PROCEEDINGS OF SCIENCE

Large N

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Some mysterious features of the strong interactions become easily understood if our usual QCD with $N = 3$ is 'close to' SU(∞) and if the latter theory is confining. $N = \infty$ theories are theoretically simpler; in particular there has been much progress in constructing weak-coupling duals in string theory. In this poster I will describe some of the things that recent lattice calculations tell us about the large-N limit of SU(N) gauge theories in 3+1 dimensions. The focus is on confinement, how close SU(∞) is to SU(3), new stable strings at larger N, the Pomeron, deconfinement, topology, 't Hooft string tensions. I also allude to other topics, such as the high-T pressure deficit, chiral physics and the phases of the theory.

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1. Some lattice results

These calculations mostly proceed by looking at SU(2), SU(3), SU(4), SU(5), ... and seeing if one can extrapolate to $N = \infty$ using the expected $O(1/N^2)$ correction.

1.1 $N = 3$ is close to $N = \infty$

The fact that for many quantities SU(3) $\simeq$ SU(∞) is demonstrated, for example, by calculations of the lightest glueball masses [1], as in Fig. 1.

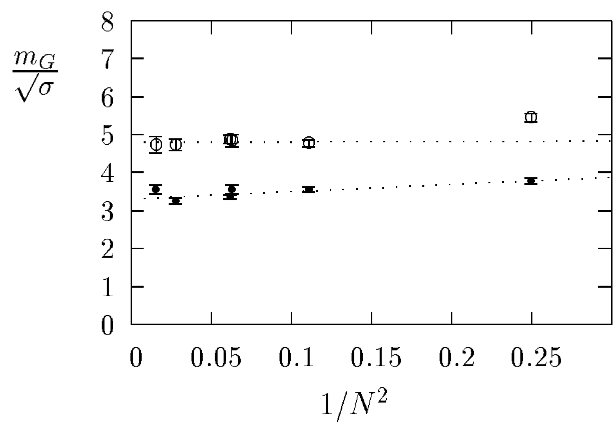

Figure 1: The lightest 0^{++}, $\bullet$, and 2^{++}, $\circ$, glueball masses expressed in units of the string tension, in the continuum limit, plotted against $1/N^2$. Dotted lines are extrapolations to $N = \infty$.

1.2 Linear confinement in SU(6) and Lüscher correction

Linear confinement at large N is demonstrated [2] for SU(6) in Fig. 2. In Fig. 1 we see that the string tension remains finite in the $N = \infty$ limit in units of the mass gap. So the $N = \infty$ theory is indeed linearly confining.

A local fit to the leading string correction, as in Fig. 3, provides good evidence that the long-distance behaviour is that of a simple bosonic string [2].

1.3 ‘t Hooft coupling, $\lambda \equiv g^2 N$, fixed for $N \to \infty$

We also see [3] in Fig. 4 that for a smooth large-N limit we need to keep $\lambda(a) = g^2(a)N$ fixed (at fixed $a\sqrt{\sigma}$) as expected from diagrams.

1.4 Pomeron is leading glueball Regge trajectory

Using novel techniques to calculate masses of high spin glueballs, one can obtain some solid evidence, as in Fig. 5, for the fact that the Pomeron is the leading glueball Regge trajectory [4]. This is for SU(3), and is a step towards $N = \infty$ where mixing and decay ambiguities disappear.

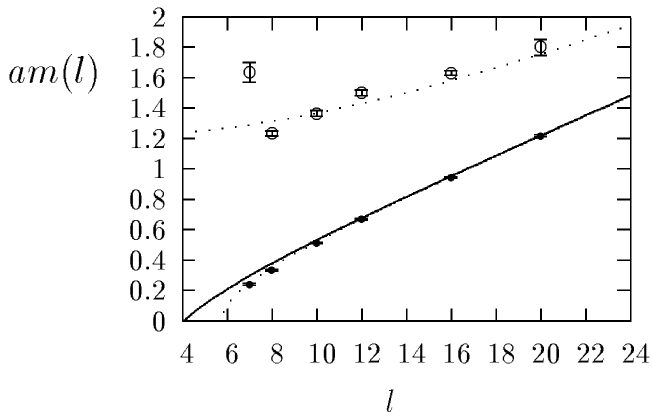

Figure 2: The masses of the lightest, •, and first excited, ∘, $k = 1$ flux loops that wind around a spatial torus of length l in the SU(6) calculation at $\beta = 25.05$. The dotted lines are the predictions of the Nambu-Goto string action.

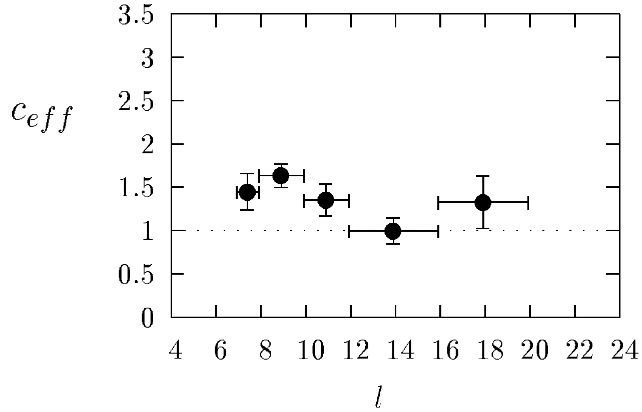

Figure 3: $am(l) = a^2\sigma l - c_{eff}\frac{\pi}{3}\frac{1}{l}$

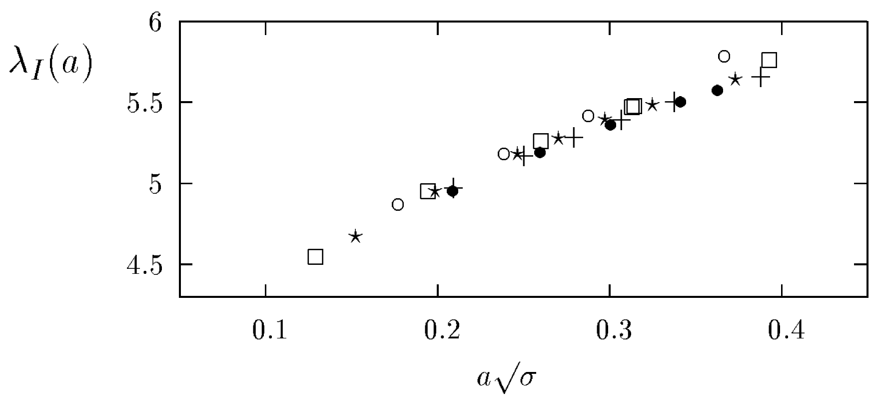

Figure 4: The value of the 't Hooft coupling on the scale a, as obtained from mean-field improved β, for $N = 2(\circ), 3(\square), 4(\star), 6(+), 8(\bullet)$, plotted against the values of a expressed in physical units.

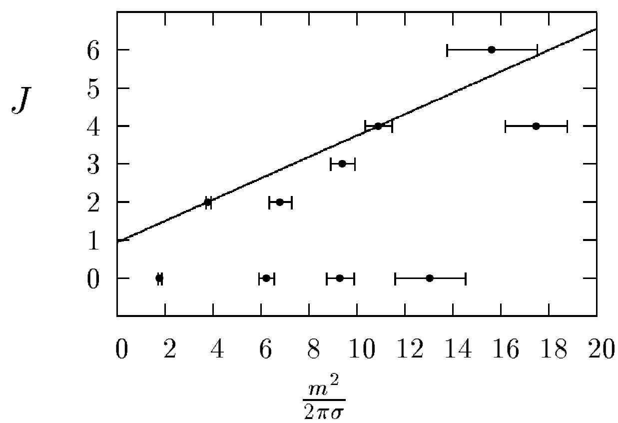

Figure 5: Chew-Frautschi plot of $PC = ++$ states in the continuum SU(3) gauge theory. The leading Regge trajectory is shown.

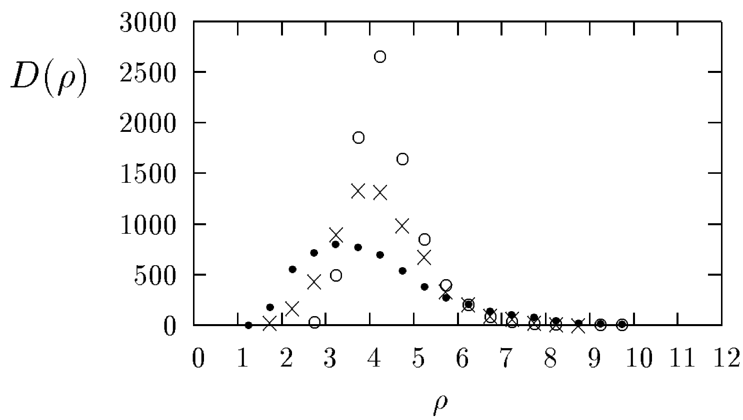

Figure 6: The 'instanton' size density, $D(\rho)$, for $N = 3(\bullet), 6(\times), 12(\circ)$ at $a \simeq 1/4.5T_c$.

1.5 Topology

The instanton size distribution seems to head to $D(\rho) \stackrel{N\to\infty}{\longrightarrow} \delta(\rho - \rho_c)$ [5, 6] as in Fig. 6.
Also:

- topological susceptibility at $N \to \infty$: [7, 8]
- no topological fluctuations at $T > T_c$ at large N: [6, 9]
- evidence for interlacing θ-vacua: [7]
- evidence that topology drives chiral symmetry breaking: [10]

1.6 Deconfinement

T_c rapidly converges to its large-N limit, as in Fig. 7, becoming more strongly first order as we see from the latent heat plot in [3, 6, 11].

1.7 k-strings

New stable confining strings appear at larger N [1, 14, 15, 12, 13] and their ratios, as listed in Table 1 from [1], can be compared to the Casimir Scaling and MQCD-inspired conjectures.

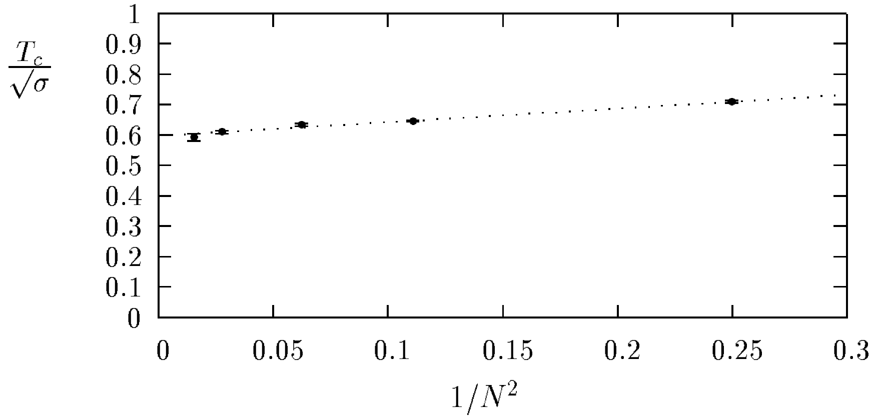

Figure 7: The deconfining temperature in units of the string tension for various SU(N) gauge theories. Large N extrapolation shown.

σ_k/σ			
(N,k)	Casimir scaling	this paper	'MQCD'
(4,2)	1.333	1.370(20)	1.414
(4,2)	1.333	1.358(33)	1.414
(6,2)	1.600	1.675(31)	1.732
(6,3)	1.800	1.886(61)	2.000
(8,2)	1.714	1.779(51)	1.848
(8,3)	2.143	2.38(10)	2.414
(8,4)	2.286	2.69(17)	2.613

Table 1: Predictions of 'Casimir Scaling' and 'MQCD' compared against calculated values of the ratio of the tension of the lightest k-string to that of the fundamental ($k = 1$) string. The second SU(4) calculation is on anisotropic lattices.

1.8 .... and more ....

Pressure deficit above T_c at large N [16]. Hunting the Hagedorn phase transition [17]. Large-N phases [18, 19]. 't Hooft string tensions [20, 21]. D=2+1 deconfinement at all N [22]. Space-time reduction at large N [19]. Chiral symmetry and quark masses at large N [23] ... Mesons and baryons at large N ... $\mathcal{N} = 1$ SUSY at $N = \infty$ ....

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