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Dynamical supersymmetry breaking and phase diagram of the lattice Wess-Zumino model

Matteo Beccaria and Gian Fabrizio De Angelis
INFN, Sezione di Lecce, and Dipartimento di Fisica dell'Università di Lecce, Via Arnesano, ex Collegio Fiorini, I-73100 Lecce, Italy
E-mail: `beccaria@le.infn.it`, `deangelis@le.infn.it`

Massimo Campostrini*
INFN, Sezione di Pisa, and Dipartimento di Fisica "Enrico Fermi" dell'Università di Pisa, Via Buonarroti 2, I-56125 Pisa, Italy
E-mail: `campostrini@df.unipi.it`

Alessandra Feo†‡
Dipartimento di Fisica, Università di Parma and INFN Gruppo Collegato di Parma, Parco Area delle Scienze 7/A, 43100 Parma, Italy
E-mail: `feo@fis.unipr.it`

We study dynamical supersymmetry breaking and the transition point by non-perturbative lattice techniques in a class of two-dimensional $N = 1$ Wess-Zumino model. The method is based on the calculation of rigorous lower bounds on the ground state energy density in the infinite-lattice limit. Such bounds are useful in the discussion of supersymmetry phase transition. The transition point is determined with this method and then compared with recent results based on large-scale Green Function Monte Carlo simulations with good agreement.

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†Speaker.
‡UPRF-2005-04

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1. Introduction

An important issue in the study of supersymmetric models is the occurrence of non-perturbative dynamical supersymmetry breaking [1]. The problem can be studied in the $N=1$ Wess-Zumino model that does not involve gauge fields and is thus a simple theoretical laboratory. Since the breaking of supersymmetry is closely related to the symmetry properties of the ground state, we will adopt a Hamiltonian formulation of the model.

Let us remind the (continuum) $N=1$ super algebra, $\{Q_\alpha, Q_\beta\} = 2(\not{P}C)_{\alpha\beta}$. Since P_i are not conserved on the lattice, the super algebra is explicitly broken by the lattice discretization. A very important advantage of the Hamiltonian formulation is the possibility of conserving exactly a subset of the full super algebra [2]. Specializing to $1+1$ dimensions, in a Majorana basis $\gamma_0 = C = \sigma_2$, $\gamma_1 = i\sigma_3$, the algebra becomes: $Q_1^2 = Q_2^2 = P^0 \equiv H$ and $\{Q_1, Q_2\} = 2P^1 \equiv 2P$. On the lattice, since H is conserved but P is not, we can pick up one of the supercharges, say, Q_1, build a discretized version Q_L and define the lattice Hamiltonian to be $H = Q_L^2$. Notice that $Q_1^2 = H$ is enough to guarantee $E_0 \geq 0$. The explicit lattice model is built by considering a spatial lattice with L sites. On each site we place a real scalar field φ_n together with its conjugate momentum p_n such that $[p_n, \varphi_m] = -i\delta_{n,m}$. The associated fermion is a Majorana fermion $\psi_{a,n}$ with $a = 1,2$ and $\{\psi_{a,n}, \psi_{b,m}\} = \delta_{a,b}\delta_{n,m}$, $\psi_{a,n}^\dagger = \psi_{a,n}$.

The continuum 2-dimensional Wess-Zumino model is defined by the supersymmetric generators involving the superpotential $V(\varphi)$,

$$Q_{1,2} = \int dx \left[p(x)\psi_{1,2}(x) - \left(\frac{\partial\varphi}{\partial x} \pm V(\varphi(x)) \right) \psi_{2,1}(x) \right], \tag{1.1}$$

where $\varphi(x)$ is a real scalar field and $\psi(x)$ is a Majorana fermion. The discretized supercharge is [2, 3]

$$Q_L = \sum_{n=1}^{L} \left[p_n \psi_{1,n} - \left(\frac{\varphi_{n+1} - \varphi_{n-1}}{2} + V(\varphi_n) \right) \psi_{2,n} \right] \tag{1.2}$$

and the Hamiltonian takes the form

$$H = Q_L^2 = \frac{1}{2}\sum_{n=1}^{L} \left[\pi_n^2 + \left(\frac{\phi_{n+1} - \phi_{n-1}}{2} + V(\phi_n) \right)^2 - (\chi_n^\dagger \chi_{n+1} + h.c.) + (-1)^n V'(\phi_n) \left(2\chi_n^\dagger \chi_n - 1 \right) \right] \tag{1.3}$$

where we replace the two Majorana fermion operators with a single Dirac operator χ satisfying canonical anticommutation rules.

The problem of predicting the pattern of supersymmetry breaking is not easy. In principle, the form of $V(\varphi(x))$ is enough to determine whether supersymmetry is broken or not. At least at tree level supersymmetry is broken if and only if V has no zeros. The Witten index [4] can help in the analysis: If $V(\varphi)$ has an odd number of zeroes then $I \neq 0$ and supersymmetry is unbroken. If $V(\varphi)$ has an even number of zeroes, when $I = 0$ we can not conclude anything. An alternative non-perturbative analysis for the case $I = 0$ is thus welcome. The simplest way to analyze the pattern of supersymmetry breaking for a given V is to compute the ground state energy E_0 through

[1] See [1] for recent reviews and a complete list of references.

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numerical simulations and/or strong coupling expansion. On the lattice, accurate numerical results are available [5, 6], although a clean determination of the supersymmetry breaking transition remains rather elusive. All the predictions for the model with cubic prepotential, $V = \varphi^3$, indicated unbroken supersymmetry. Dynamical supersymmetry breaking in the model with quadratic prepotential $V = \lambda_2\varphi^2 + \lambda_0$ was studied performing numerical simulations [5] along a line of constant λ_2, confirming the existence of two phases: a phase of broken supersymmetry with unbroken discrete Z_2 at high λ_0 and a phase of unbroken supersymmetry with broken Z_2 at low λ_0, separated by a single phase transition.

On the other hand, from the strong coupling analysis what comes out is the following: for odd q, strong coupling and weak coupling expansion results agree and supersymmetry is expected to be unbroken [5]. This conclusion gains further support from the non vanishing value of the Witten index [4]. For even q in strong coupling, the ground state has a positive energy density also for $L \to \infty$ and supersymmetry appears to be broken. In particular, for $V = \lambda_2\varphi^2 + \lambda_0$, weak coupling predicts unbroken supersymmetry for $\lambda_0 < 0$, whereas strong coupling prediction gives broken supersymmetry for all λ_0.

2. Numerical Simulations and Discussion

We used two different approaches to investigate the pattern of dynamical supersymmetry breaking. In the first one, [5, 6], the numerical simulations were performed using the Green Function Monte Carlo (GFMC) algorithm and strong coupling expansion. The GFMC is a method that computes a numerical representation of the ground state wave function on a finite lattice with L sites in terms of the states carried by an ensemble of K walkers. Numerical results using the GFMC algorithm for the odd prepotential confirm unbroken supersymmetry.

A more interesting case is the even prepotential. When $V = \lambda_2\varphi^2 + \lambda_0$ and for fixed $\lambda_2 = 0.5$, we may expect (in the $L \to \infty$ limit) a phase transition at $\lambda_0 = \lambda_0^{(c)}(\lambda_2)$ separating a phase of broken supersymmetry and unbroken Z_2 (high λ_0) from a phase of unbroken supersymmetry and broken Z_2 (low λ_0).

The usual technique for the study of a phase transition is the crossing method applied to the Binder cumulant, B. The crossing method consists in plotting B vs. λ_0 for several values of L. The crossing point $\lambda_0^{\rm cr}(L_1, L_2)$, determined by the condition $B(\lambda_0^{\rm cr}, L_1) = B(\lambda_0^{\rm cr}, L_2)$ is an estimator of $\lambda_0^{(c)}$. The value obtained is showed in Fig. 1 and corresponds to $\lambda_0^{(c)} = -0.48 \pm 0.01$ [5]. The main source of systematic errors in this method is the need to extrapolate to infinity both K and L. For this reason, an independent method to test the numerical results of [5] is welcome.

The second method is based on the calculation of rigorous lower bounds on the ground state energy density in the *infinite-lattice* limit [7, 8]. Such bounds are useful in the discussion of supersymmetry breaking as follows: The lattice version of the Wess-Zumino model conserves enough supersymmetry to prove that the ground state has a non negative energy density $\rho \geq 0$, as its continuum limit. Moreover the ground state is supersymmetric if and only if $\rho = 0$, whereas it breaks (dynamically) supersymmetry if $\rho > 0$. Therefore, if an exact positive lower bound $\rho_{\rm LB}$ is found with $0 < \rho_{\rm LB} \leq \rho$, we can claim that supersymmetry is broken.

The idea is to construct a sequence $\rho^{(L)}$ of exact lower bounds representing the ground state energy densities of modified lattice Hamiltonians describing a cluster of L sites and converging to

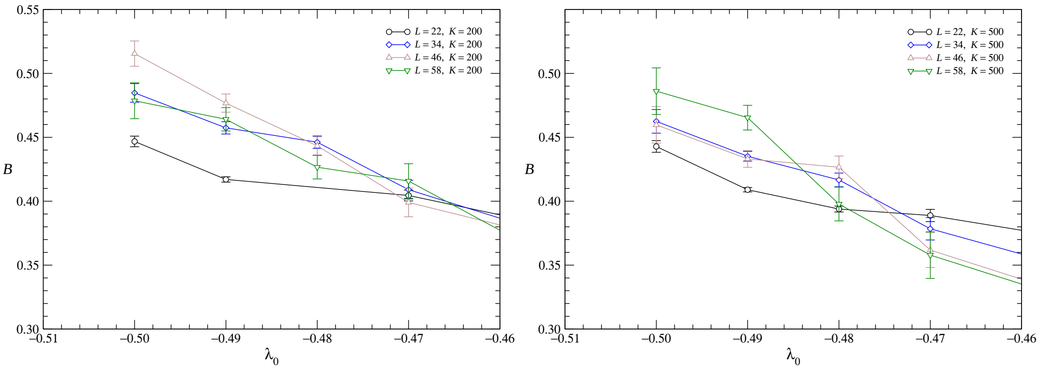

Figure 1: The Binder cumulant B vs. λ_0 for $K = 200$ and $K = 500$. Here $\lambda_0^{(c)} = -0.48 \pm 0.01$.

$\rho^{(L)} \to \rho$ in the limit $L \to \infty$. The bounds $\rho^{(L)}$ can be computed numerically on a finite lattice with L sites. The relevant quantity for our analysis is the ground state energy density ρ evaluated on the infinite lattice limit $\rho = \lim_{L\to\infty} \frac{E_0(L)}{L}$. It can be used to tell between the two phases of the model: supersymmetric with $\rho = 0$ or broken with $\rho > 0$.

In Ref. [7] we presented how to build a sequence of bounds $\rho^{(L)}$ which are the ground state energy density of the Hamiltonian H with modified couplings on a cluster of L sites: given a translation-invariant Hamiltonian H on a regular lattice it is possible to obtain a lower bounds on its ground state energy density from a cluster decomposition of H, i.e., given a suitable finite sublattice Λ, it is possible to introduce a modified Hamiltonian $\widetilde{H}$ restricted to Λ such that its energy density ρ_Λ bounds ρ from below. The difference between H and $\widetilde{H}$ amounts to a simple rescaling of its coupling constants. The only restriction on H being that its interactions must have a finite range [7].

We compute numerically $\rho^{(L)}$ at various values of the cluster L: if we find $\rho^{(L)} > 0$ for some L we conclude that we are in the broken phase. We know that $\rho^{(L)} \to \rho$ for $L \to \infty$ and the study of $\rho^{(L)}$ as a function of *both* L and the coupling constants permit the identification of the phase in all cases. The calculation of $\rho^{(L)}$ is numerically feasible because it requires to determine the ground state energy of a Hamiltonian quite similar to H and defined on a finite lattice with L sites.

To test the effectiveness of the proposed bound and its relevance to the problem of locating the supersymmetry transition in the Wess-Zumino model we study in detail the case of a quadratic prepotential $V = \lambda_2 \varphi^2 + \lambda_0$ at a fixed value $\lambda_2 = 0.5$ [7]. An argument by Witten [4] suggest the existence of a negative number λ_0^* such that $\rho(\lambda_0)$ is positive when $\lambda_0 > \lambda_0^*$ and it vanishes for $\lambda_0 < \lambda_0^*$. λ_0^* is the value of λ_0 in which dynamical supersymmetry breaking occurs. In Fig. 2 we show a qualitative pattern of the curves representing $\rho^{(L)}(\lambda_0)$. We see that a single zero is expected in $\rho^{(L)}(\lambda_0)$ at some $\lambda_0 = \lambda_0(L)$. Since $\lim_{L\to\infty} \rho^{(L)} = \rho$, we expect that $\lambda_0(L) \to \lambda_0^*$ for $L \to \infty$ allowing for a determination of the critical coupling λ_0^*. The continuum limit of the model is obtained by following a Renormalization Group trajectory that, in particular, requires the limit $\lambda_2 \to 0$ [5].

The properties of the bound $\rho^{(L)}(\lambda_0)$ guarantee that for L large enough it must have a single zero $\lambda_0^*(L)$ converging to λ_0 as $L \to \infty$. In any case for each L we can claim that $\lambda_0^* > \lambda_0^*(L)$. To obtain the numerical estimate of $\rho^{(L)}(\lambda_0)$ we used the world line path integral (WLPI) algorithm.

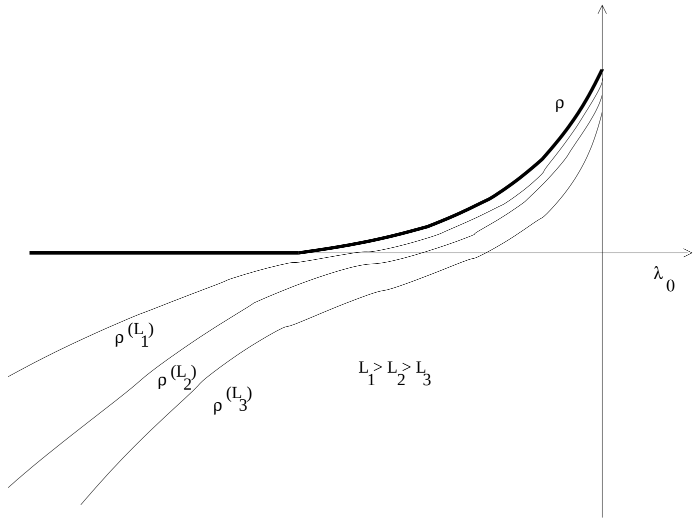

Figure 2: Qualitative plot of the functions $\rho(\lambda_0)$ and $\rho^{(L)}(\lambda_0)$.

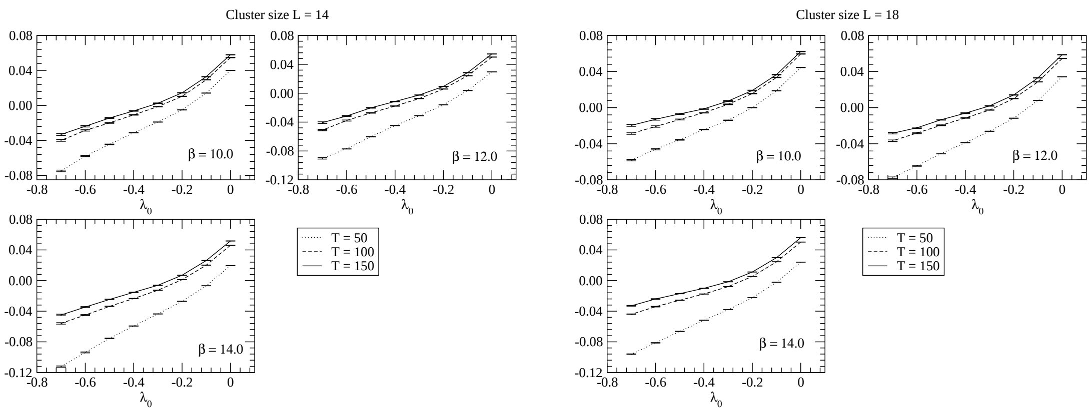

Figure 3: Plot of the energy lower bound $\rho^{(L)}(\beta,T)$ at $L = 14$ and $L = 18$.

The WLPI algorithm computes numerically the quantity $\rho^{(L)}(\beta,T) = \frac{1}{L}\frac{\mathrm{Tr}\{H\ (e^{-\frac{\beta}{T}H_1}e^{-\frac{\beta}{T}H_2})^T\}}{\mathrm{Tr}\{(e^{-\frac{\beta}{T}H_1}e^{-\frac{\beta}{T}H_2})^T\}}$ where the Hamiltonian for a cluster of L sites is written as $H = H_1 + H_2$, by separating in a convenient way the various bosonic and fermionic operators in the subhamiltonians H_1 and H_2 [2]. The desired lower bound is obtained by the double extrapolation $\rho^{(L)} = \lim_{\beta\to\infty}\lim_{T\to\infty}\rho^{(L)}(\beta,T)$, with polynomial convergence $\sim 1/T$ in T and exponential in β. Numerically, we determined $\rho^{(L)}(\beta,T)$ for various values of β and T and a set of λ_0 that should include the transition point, at least according to the GFMC results. In Fig. 3 we plot the function $\rho^{(L)}(\beta,T)$ for the cluster sizes $L = 14$ and $L = 18$, various β and $T = 50$, 100, 150. Here we see that the energy lower bound behaves as expected: it is positive around $\lambda_0 = 0$ and decreases as λ_0 moves to the left. At a certain unique point $\lambda_0^*(L)$, the bound vanishes and remains negative for $\lambda_0 < \lambda_0^*(L)$. This means that supersymmetry breaking can

[2] we do not report H_1 and H_2 here, see Ref. [7] for details.

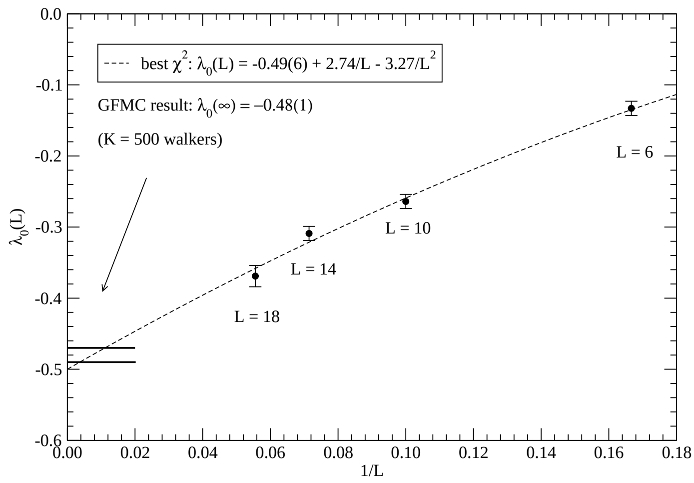

Figure 4: Plot of $\lambda_0(L)$ vs. $1/L$ for $L = 6, 10, 14, 18$. The best fit with a quadratic polynomial in $1/L$ gives $\lambda_0^* = -0.49 \pm 0.06$ that should compared with the best GFMC result obtained with $K = 500$ walkers.

be excluded for $\lambda_0 > \min_L \lambda_0^*(L)$. Also, consistency of the bound means that $\lambda_0^*(L)$ must converge to the infinite-volume critical point as $L \to \infty$. Since the difference between the exact Hamiltonian and the one used to derive the bound is $\mathscr{O}(1/L)$, we can fit $\lambda_0^*(L)$ with a polynomial in $1/L$. This is shown in Fig. 4 where we also show the GFMC result. The best fit with a parabolic function gives $\lambda_0^* = -0.49 \pm 0.06$ [7] quite in agreement with the previous $\lambda_{0,\text{GFMC}}^* = -0.48 \pm 0.01$ [5]. In conclusion, both methods reported here are quite in agreement and confirm the existence of two phases separated by a single phase transition at λ_0 for the quadratic prepotential.

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