

Perturbative renormalization in parton distribution functions using improved actions

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We calculate the 1-loop renormalization of a set of extended fermionic bilinears which form a basis corresponding to moments of the parton distribution functions.

We use the overlap action for fermions and Luescher-Weisz (LW) action for gluons.

Our results are presented as a function of the overlap parameter ρ and the parameters entering the LW action.

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1. Introduction.

The description of physics in the Bjorken limit involves the operator product expansion which has the standard form:

$$J(x)J(0) \sim \sum_{n,i,l} C_l^{n,i}(x^2)x^{\mu_1} \dots x^{\mu_n} O_{\mu_1 \dots \mu_n}^{(n,i)}(0) \quad (1.1)$$

The forward matrix elements of the local operators $O^{(n,i)}$ appearing in this expansion are directly related to the moments of hadron structure functions. The dominant contribution in the expansion is given by operators whose twist (dimension minus spin) equals two, which in the flavor non-singlet case means the symmetric traceless operators[1]

$$O_{\mu\mu_1 \dots \mu_n} = \bar{\psi} \gamma_{\{\mu} D_{\mu_1} \dots D_{\mu_n\}} \frac{\lambda^a}{2} \psi \quad (1.2)$$

$$O_{\mu\mu_1 \dots \mu_n}^{(5)} = \bar{\psi} \gamma_{\{\mu} \gamma_5 D_{\mu_1} \dots D_{\mu_n\}} \frac{\lambda^a}{2} \psi \quad (1.3)$$

where λ^a are flavor matrices. In this work we compute the renormalization factors of the following quantities on the lattice.

a) Self-energy.

$$S_N^{-1} = i\gamma_\mu P_\mu \left(1 - \frac{g^2 C_F}{16\pi^2} \Sigma_1\right) \quad (1.4)$$

b) Quark bilinears.

$$O_X = \bar{\psi}(x)\Gamma^X\psi(x) \quad (1.5)$$

c) Operators which measure the first moment of quark momentum distributions.

$$O_{V_2} = \bar{\psi} \gamma_{\{1} D_{4\}} \psi \quad (1.6)$$

$$O'_{V_2} = \bar{\psi} \gamma_4 D_4 \psi - \frac{1}{3} \sum_{i=1}^3 \bar{\psi} \gamma_i D_i \psi \quad (1.7)$$

d) Operators which measure the first moment of quark helicity distributions.

$$O_{A_2} = \bar{\psi} \gamma_{\{1} \gamma_5 D_{4\}} \psi \quad (1.8)$$

$$O'_{A_2} = \bar{\psi} \gamma_4 \gamma_5 D_4 \psi - \frac{1}{3} \sum_{i=1}^3 \bar{\psi} \gamma_i \gamma_5 D_i \psi \quad (1.9)$$

Work is in progress using operators which measure the second moment of quark distributions. A longer write-up of our results can be found in our Ref.[2].

2. Computational details.

We denote the lattice action by

$$S = S_G + S_F \quad (2.1)$$

where S_G is the gluon action, and S_F is the fermion action. The gluon action we consider is written in standard notation:

$$S_{gl} = -\frac{1}{g^2} \left(c_0 \sum_{\text{plaquette}} \text{Tr}U_{pl} + c_1 \sum_{\text{rectangle}} \text{Tr}U_{rtg} + c_2 \sum_{\text{chair}} \text{Tr}U_{chr} + c_3 \sum_{\text{parallelogram}} \text{Tr}U_{plg} \right) \quad (2.2)$$

where U_{pl} is the standard plaquette, while the remaining U 's cover all possible closed loops containing up to six links. The coefficients c_0, c_1, c_2, c_3 satisfy the normalization condition:

$$c_0 + 8c_1 + 16c_2 + 8c_3 = 1 \quad (2.3)$$

As in Ref.[3], we have used the values of c_0, c_1, c_2, c_3 , shown in Table I, where ‘‘Plaquette’’ is the standard Wilson action for gluons, ‘‘Symanzik’’ is the tree-level improved action [4] and TILW is the tadpole improved Luescher-Weisz action [5, 6].

action	c_0	c_1	c_2	c_3
Plaquette	1.0	0.0	0.0	0.0
Symanzik	1.66666	-0.083333	0.0	0.0
TILW1 $\beta = 8.60$	2.31681	-0.151791	0.0	-0.0128098
TILW2 $\beta = 8.45$	2.34602	-0.154846	0.0	-0.0134070
TILW3 $\beta = 8.30$	2.38698	-0.159128	0.0	-0.0142442
TILW4 $\beta = 8.20$	2.41278	-0.161827	0.0	-0.0147710
TILW5 $\beta = 8.10$	2.44654	-0.165353	0.0	-0.0154645
TILW6 $\beta = 8.00$	2.48917	-0.169805	0.0	-0.0163414
Iwasaki [7]	3.648	-0.331	0.0	0.0
DBW2 [8]	12.2688	-1.4086	0.0	0.0

Table 1: The values of coefficients c_0, c_1, c_2, c_3

The action for massless overlap fermions is given by [9]

$$S_f = \sum_f \sum_{x,y} \bar{\psi}_x^f D_N(x,y) \psi_y^f \quad (2.4)$$

with

$$D_N = \rho [1 + X(X^\dagger X)^{-\frac{1}{2}}] \quad (2.5)$$

and: $X = D_w - \rho$. Here D_w is the massless Wilson-Dirac operator with $r = 1$, and ρ is a free parameter whose value must be in the range $0 < \rho < 2$ in order to guarantee the correct pole structure of D_N .

3. Calculations and Results.

Self energy

Let us consider the massless quark propagator S_N first. The inverse of S_N can be written

$$S_N^{-1} = i\gamma_\mu p_\mu \left(1 - \frac{g^2 C_F}{16\pi^2} \Sigma_1\right) \quad (3.1)$$

with $C_F = (N^2 - 1)/2N$, and $\Sigma_1(a, p) = \log(a^2 p^2) + b_\Sigma$ (Feynman gauge). Our results for b_Σ are shown in Fig.1.

Quark bilinears

Let us consider local operators of the form

$$O_X = \bar{\psi}(x) \gamma^X \psi(x) \quad (3.2)$$

with $\Gamma^S = 1$, $\Gamma^P = \gamma_5$, $\Gamma^V = \gamma_\mu$, $\Gamma^A = \gamma_\mu \gamma_5$, $\Gamma^T = \sigma_{\mu\nu} \gamma_5$, i.e. $X = S, P, V, A$ and T . We denote the amputated Green's function of the operator O_X by Λ^X . The final results are (in Feynman gauge):

$$\Lambda^{S,P} = \{1, \gamma_5\} + \frac{g^2 C_F}{16\pi^2} [-4\log(a^2 p^2) + b_{S,P}] \{1, \gamma_5\} \quad (3.3)$$

$$\Lambda_{\mu}^{V,A} = \{\gamma_\mu, \gamma_\mu \gamma_5\} + \frac{g^2 C_F}{16\pi^2} [-\gamma_\mu (\log(a^2 p^2) + b_{V,A}) + 2\frac{p_\mu \not{p}}{p^2}] \{1, \gamma_5\} \quad (3.4)$$

$$\Lambda_{\mu,\nu}^T = \sigma_{\mu,\nu} \gamma_5 + \frac{g^2 C_F}{16\pi^2} b_T \sigma_{\mu,\nu} \gamma_5 \quad (3.5)$$

Where $b_S = b_P$ and $b_V = b_A$. Only one diagram (“Vertex”) contributes to Λ^X .

Our results for b_Σ , $b_{S,P}$, $b_{V,A}$ and b_T provide a cross check and an extension of results appearing in Refs.[10, 3]. They are shown in Figures 1, 2, 3, 4 for typical values of the overlap parameter ρ .

Tables of numerical results are presented in our ref [2]. In all cases which can be compared, our results agree with those of Refs.[10, 3]. There is one exception, regarding the values of b_T for cases other than the standard plaquette b_T^{plaq} : In these cases, our results for $b_T - b_T^{plaq}$ have the opposite sign compared to Ref. [3].

First moment of quark distributions.

We have calculated the 1-loop renormalization coefficients of the operators O_{V_2}, O'_{V_2} which are the symmetric off-diagonal, and the traceless diagonal parts, respectively, of the operator:

$$O_{\mu,\nu} = \bar{\psi} \gamma_{\{\mu} D_{\nu\}} \psi \quad (3.6)$$

The axial counterparts O_{A_2}, O'_{A_2} of the above operators renormalize in the same way. Using the notation of Ref.[11], we find for the amputated Green's function of $O_{\mu,\nu}$ in the Feynman gauge:

$$\Lambda_{\mu,\nu}(a, p) = \gamma_{\{\mu} p_{\nu\}} + \frac{g^2 C_F}{16\pi^2} \left[\left(\frac{5}{3} \log(a^2 p^2) + b \right) \gamma_{\{\mu} p_{\nu\}} + b' \delta_{\mu\nu} \gamma_\nu p_\nu - \frac{4}{3} \frac{p_\mu p_\nu}{p^2} \not{p} \right] \quad (3.7)$$

The quantities b, b' correspond to $(b_1 + b_2)$, b_4 of Ref.[3], respectively. A total of four diagrams (“Tadpole”, “Vertex”, left and right “Sails”) contribute in this case. The rational coefficients 5/3 and -4/3 in Eq. (3.7) check with those of Ref.[11]. The values of b and b' are shown in Figures 5 and 6.

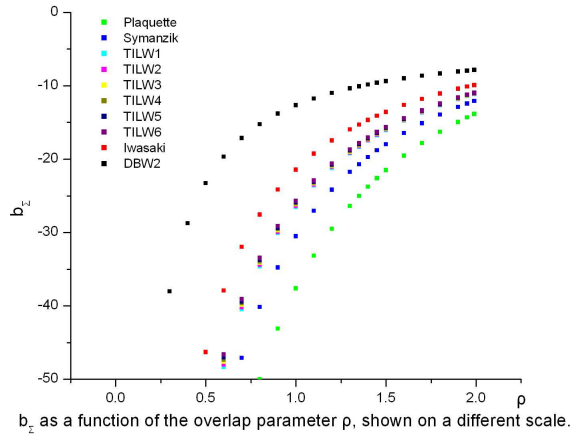


Figure 1

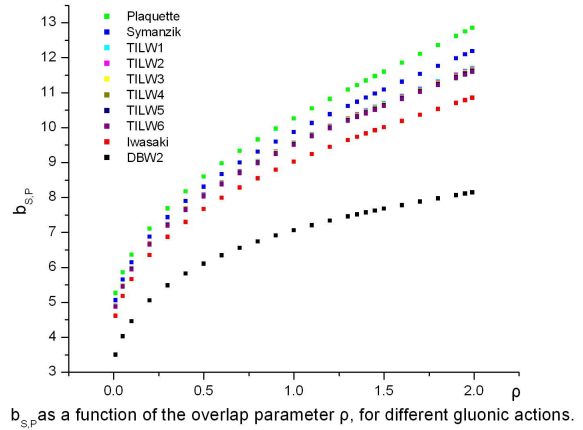


Figure 2

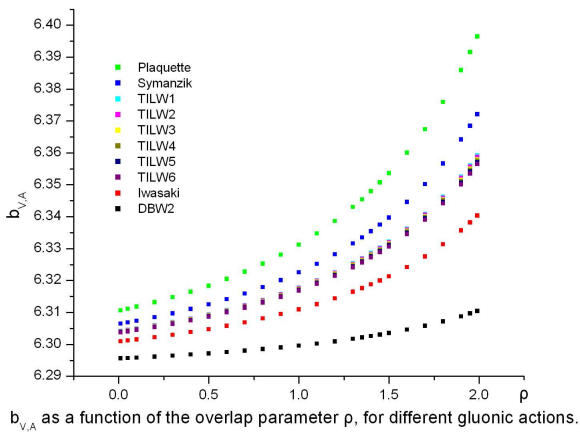


Figure 3

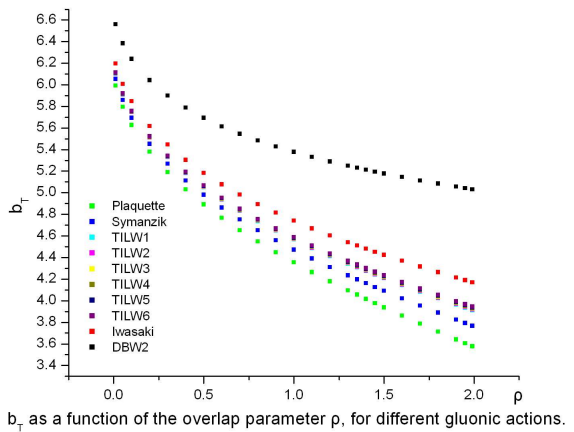


Figure 4

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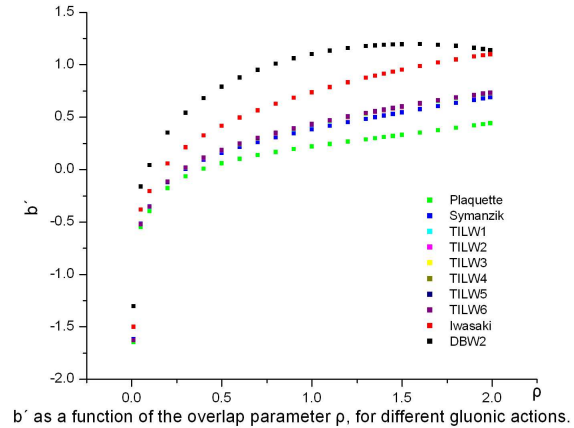
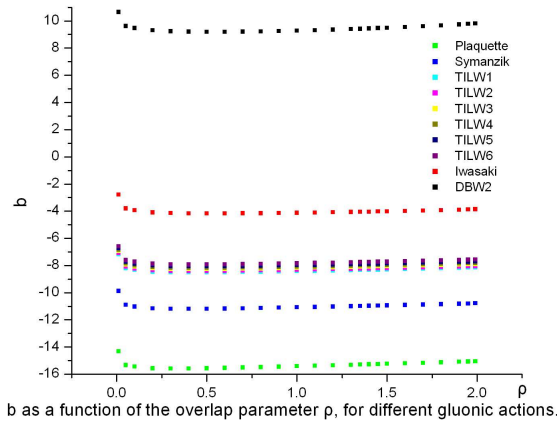


Figure 5

Figure 6

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