m$ correction to M_{b}, are presented in a companion talk.

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1. Introduction

Although HQET is the most natural effective theory for heavy-light systems, its lattice regularized version has practically only been used at lowest order. Indeed, a strategy to overcome the problem of power divergent mixings [1], was only found rather recently [2]. Its potential was demstrated by a computation of the b-quark mass to lowest non-trivial order in $1/m$, the static approximation. Here we fill the formalism of [2], sketched in the abstract, with practicable definitions in terms of Schrödinger functional correlation functions and give a concrete formula for the $1/m$-correction to the quark mass.

Neglecting $1/m^2$ corrections – as throughout this report – we write the HQET Lagrangian

$$\mathscr{L}_{\rm HQET} \,=\, \mathscr{L}_{\rm stat}(x) - \omega_{\rm spin}\mathscr{O}_{\rm spin}(x) - \omega_{\rm kin}\mathscr{O}_{\rm kin}(x) \tag{1.1}$$

$$\mathscr{O}_{\rm spin} = \bar\psi_{\rm h}\boldsymbol{\sigma}\mathbf{B}\psi_{\rm h}\,, \quad \mathscr{O}_{\rm kin} = \bar\psi_{\rm h}\mathbf{D}^2\psi_{\rm h}\,, \tag{1.2}$$

such that the classical values for the coefficients are $\omega_{\rm kin} = \omega_{\rm spin} = 1/(2m)$. Since expectation values

$$\langle\mathscr{O}\rangle \,=\, \langle\mathscr{O}\rangle_{\rm stat} + \omega_{\rm kin}\langle\mathscr{O}\rangle_{\rm kin} + \omega_{\rm spin}\langle\mathscr{O}\rangle_{\rm spin}\,, \tag{1.3}$$

$$\langle\mathscr{O}\rangle_{\rm kin} = \sum_x \langle\mathscr{O}\mathscr{O}_{\rm kin}(x)\rangle_{\rm stat}\,, \quad \langle\mathscr{O}\rangle_{\rm spin} = \sum_x \langle\mathscr{O}\mathscr{O}_{\rm spin}(x)\rangle_{\rm stat} \tag{1.4}$$

are defined through *insertions* of the higher dimensional terms $\mathscr{O}_{\rm kin}, \mathscr{O}_{\rm spin}$ *in the static theory*, they are renormalizable by power counting. However, in order to have a well defined continuum limit the bare, dimensionful, couplings $\omega_{\rm kin}, \omega_{\rm spin}$ have to be determined non-perturbatively [1, 2]. In the framework of lattice QCD, this is possible by matching a number of observables, Φ_i, $i = 1\ldots n$, between QCD and HQET, thus retaining the predicitivity of QCD. It is essential to note that this matching can be carried out in a finite volume of linear extent $L_1 \simeq 0.4\,\mathrm{fm}$, where heavy quarks can be simulated with a relativistic action [2, 3, 4].

Since the lowest order theory is spin-symmetric, it is trivial to form spin-averages which are independent of $\omega_{\rm spin}$. One thus expects that $n = 2$ is sufficient for a computation of the quark mass (in addition to $\omega_{\rm kin}$ there is an overall (state-independent) shift of energy levels, which we denote by $m_{\rm bare}$). For unexplained notation we refer to [2].

2. Basic observables

We consider the spin-symmetric combination

$$f_1^{\rm av}(\theta, T) \,=\, Z_\zeta^4\,\{f_1(\gamma_5)\}^{1/4}\,\{f_1(\gamma_1)\}^{3/4}\,, \tag{2.1}$$

formed from the boundary to boundary correlation functions

$$f_1(\Gamma) \,=\, -\frac{a^{12}}{2L^6}\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}} \left\langle \overline{\zeta}_{\rm l}{}'(\mathbf{u})\Gamma\zeta_{\rm b}'(\mathbf{v})\,\overline{\zeta}_{\rm b}(\mathbf{y})\Gamma\zeta_{\rm l}(\mathbf{z})\right\rangle\,, \tag{2.2}$$

of the QCD Schrödinger functional of size $T\times L^3$ and a periodicity phase θ [5] for the quark fields. Replacing the b-quark field by the effective field $\psi_{\rm h}$, using eq.(1.3,1.4), and accounting for the multiplicative renormalization of the boundary quark fields $\zeta\,,\,\overline{\zeta}$ one finds the $1/m$ expansion

$$f_1^{\rm av} \,=\, Z_{\zeta_{\rm h}}^2 Z_\zeta^2\,{\rm e}^{-m_{\rm bare}T}\,\{f_1^{\rm stat} + \omega_{\rm kin} f_1^{\rm kin}\}\,, \tag{2.3}$$

where the aformentioned energy shift $m_{\rm bare}$ enters. Deviating from the choice in [2], we now define[1]

$$\Phi_1(L,M) = \ln\left(f_1^{\rm av}(\theta,T)/f_1^{\rm av}(\theta',T)\right) - \ln\left(f_1^{\rm stat}(\theta,T)/f_1^{\rm stat}(\theta',T)\right) \quad \text{for} \quad T = L/2\,, \tag{2.5}$$

$$\Phi_2(L,M) = \tfrac{L}{2a}\ln\left(f_1^{\rm av}(\theta,T-a)/f_1^{\rm av}(\theta,T+a)\right), \tag{2.6}$$

with the expansion

$$\Phi_1(L,M) = \omega_{\rm kin}R_1^{\rm kin}\,, \quad \Phi_2(L,M) = L\left(m_{\rm bare}+\Gamma_1^{\rm stat}+\omega_{\rm kin}\Gamma_1^{\rm kin}\right) \tag{2.7}$$

$$R_1^{\rm kin} = \frac{f_1^{\rm kin}(\theta,T)}{f_1^{\rm stat}(\theta,T)} - \frac{f_1^{\rm kin}(\theta',T)}{f_1^{\rm stat}(\theta',T)}\,, \tag{2.8}$$

$$\Gamma_1^{\rm stat} = \tfrac{1}{2a}\ln\left(f_1^{\rm stat}(\theta,T-a)/f_1^{\rm stat}(\theta,T+a)\right), \tag{2.9}$$

$$\Gamma_1^{\rm kin} = \tfrac{1}{2a}\left(\frac{f_1^{\rm kin}(\theta,T-a)}{f_1^{\rm stat}(\theta,T-a)} - \frac{f_1^{\rm kin}(\theta,T+a)}{f_1^{\rm stat}(\theta,T+a)}\right). \tag{2.10}$$

3. Step scaling functions

We choose $L_1 \approx 0.4$fm, where a computation of $\Phi_i(L_1,M_{\rm b})$ is possible in lattice QCD (while at significantly larger values, L_1/a would have to be too large in order to control a^2 effects). From eq. (2.7) one then gets $\omega_{\rm kin}, m_{\rm bare}$ for lattice spacings $a = \frac{a}{L_1}\times 0.4$fm. On the other hand, contact to physical observables, e.g. the B-meson mass is made in large volume, where finite size effects are exponentially small. For reasonable values $a/L_1 = 1/12$ and $L_\infty \simeq 1.5$fm at the same lattice spacing, one needs $L_\infty/a \sim 50$. This situation is avoided by first computing step scaling functions which connect $\Phi_i(L_1,M)$ to $\Phi_i(L_2,M), L_2 = 2L_1$ and then connecting to large volume.

With the Schrödinger functional coupling, $u = \bar{g}^2(L)$, everywhere, the continuum step scaling functions σ are defined by

$$\Phi_1(2L,M) = \sigma_1^{\rm kin}(u)\Phi_1(L,M)\,, \quad \sigma_1^{\rm kin}(u) = \lim_{a/L\to 0}\left.\frac{R_1^{\rm kin}(2L)}{R_1^{\rm kin}(L)}\right|_{u=\bar{g}^2(L)} \tag{3.1}$$

and

$$\begin{aligned}\Phi_2(2L,M) - 2\Phi_2(L,M) &= \sigma_{\rm m}(u) + \left[\omega_{\rm kin}\,2L\left(\Gamma_1^{\rm kin}(2L)-\Gamma_1^{\rm kin}(L)\right)\right] \\ &= \sigma_{\rm m}(u)+\sigma_2^{\rm kin}(u)\,\Phi_1(L,M)\,, \quad \sigma_2^{\rm kin}(u) = \lim_{a/L\to 0} 2L\left.\frac{\Gamma_1^{\rm kin}(2L)-\Gamma_1^{\rm kin}(L)}{R_1^{\rm kin}(L)}\right|_{u=\bar{g}^2(L)}.\end{aligned} \tag{3.2}$$

Here the static step scaling function

$$\sigma_{\rm m}(u) = \lim_{a/L\to 0} 2L\left[\Gamma_1^{\rm stat}(2L)-\Gamma_1^{\rm stat}(L)\right]_{u=\bar{g}^2(L)}\,, \tag{3.3}$$

is not identical to $\sigma_{\rm m}(u)$ defined earlier [2], since $\Gamma_1^{\rm stat}$ differs from $\Gamma^{\rm stat}$ defined there. Note that the step scaling functions are independent of M, but $\Phi_i(L,M)$ have a mass dependence from fixing $\Phi_i(L_1,M)$ in the full theory.

[1] In the static computation of [2] the logarithmic derivative Γ of the correlation function $f_{\rm A}$ of the axial current with a boundary operator was used as a quantity to match effective theory and QCD. Including $1/m$ terms its expansion reads

$$f_{\rm A} = Z_{\rm A}^{\rm HQET}Z_{\zeta_{\rm h}}Z_\zeta\,{\rm e}^{-m_{\rm bare}x_0}\left\{f_{\rm A}^{\rm stat}+c_{\rm A}^{\rm HQET}f_{\delta{\rm A}}^{\rm stat}+\omega_{\rm kin}f_{\rm A}^{\rm kin}+\omega_{\rm spin}f_{\rm A}^{\rm spin}\right\}, \tag{2.4}$$

with the term $f_{\delta{\rm A}}^{\rm stat}$ due to the $1/m$ correction to the static axial current. While $\omega_{\rm spin}$ represents no problem, an extra observable is needed to fix $f_{\delta{\rm A}}^{\rm stat}$. Here, we avoid this complication by working exclusively with $f_1^{\rm av}$.

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4. Large volume

The connection of Φ_i to the spin-averaged B-meson mass, $m_{\rm B}$, is

$$
\begin{aligned}
Lm_{\rm B} \,-\, \Phi_2(L,M) &= \left[L\,(E^{\rm stat} - \Gamma_1^{\rm stat}(L))\right] + \left[L\,\omega_{\rm kin}\,(\hat{E}^{\rm kin} - \Gamma_1^{\rm kin}(L))\right] \\
&= \left[L\,(E^{\rm stat} - \Gamma_1^{\rm stat}(L))\right] + \rho(u)\Phi_1(L,M)\,, \quad \rho(u) = \lim_{a/L\to 0} L\,\left.\frac{\hat{E}^{\rm kin} - \Gamma_1^{\rm kin}(L)}{R_1^{\rm kin}(L)}\right|_{u=\bar{g}^2(L)}\,.
\end{aligned}
\tag{4.1}
$$

Here we have used the abbreviations

$$
E^{\rm stat} = \lim_{L\to\infty}\Gamma_1^{\rm stat}(L)\,, \qquad \hat{E}^{\rm kin} = \lim_{L\to\infty}\Gamma_1^{\rm kin}(L)\,, \tag{4.2}
$$

where $E^{\rm stat}$ is the (unrenormalized) energy in large volume in the spin-averaged B-channel in static approximation and $\omega_{\rm kin}\hat{E}^{\rm kin}$ is its $1/m$ correction. The hat on $\hat{E}^{\rm kin}$ is to remind us that this quantity turns into an energy only upon multiplication with the dimensionful $\omega_{\rm kin}$. Its numerical evaluation has already been investigated in [6]. We use $[\ldots]$ braces to indicate combinations which have a continuum limit by themselves. For example, the two terms in eq. (4.1) can be computed with different regularizations if this is useful.

5. Final equation

The above equations are now easily combined to yield the $1/m$ correction, $m_{\rm B}^{(1)}$, to the (spin-averaged) B-meson mass via ($L_2 = 2L_1$),

$$
\begin{aligned}
m_{\rm B} \;&=\; m_{\rm B}^{\rm stat} + m_{\rm B}^{(1)} = m_{\rm B}^{\rm stat} + m_{\rm B}^{(1a)} + m_{\rm B}^{(1b)}\,, && (5.1)\\
L_2\,m_{\rm B}^{\rm stat}(M) \;&=\; \left[L_2\,(E^{\rm stat} - \Gamma_1^{\rm stat}(L_2))\right] + \sigma_{\rm m}(u_1) + 2\,\Phi_2(L_1,M) && (5.2)\\
L_2\,m_{\rm B}^{(1a)}(M) \;&=\; \sigma_2^{\rm kin}(u_1)\,\Phi_1(L_1,M)\,, \qquad u_i = \bar{g}^2(L_i) && (5.3)\\
L_2\,m_{\rm B}^{(1b)}(M) \;&=\; \left[L_2\,(\hat{E}^{\rm kin} - \Gamma_1^{\rm kin}(L_2))\omega_{\rm kin}\right] = \rho(u_2)\,\sigma_1^{\rm kin}(u_1)\Phi_1(L_1,M)\,.
\end{aligned}
$$

Again, terms in braces have a continuum limit. While $m_{\rm B}^{(1a)}$ is purely derived from finite volume, the term $m_{\rm B}^{(1b)}$ involves a large volume computation.

Starting from $M_{\rm b}^{\rm stat}$, the solution of the leading order equation,

$$
m_{\rm B}^{\rm exp} \;=\; m_{\rm B}^{\rm stat}(M_{\rm b}^{\rm stat})\,, \tag{5.4}
$$

and the slope

$$
S \;=\; \left.\frac{\rm d}{{\rm d}M} m_{\rm B}^{\rm stat}\right|_{M=M_{\rm b}^{\rm stat}} = \frac{1}{L_1}\left.\frac{\rm d}{{\rm d}M}\Phi_2(L_1,M)\right|_{M=M_{\rm b}^{\rm stat}}\,, \tag{5.5}
$$

we finally obtain the first order correction $M_{\rm b}^{(1)}$ to the RGI b-quark mass

$$
M_{\rm b} \;=\; M_{\rm b}^{\rm stat} + M_{\rm b}^{(1)}\,, \quad M_{\rm b}^{(1)} = -\frac{1}{S} m_{\rm B}^{(1)}\,. \tag{5.6}
$$

The final uncertainty for $M_{\rm b}$ *due to the* $1/m$ *expansion* is of order $\mathrm{O}(\Lambda_{\rm QCD}^3/M_{\rm b}^2)$, which translates into a numerical estimate of MeV scale. It is thus clear that other sources of error will dominate in a practical calculation. Note that the precise value for m_B matters. One should use the spin-averaged mass $m_B^{\text{experimental}} = \frac{1}{4}m_{B^0} + \frac{3}{4}m_{B_0^*} = [\frac{1}{4}5279 + \frac{3}{4}5325]\,\text{MeV} = 5314\,\text{MeV}$ if one can extrapolate E to the chiral limit of the light quark or

$$m_B^{\text{experimental}} = m_{B_s} + \frac{3}{4}m_{B_0^*} - \frac{3}{4}m_{B_0} = [5370 + \frac{3}{4}(5325 - 5279)]\,\text{MeV} = 5405\,\text{MeV} \quad (5.7)$$

if one works directly with a strange quark (as light quark). The latter formula neglects the dependence of the spin splitting on the light quark mass.

6. Remarks

The following facts are worth noting.

- The $1/m$ expansion in heavy light systems is an expansion in terms of Λ_{QCD}/m, where all external scales have to be of order Λ_{QCD}. This applies in particular to our scale L_1^{-1}. Indeed, numerically it is rather close to Λ_{QCD} and explicit investigations [3, 4] have shown that the $1/m$-expansion is well behaved even when L^{-1} is a factor two larger.
- In our static computation [2, 3], we made the more natural choice Γ instead of Γ_1. Although it is advantageous to use Γ_1 when one includes the $1/m$ terms, the strategy can easily be formulated with Γ, at the expense of introducing a third quantity Φ_i to fix c_A^{HQET}. Since this will certainly be required for the computation of the $1/m$-correction to F_B, we will follow also that approach.
- Note that at each order k in the expansion, the result is ambiguous by terms of order $1/m^{k+1}$. Thus both $M_b^{(1)}$ and M_b^{stat} have an order $1/m$ ambiguity (e.g. they change when L_1 is changed), while in their sum $M_b = M_b^{\text{stat}} + M_b^{(1)}$ the ambiguity is reduced to $1/m^2$.
- In the present formulation of the effective theory, the $1/m$-terms approach the continuum with an asymptotic rate $\propto a$, in contrast to the leading order terms where this is $\propto a^2$ [2].
- Let us comment just on one numerical result at that point. The computation of $\sigma_2^{\text{kin}}(u_1)$, eq. (3.2), involves the difference of $\Gamma_1^{\text{kin}}(2L) - \Gamma_1^{\text{kin}}(L)$, where power divergent contributions cancel. As a typical case we choose $L/a = 12\,,\ T/a = 6$, and the static action HYP2 (see [7]), where our simulations yield $a^2\Gamma_1^{\text{kin}}(2L) = 0.5631(6)\,,\ a^2\Gamma_1^{\text{kin}}(L) = 0.5595(2)$, demonstrating a considerable cancellation. A detailed account of numerical results is presented in [8].

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