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Scaling test of quenched Wilson twisted mass QCD at maximal twist

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We present the results of an extended scaling test of quenched Wilson twisted mass QCD. We fix the twist angle by using two definitions of the critical mass, the first obtained by requiring the vanishing of the pseudoscalar meson mass $m_{\rm PS}$ for standard Wilson fermions and the second by requiring restoration of parity at non-zero value of the twisted mass μ and subsequently extrapolating to $\mu \to 0$. Depending on the choice of the critical mass we simulate at values of $\beta \in [5.7, 6.45]$, for a range of pseudoscalar meson masses 250 MeV $\lesssim m_{\rm PS} \lesssim$ 1 GeV and we perform the continuum limit for the pseudoscalar meson decay constant $f_{\rm PS}$ and various hadron masses (vector meson m_V, baryon octet $m_{\rm oct}$ and baryon decuplet $m_{\rm dec}$) at fixed value of $r_0 m_{\rm PS}$. For both definitions of the critical mass, lattice artifacts are consistent with O(a) improvement. However, with the second definition, large O(a^2) discretization errors present at small quark mass with the first definition are strongly suppressed. The results in the continuum limit are in very good agreement with those from the Alpha and CP-PACS Collaborations.

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1. Introduction

Twisted mass QCD (tmQCD), whose action reads

$$S[U,\psi,\bar{\psi}] = a^4 \sum_x \bar{\psi}(x)(D_W + m_0 + i\mu\gamma_5\tau_3)\psi(x) \ , \tag{1.1}$$

has been proposed as an alternative to Wilson QCD because it is not affected by the problem of unphysical zero modes and can lead to simplifications of the operator mixing pattern [1]. At maximal twist (i.e. by setting m_0 to its critical value m_c up to O(a), with μ now the bare quark mass) it has been shown [2] that parity even correlators (and thus energies and matrix elements) are automatically O(a) improved. Due to these properties, tmQCD is a very interesting candidate for dynamical simulations at small quark masses [3]. However, right at small quark masses $\mu \lesssim a$ [1], large lattice artifacts have been observed when using a definition of m_c obtained by requiring the vanishing of $m_{\rm PS}^2$ with standard Wilson fermions (in the following we will call it $m_c^{\rm pion}$) [4]. This kind of lattice artifacts are obviously affecting also dynamical simulations. An analysis based on Wilson Chiral Perturbation Theory (WχPT) [5] suggested a definition of m_c suitable to reach quark masses $\mu \sim a^2$. This definition is obtained by requiring the vanishing of the PCAC quark mass

$$m_{\rm PCAC} = \lim_{x_0\to\infty} \frac{\sum_{\mathbf{x}}\langle \partial_0 A_0^a(x)\, P^a(0)\rangle}{2\sum_{\mathbf{x}}\langle P^a(x) P^a(0)\rangle} \tag{1.2}$$

where $P^a = \bar{\psi}\gamma_5\frac{\tau^a}{2}\psi$, $A_\mu^a = \bar{\psi}\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi$.

An analysis *à la* Symanzik beyond O(a) shows the presence of O(a^2) cutoff effects enhanced at small pion mass, the most dangerous of which are of order $(a/m_{\rm PS}^2)^{2k}$ $k \geq 1$ which signal the presence of possibly large lattice artifacts in the $m_{\rm PS}^2 \lesssim a$ regime [6] (see also [7]). The result of this analysis is that there are two ways of reducing these large lattice artifacts: fixing the O(a) ambiguity in m_c as proposed in Refs. [5] (we will call this definition $m_c^{\rm PCAC}$) or add the clover term with non-perturbatively determined c_{SW} coefficient (and the corresponding value of m_c). We investigate here the first proposal (see Refs. [8] for the second possibility).

2. Determination of $m_c^{\rm PCAC}$

$m_{\rm PCAC}$ depends, in the neighborhood of a given estimate of m_c (e.g. $m_c^{\rm pion}$), smoothly on both m_0 and μ. Moreover, in the quenched approximation, a multiple mass solver can be used to compute the fermion propagator for different μ values at a given value of m_0. In view of these two facts, the procedure we adopted to determine $m_c^{\rm PCAC}$ is the following [9]:

1. choose n values of μ (with $\mu > a^2$) and n' values of m_0 (in the vicinity of $m_c^{\rm pion}$) which cover the range where $m_{\rm PCAC}(\mu)$ is close to zero (in the present case $n = 9$ and $n' = 4$).

2. find the values of $m_c(\mu)$ at which $m_{\rm PCAC}(\mu)$ is zero (see Fig. 1.a)

3. extrapolate $m_c(\mu)$ to $\mu = 0$ (see Fig. 1.a)[2].

In Ref. [6] one can find a theoretical analysis that justifies this procedure and shows that the value of $m_c^{\rm PCAC}$ in which we are interested can be obtained by a linear extrapolation from the region $\mu > a^2$ down to $\mu = 0$. $m_c^{\rm PCAC}$ has been determined for $\beta \in \{5.7, 5.85, 6.0, 6.2\}$ using statistics of O(100)-O(200) gauge configurations.

[1] powers of $\Lambda_{\rm QCD}$ required to match physical dimensions are in the following understood

[2] in Ref. [10] for each simulated value of μ the corresponding value of $m_c(\mu)$ were used.

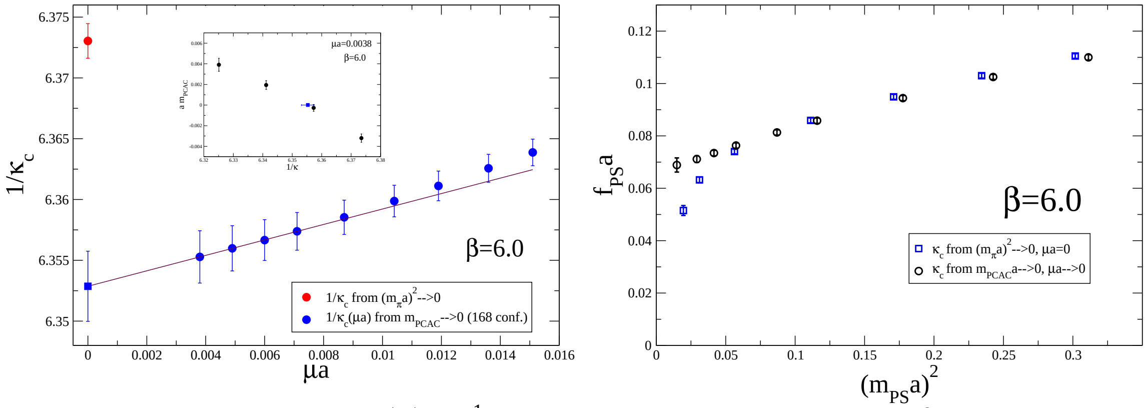

Figure 1: a. Determination of $m_c(\mu)$ ($\kappa_c^{-1} = 2am_c + 8$) for given values of μ and β and extrapolation of $m_c(\mu)$ to $\mu = 0$. b. $f_{\rm PS}$ as function of $m^2_{\rm PS}$ at $\beta = 6.0$ for the two definitions of m_c ($m_c^{\rm pion}$ and $m_c^{\rm PCAC}$).

3. Chiral behaviour at fixed a

The effectiveness of $m_c^{\rm PCAC}$ in reducing the large lattice artifacts observed at small quark mass is evident when considering the chiral behaviour of two simple observables: $f_{\rm PS}$ and $m_{\rm PS}$. $f_{\rm PS}$ can be extracted, without need of renormalization constants, by using the exact lattice PCVC relation $\langle \partial^*_\mu \tilde{V}^a_\mu(x) O(0)\rangle = -2\mu\varepsilon^{3ab}\langle P^b(x)O(0)\rangle$ (where ∂^*_μ is the lattice backward derivative and $\tilde{V}^a_\mu$ the point-splitted vector current). It follows that, at maximal twist,

$$f_{\rm PS} = \frac{2\mu}{m^2_{\rm PS}} |\langle 0|P^a|PS\rangle| \; . \tag{3.1}$$

In Fig. 1.b one can compare the chiral behaviour of $f_{\rm PS}$ obtained with either $m_c^{\rm pion}$ or $m_c^{\rm PCAC}$. One sees immediately that $m_c^{\rm PCAC}$ reduces the large lattice artifacts present at small μ when using $m_c^{\rm pion}$ ($f_{\rm PS}$ is predicted to be linear in $m^2_{\rm PS}$ at one loop in quenched χPT).

Using the integrated PCVC relation, it is also possible to prove [6] that, by using $m_c^{\rm PCAC}$ and in the region $\mu > a^2$, m^2_{PS} is linear with μ up to small a^4 cut-off effects [3] (i.e. O(a^2) lattice artifacts are proportional to μ). This is qualitatively confirmed by our data as shown in Fig. 2.a.

4. Scaling behaviour

We present now results concerning the scaling behaviour of $f_{\rm PS}r_0$, $m_V r_0$, $m_{\rm oct}r_0$ and $m_{\rm dec}r_0$ at fixed value of $m_{\rm PS}r_0$ (where r_0 is the Sommer scale). More details concerning meson quantities can be found in Ref. [11]. m_V has been extracted by using either the spatial component of the axial vector or the temporal component of the tensor as interpolating operators (in the following we will quote only the latter, which systematically present lower statistical errors). $m_{\rm oct}$ and $m_{\rm dec}$ have been extracted by using respectively $\varepsilon^{ABC}((d^A)^T C\gamma_5 u^B)u^C_\alpha$ and $\varepsilon^{ABC}((u^A)^T C\gamma_k u^B)u^C_\alpha$ as interpolating operators. The parameters of the simulations can be found in Tab. 1. We have simulated quark masses μ corresponding to 235MeV $\le m_{PS} \le$ 1.0GeV (where the scale r_0, as will be explained below, has been fixed through the ρ mass). The scaling behaviour of $f_{\rm PS}r_0$, as shown in Fig. 2.b, is clearly linear in $(a/r_0)^2$. However, $m_c^{\rm pion}$ gives large O(a^2) effects at small masses, effects that are

[3] chiral logs and other O($m^2_{\rm PS}$) contributions are here assumed to be negligible

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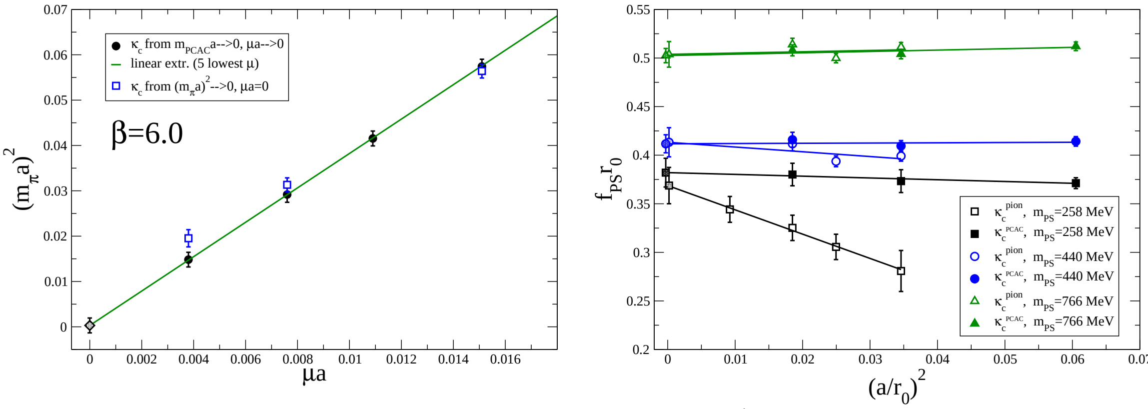

Figure 2: a. $m_{\rm PS}$ as function of μ at $\beta = 6.0$ for both definitions $m_c^{\rm pion}$ and $m_c^{\rm PCAC}$. b. Scaling behaviour of $f_{\rm PS}r_0$ for 3 fixed values of $m_{\rm PS}r_0$. For each definition of m_c an indipendent fit has been performed.

drastically reduced by the use of $m_c^{\rm PCAC}$. This obviously influences the scaling region which starts at $\beta = 6.0$ for $m_c^{\rm pion}$ and at $\beta = 5.85$ for $m_c^{\rm PCAC}$. We perform indipendent continuum extrapolations for the two choices of m_c and the results are in good agreement. In the case of $m_c^{\rm pion}$, due to the highest slope for the lowest quark mass, we needed an additional point at $\beta = 6.45$ in order to control the extrapolation. $m_V r_0$, $m_{\rm oct} r_0$ and $m_{\rm dec} r_0$ have been thus computed by using only $m_c^{\rm PCAC}$.

Since we have simulated down to $m_{\rm PS}$ of 235 MeV, finite size effects (FSE) can be quite relevant. In order to check for FSE we performed two additional simulations at $\beta = 5.85$ on volumes of $12^3 \times 32$ and $14^3 \times 32$ in order to extend the results of Ref. [13] at smaller masses. For meson quantities ($m_{\rm PS}$, $f_{\rm PS}$ and m_V) FSE are negligible for all the quark masses starting from the third smallest one; on the two smallest masses they are in practice below the statistical accuracy of our data. For baryon masses, instead, FSE are very large for the two smallest masses and still relevant on the next three. Since the sensitivity required to study FSE on the smallest two masses is computationally very expensive, we chose here to correct only the data from the third value of μ on (corresponding to $m_{\rm PS} = 375$ MeV). The results for the scaling behaviour of $m_{\rm oct} r_0$ are shown in Fig. 3.a from which the lattice artifacts appear to be O(a^2) down to $\beta = 5.85$ and of relatively small size.

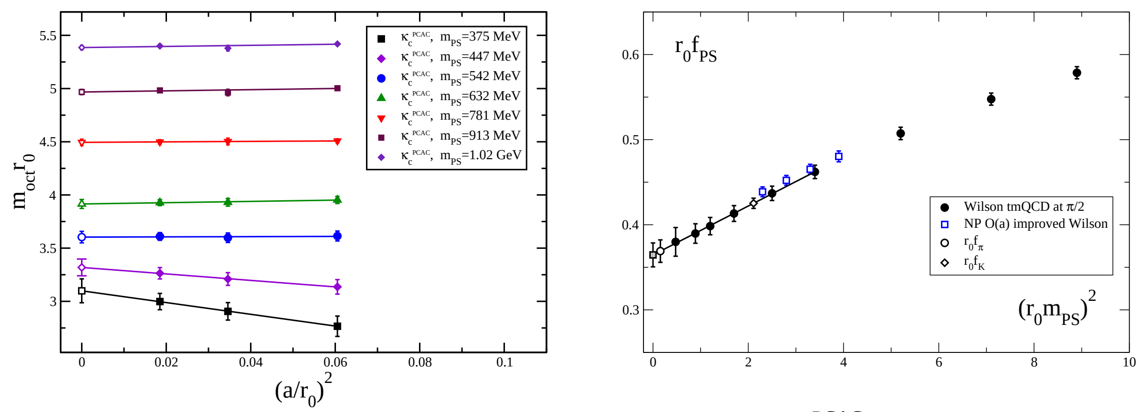

Figure 3: a. Scaling behaviour of $m_{\rm oct} r_0$ for 7 fixed values of $m_{\rm PS} r_0$ (only $m_c^{\rm PCAC}$ used). b. Continuum limit of $f_{\rm PS} r_0$ as function of $(m_{\rm PS} r_0)^2$ (only $m_c^{\rm PCAC}$ used). The empty squares are results taken from [12]

β	5.70	5.85	6.00	6.10	6.20	6.45
a (fm)	0.171	0.123	0.093	0.079	0.068	0.048
L/a	12	16	16	20	24	32
T/a	32	32	32	40	48	64
$N_{\rm conf}$ ($m_c^{\rm pion}$)	600	378	387	300	260	182
$N_{\rm conf}$ ($m_c^{\rm PCAC}$)	600	500	400		300	

Table 1: Parameters of the simulations

5. Continuum limit

Results for the continuum limit of $f_{\rm PS}r_0$, $m_V r_0$, $m_{\rm oct}r_0$ and $m_{\rm dec}r_0$ are presented in Fig. 3.b and 4.a. Our determinations of $f_{\rm PS}$ and m_V are in good agreement with those from the ALPHA Coll. [12] with non-perturbatively O(a) improved Wilson fermions (the comparison for $f_{\rm PS}$ is shown in Fig. 3.b). For the chiral extrapolation we used the form f_{PS}, $m_V \sim A + Bm_{PS}^2$ and $m_{\rm oct}$, $m_{\rm dec} \sim A + Bm_{PS} + Cm_{PS}^2$. In order to compare with the results of the CP-PACS Coll. [14], we fixed the scale r_0 through the ρ mass m_ρ, obtaining $r_0 = 0.576\,fm$ [4]. As a prediction we get f_π, m_N, m_Δ and (working in the $SU(3)$ symmetric limit) f_K and m_{K^*}. The results can be found in Fig. 4.b and Tab. 2 together with those of Ref. [14] and turn out to be in very good agreement. Notice however that in the present work quantities are O(a) improved and pseudoscalar masses significantly smaller than those in Ref. [14] (where standard Wilson fermions were used) have been simulated.

6. Conclusions and Outlook

In the present study we have extrapolated to the continuum (taking into account possible FSE) meson quantities (f_{PS} and m_V) and baryon masses ($m_{\rm oct}$ and $m_{\rm dec}$) down to pseudoscalar messon masses of 235 MeV and 375 MeV respectively. We have presented a strong evidence that lattice artifacts are O(a^2) for both definitions of m_c ($m_c^{\rm pion}$ and $m_c^{\rm PCAC}$) and moreover that the use of $m_c^{\rm PCAC}$ drastically reduces the chirally enhanced O(a^2) lattice artifacts present at small quark masses when

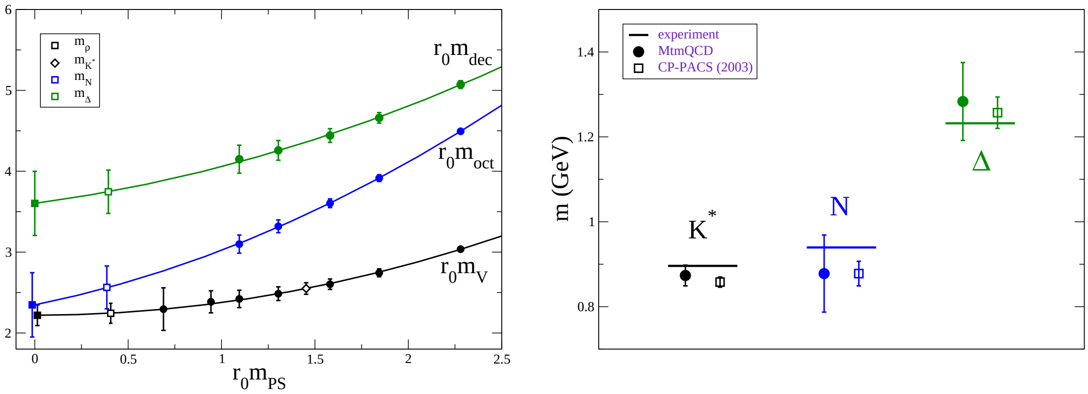

Figure 4: a. Continuum limit of $m_V r_0$, $m_{\rm oct}r_0$, $m_{\rm dec}r_0$ as function of $m_{\rm PS}r_0$ (only $m_c^{\rm PCAC}$ used). b. Comparison of our results for m_{K^*}, m_N, m_Δ with those of Ref. [14].

[4] had we fixed the scale through f_K we would have obtained $r_0 = 0.508\,fm$

	f_π (MeV)	f_K (MeV)	f_K/f_π
exp.	132	160	1.22
tmQCD	126(5)	146(3)	1.15(5)
CP-PACS	120(6)	139(5)	1.16(3)

Table 2: Pseudoscalar meson decay constants from the present work and from Ref. [14] (in Ref. [14], tadpole-improved perturbation theory has been used to renormalize the axial current).

using $m_c^{\rm pion}$. Our results for the continuum extrapolated quantities are in good agreement with those from the ALPHA [12] and CP-PACS [14] Collaborations. This is very encouraging in view of future dynamical simulations [3]. There are however other aspects of tmQCD that it is worth to investigate, for instance the problem of isospin breaking (see Ref. [15] for an exploratory study in the quenched approximation). This problem is practically very important for phenomenological applications of tmQCD and also strictly related to the phase structure of tmQCD in the neighborhood of the critical point [5, 16] and thus directly relevant for dynamical symulations.

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