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Evidence for the BFKL Pomeron from String/Gauge Duality

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The high energy limit for hadronic scattering in QCD has two traditional forms: the so called soft Pomeron Regge trajectory for the tensor glueball state and the hard BFKL Pomeron for gluon exchange computed in leading order at weak coupling. This talk will show how AdS/CFT duality provides a natural synthesis of both approaches and will present the BFKL intercept in leading order at strong coupling.

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1. Introduction

One of the most striking aspects of high-energy hadronic scattering is the rise in the total cross section, $\sigma_T(s)$, at the highest available energies to date. In Regge language, this requires a leading J-plane singularity, referred to as the Pomeron, with vacuum quantum numbers and an intercept above $j=1$, for the forward elastic amplitude $\mathscr{A}(s,t)$. In the large N expansion for QCD, the Pomeron is the first term or cylinder exchange diagram in the t-channel. At present energies, there appear to be relatively small unitarity corrections, although they must ultimately enter to satisfy the Froissart bound, $\sigma_T = O(m_\pi^{-2}\log^2(s))$. However there are currently two seemingly conflicting theoretical interpretations of high energy amplitudes in QCD: One is the so-called Balitsky-Fadin-Kuraev-Lipatov (BFKL) or "Hard" Pomeron, based on leading $g^2N\log(s)$ perturbative approximation and the other is the "Soft" Pomeron or the traditional Regge pole in the J-plane at $j=\alpha_P(t)$ that interpolates the glueball resonances for even integer values of j.

The central focus of this talk is present a natural synthesis of both approaches based on AdS/CFT duality. The resulting J-plane structure illustrated in Fig. 1 has both the soft Pomeron Regge pole and a hard Pomeron cut located at $\alpha_{BFKL}(0) = 2-2/\sqrt{g^2N}$ in leading order in strong coupling. The reader is referred to a forth coming article by Brower, Polchinski, Strassler and Tan [1] for a much more rigorous and complete discussion based on the operator products expansion in the conformal gauge. We begin in the next section by briefly reviewing key features of

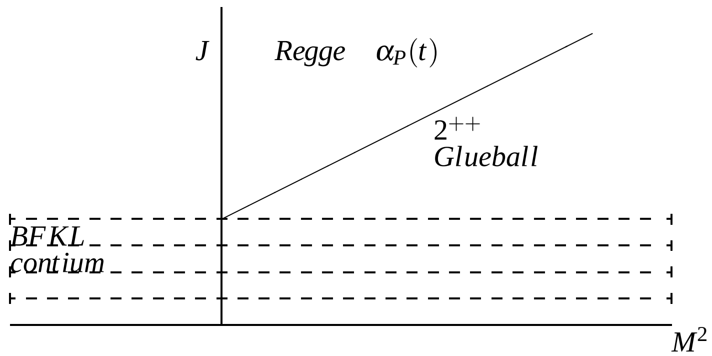

Figure 1: Soft Pomeron Regge pole at $j=\alpha_P(t)$ terminates at the BFKL cut with intercept at $j=\alpha_{BFKL}(0)$

the perturbative (BFKL) Pomeron and the non-perturbative (Regge pole) Pomeron, emphasizing both their differences and similarities. We next show how these key features can be unified in a curved-space string theory in a light-cone description.

2. Diffusion in Impact Space and Virtuality

Due to a linear confining potential, the QCD spectrum is expected to contains states with arbitrarily high spin and masses, lying on nearly parallel Regge trajectories, in the large N limit. Also the classic Regge behavior, $\mathscr{A} \sim s^{\alpha(t)}$ for a linear trajectory $\alpha(t) = \alpha_0 + \alpha' t$ satisfies a diffusion equation in impact parameter space,

$$[\,\partial_y - \alpha_0 - \alpha'\partial^2_{x^\perp}\,]\,K(y;x^\perp,x'^\perp) = \delta^2(x^\perp - x'^\perp)\;\delta(y)\,, \tag{2.1}$$

as can be seen by taking the Fourier transform: $\int d^2k_\perp e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp} s^{\alpha_0-\alpha' k_\perp^2} \sim s^{\alpha_0} e^{-x_\perp^2/4\alpha'\ln s}/(\alpha'\ln s)$. On the other hand, the leading log summation of perturbative diagrams by Balitsky and Lipatov and by Fadin and Kuraev for scattering at small fixed angles and high energies ($s \gg -t \gg \Lambda_{QCD}$), exhibits an entirely different diffusion process. In the forward direction the BFKL elastic amplitude [2],

$$A(s,0) \simeq \int \frac{dk_\perp}{k_\perp} \int \frac{dk'_\perp}{k'_\perp} \Phi_{12}(k_\perp) K(s;k_\perp,k'_\perp) \Phi_{34}(k'_\perp), \tag{2.2}$$

is given in terms of the hadronic impact factors Φ_{ij} and a t-channel 2-gluon interaction kernel K. The variables $k_\perp = \sqrt{k^2}$ are a measure of the off-shell dependence of the gluons referred as "virtuality". The exact kernel is known and well approximated by

$$K(s,k_\perp,k'_\perp) = \frac{s^{\omega_0}}{\sqrt{\pi \ln s}} e^{-\left[(\ln k'_\perp - \ln k_\perp)^2/4\mathscr{D}\ln s\right]}, \tag{2.3}$$

where $\omega_0 = \lambda \ln 2/\pi^2$, and $\mathscr{D} = 14\zeta(3)\lambda/\pi$ with the 'tHooft coupling $\lambda = g^2 N$. We recognize K as a the kernel for 1-d diffusion occurring in $\ln k_\perp$ over a time $\ln s$. The diagrams contributing to the BFKL equation (2.3) is consistent with the large N limit and conformal invariance as exhibited by its dependence on the 'tHooft coupling and the scale invariance of the kernel ($k_\perp \to \eta k_\perp$) respectively.

Immediately one may ask if there isn't a single equation combining diffusion in both impact space and conformal "virtuality". To examine this we turn to an AdS^5 model for the QCD string,

$$ds^2 = \frac{r^2}{R_{ads}^2}(d\vec{x}^2 - dx^{0\,2}) + \frac{R_{ads}^2}{r^2} dr^2 \quad , \quad r \in [r_{min}, \infty]\,, \tag{2.4}$$

inspired by AdS/CFT holography [4]. To accommodate the nearly conformal character of QCD, the target space geometry in the UV should be asymptotically near to AdS up to logarithmic terms due to asymptotic freedom. To accommodate confinement the simplest choice, suggested by Polchinski and Strassler, is a pure AdS^5 space with a hardwall cut-off at $\Lambda_{qcd} = r_{min}/R_{ads}^2$. The resulting diffusion kernel (at large r away from the IR cut-off) is

$$K(s;r,r') = \frac{s^{\omega_0}}{\sqrt{\pi \ln s}} e^{-\left[(\ln r' - \ln r)^2/4\mathscr{D}\ln s\right]}, \tag{2.5}$$

where now $\omega_0 = 2 - 2/\sqrt{\lambda}$, and $\mathscr{D} = 1/2\sqrt{\lambda}$. Comparing this with Eq. (2.3), one sees in this context that the spatial coordinate r of the string theory should be identified with the momentum $k_\perp$ of the gauge theory, which has an another example of the well known UV/IR holographic correspondence. In the next section, we will explain how this result follow for string scattering in AdS^5 using Mandlestam's light-cone gauge path integral.

3. Description in light-cone gauge

The light-cone gauge for string theory eliminates all spurious degrees of freedom in favor of the transverse bosonic fields $X_\perp(\sigma,\tau) = (X_1, X_2)$ and $R(\sigma,\tau)$ by fixing $X^+(\sigma,\tau) \equiv (X^0 + X^3)/\sqrt{2} = \tau$, $P^+(\sigma,\tau) = \text{const}$ and the first derivatives of $X^-(\sigma,\tau) = (X^0 - X^3)/\sqrt{2}$ as quadratic functions of the transverse fields via the Virasoro constraints. For example, elastic scattering amplitude $(p_1, p_3 \to -p_2, -p_4)$, is given by the light-cone path integral,

$$\mathscr{A}(s,t)\delta^2(p_1^\perp + p_2^\perp + p_3^\perp + p_4^\perp) = \mathscr{N} \int dT \int \mathscr{D}X_\perp DY G^{1/2}[Z]\; V_1 V_2 V_3 V_4\;\; e^{-\int d\tau \int_0^{p^+} d\sigma \mathscr{L}[X_\perp, Z]} \tag{3.1}$$

where for the AdS radial fluctuation we use the reciprocal field $Z(\sigma,\tau) = R^2_{ads}/R(\sigma,\tau)$. Scattering takes place on the world sheet illustrated in Fig. 2, with Neumann (open) or periodic (closed) boundary condition on the edges. The modulus T is the time τ in the interaction region. Closed strings have one additional modulus to enforce level matching.

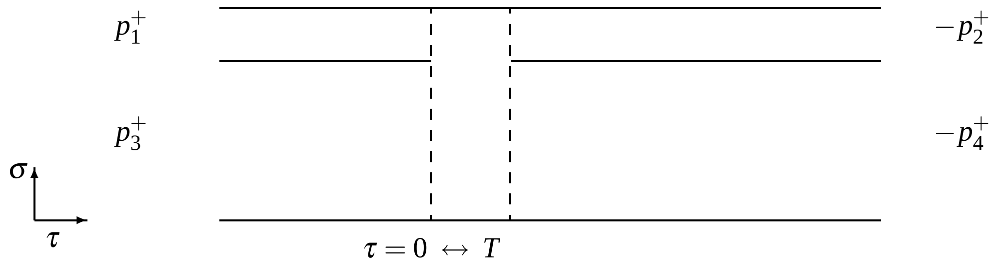

Figure 2: The light-cone world sheet domain, $X^+ = \tau \in [-\infty, \infty]$, $\sigma \in [0, p^+]$,with $p^+ = p_1^+ + p_3^+$ for elastic scattering in the brickwall frame.

It is natural to choose the width of the world sheet to be the total conserved $p^+ = p_1^+ + p_3^+$. The in/out external particles are inserted by vertex operators V_i at $\tau = \mp\infty$ respectively. The light-cone Lagrangian in AdS space [3] is

$$L = \frac{1}{2}\int_0^{p^+} d\sigma [\dot{X}_\perp^2 + \dot{Z}^2 + \frac{1}{(2\pi\alpha'_{eff}[Z])^2}({X'_\perp}^2 + Z'^2)]\,, \tag{3.2}$$

where the effective slope is $\alpha'_{eff}[Z] = \alpha' Z^2/R^2_{ads}$. Ignoring the IR cut-off, we have a residual conformal invariance: $Z \to \eta Z, X_\perp \to \eta X_\perp, \tau \to \eta\tau, \sigma \to \sigma/\eta$. To exploit this we change variables to $U(\sigma,\tau) = -\ln[Z(\sigma,\tau)/R_{ads}]$ and make a semi-classical expansions around the zero modes $U = u$, $X_\perp = x_\perp$. To Gaussian order the essential new feature relative to flat space is an effective string slope, $\alpha'_{eff}(u) = \alpha' e^{-2u}$, that depends locally on position in the extra dimension.

3.1 Regge limit in flat space

Let us begin by considering the classical Regge diffusion in flat space by freezing the radial field: $R(\sigma,\tau) = r_{min}$, which technically replaces α' for strings by $\alpha'_{qcd} = \alpha' R^2_{ads}/r^2_{min}$. With this constraint, the theory is exactly Gaussian with a normal mode expansion for each external state,

$$X_\perp(\sigma,\tau) = x_\perp + i\,\frac{p_\perp}{p^+}\tau + \sqrt{\frac{2}{p^+}}\sum_{n=1}^\infty X_n(\tau)\cos(\omega_n\sigma/c) \quad , \quad X_n(\tau) = \frac{a_n^\dagger e^{-\omega_n\tau} + a_n e^{\omega_n\tau}}{\sqrt{2\omega_n}}\,, \tag{3.3}$$

with frequencies $\omega_n = n/(2\alpha' p^+)$ dependent inversely on p^+. Consequently in the light-cone frame strings with small/large longitudinal boosts appear to be more strongly/weakly bound.

Now the Regge limit with $p_3^+ \simeq 1/p_1^+ \sim \sqrt{s}$ is dominated by a vanishing interaction time $T \sim 1/\sqrt{s}$. To zeroth order, the two separate strings momentarily join at a point with Dirichlet boundary condition and the Regge amplitude factorizes,

$$\mathscr{A}(s,t)\delta^2(p_1^\perp + p_2^\perp + p_3^\perp + p_4^\perp) \simeq \int \frac{d^2k^\perp}{(2\pi)^2}\, V_{12}(k^\perp)\;\; F_{34}(-k^\perp)\;, \tag{3.4}$$

in terms of “short” and “long” string form factors V_{12} and F_{34}. The expression is asymmetric as a natural consequence of the “infinite momentum” frame.

For the “short” string ($p_1^+ \sim 1/\sqrt{s}$), the excitation frequencies in the wave function grow forcing it to interact like a rigid point-like object: $V_{12}(k) \sim (2\pi)^2\delta^2(p_1^\perp + p_2^\perp - k^\perp)$. On the other hand, the “long” string ($p_3^+ \sim \sqrt{s}$) becomes more weakly bound and it has a non-trivial form factor that must be treated to first order in T. It can be evaluate by expanding in normal modes [1],

$$F_{34}(-k^\perp) \simeq 2(2\pi)^2\delta^2(p_3^\perp + p_4^\perp + k^\perp)\;\int dT\;T^{-2}p^+e^{-p^-T}\;\exp[\,-\sum_n \frac{\alpha' k_\perp^2}{n + n^2T/2\alpha' p_3^+}\,]. \tag{3.5}$$

The sum in the exponent, at large $p_3^+ \simeq s/2p^-$, leads to logarithmic growth in impact parameter or so called Regge shrinkage of the form factor, giving the final result,

$$\mathscr{A}(s,t) \simeq \Gamma[-1-\alpha' t]\;(-\alpha' s)^{1+\alpha' t}\;, \tag{3.6}$$

where $t = -k_\perp^2$. As explained above the s-dependent term is the solution to the diffusion Eq. 2.1.

3.2 Regge limit in warped spacetime

Again diffusion takes place only for the constituent (partons or string bits) in the “long” boosted string. In the semi-classical approximation the calculation is analogous to the flat space example, except that in Eq. 3.5 α' is replaced by an effective Regge slope, $\alpha'_{eff}(u)$, and a new term ν^2 is added to $\alpha'_{eff}(u)k_\perp^2$, where ν is the "momentum" conjugate to the radial co-ordinate u. For $k_\perp^2 = 0$, the ν^2 factor clearly corresponds to diffusion in the radial direction: $u \sim \ln r$. For non-zero $k_\perp^2 \neq 0$, ν must be represented as an operator, $\nu = i\partial_u$, conjugate to u. In fact, to maintain general covariance, one also needs to go to the one-loop order beyond the Gaussian approximation. Alternatively, one can fix the operator ordering by comparison with the covariant vector field equation in AdS space at $j = 1$. Transforming to the J-plane, the diffusion equation for the open string is

$$[j - 1 - \alpha'\, e^{-2u}\partial_x^2 - \frac{1}{\sqrt{\lambda}}(\partial_u^2 - 1)]\, K(j;x,u,x',u') = \delta^2(x-x')\delta(u-u')\;, \tag{3.7}$$

to leading order in $\alpha' = R_{ads}^2/\sqrt{g^2N}$. We note that diffusion in u suppresses the corresponding diffusion in impact parameter space, giving rise to the BFKL cut at $t < 0$ for the open string Regge exchange starting at $j = 1 - 1/\sqrt{\lambda}$.

The generalization to the closed string is essential trivial. One changes to periodic boundary condition on the world sheet in Fig 2 and introduces an additional modulus that rotates the Riemann surface around a cut on the s-channel intermediate closed string. This has the effect of replacing $\alpha' \to \frac{1}{2}\alpha'$. Again we can also avoid calculating beyond the Gaussian approximation by match in the strong coupling limit, at $j = 2$, with the equation for tensor glueballs. The final result for our diffusion kernel, in a j-t representation, is

$$[j - 2 - \frac{\alpha'}{2}\, t\, e^{-2u} - \frac{1}{2\sqrt{\lambda}}(\partial_u^2 - 4))]\, K(j,t;u,u') = \delta(u-u')\;. \tag{3.8}$$

4. Discussion

The spectral representation for diffusion kernel $K(j,t;u,u')$ is expressed in terms of the complete set of solutions to the Schrodinger equation, $(-\frac{1}{2\sqrt{\lambda}}\partial_u^2 + V_{eff}(u,t))\psi(u) = E\psi(u)$, where $V_{eff}(u,t)$ is bounded from below and $E \equiv 2 - j$. This determines the spectrum in j as a function of t. Using the hardwall model in Eq.2.4 with $V_{eff} = 2/\sqrt{\lambda} - \alpha' t e^{-2u}/2$, the solution can be obtained in terms of Bessel functions. The general structure for the leading J-plane singularities is depicted in Fig. 1. Let us comment briefly on some of the key features.

First we concentrate on the region for t large and negative. Since the effective potential approaches a constant, $2/\sqrt{\lambda}$, the spectrum has a continuum, which corresponds to a branch cut at $j = 2 - 2/\sqrt{\lambda}$, which is the location of the BFKL in the strong coupling limit. We next examine whether there exists bound states, i.e. poles in j above the BFKL cut, as one increases t. For small negative t, the BFKL cut persists and the physics becomes more sensitive to the confinement deformation, which in the hardwall model is crudely represented by the cutoff at $r_{min} = R_{ads} exp[u_{min}]$. Since the potential is monotonic in the hard wall model, there is clearly no bound state and the leading J-plane singularity is the BFKL cut. Finally for $t > 0$ the effective potential has a minimum at u_{min} and bound states can now be formed. As one increases t, more and more bound states appear and the leading trajectory is approximately linear, as depicted in Fig. 1. In particular, at $j = 2$, it determines the mass of the leading tensor glueball. As one decreases t towards the forward limit, the leading trajectory continue to decrease approximately linearly.

Close to $t = 0$, the physics is clearly sensitive to details of confinement deformation in the IR and is obviously model dependent. The effect of asymptotic freedom can also be taken into account for t large and negative. This one of as series of issues will be addressed in the forthcoming publications [1] in much greater detail. After decades of effort in studying QCD at high energy, string/gauge duality has begun to shed new light on how to reconcile BFKL, which has its origin in a perturbative analysis, and the soft Pomeron, which is a non-perturbative consequence of confinement. The close connection between the BFKL equation for the hard Pomeron and GLAP equation for structure functions raises the challenge to lattice methods on how to compute the non-perturbative impact factors in Eq. 2.2 essential to an overall normalization of the hard Pomeron component for physical hadron scattering at high energies. Much more can be done. The work has just begun.

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